# galpy Documentation 

Release v1.1

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galpy is a Python 2 and 3 package for galactic dynamics. It supports orbit integration in a variety of potentials, evaluating and sampling various distribution functions, and the calculation of action-angle coordinates for all static potentials.
galpy Documentation, Release v1.1

## CHAPTER 1

## Quick-start guide

### 1.1 Installation

galpy can be installed using pip as:

```
pip install galpy
```

or to upgrade without upgrading the dependencies:

```
> pip install -U --no-deps galpy
```

Some advanced features require the GNU Scientific Library (GSL; see below). If you want to use these, install the GSL first (or install it later and re-install using the upgrade command above).

The latest updates in galpy can be installed using:

```
pip install -U --no-deps git+git://github.com/jobovy/galpy.git#egg=galpy
```

or:

```
pip install -U --no-deps --install-option="--prefix=~/local" git+git://github.com/
uobovy/galpy.git #egg=galpy
```

for a local installation. The latest updates can also be installed from the source code downloaded from github using the standard python setup.py installation:

```
python setup.py install
```

or:

```
python setup.py install --prefix=~/local
```

for a local installation. A basic installation works with just the numpy/scipy/matplotlib stack. Some basic tests can be performed by executing:

### 1.1.1 Advanced installation

Certain advanced features require the GNU Scientific Library (GSL), with action calculations requiring version 1.14 or higher. On a Mac you can make sure that the correct architecture is installed using Homebrew as:

```
> brew install gsl --universal
```

You should be able to check your version using:

```
> gsl-config --version
```

Other advanced features, including calculating the normalization of certain distribution functions using GaussLegendre integration require numpy version 1.7 .0 or higher.
galpy uses OpenMP to parallelize various of the computations done in C. galpy can be installed without OpenMP by specifying the option -- no-openmp when running the python setup.py commands above or when using pip as follows:

```
> pip install -U --no-deps --install-option="--no-openmp" git+git://github.com/jobovy/
\hookrightarrowgalpy.git#egg=galpy
```

or:

```
> pip install -U --no-deps --install-option="--prefix=~/local" --install-option="--no-
\hookrightarrowopenmp" git+git://github.com/jobovy/galpy.git #egg=galpy
```

for a local installation. This can be especially useful if one is using the clang compiler, which is the new default on macs with OS X $(>=10.8)$, but does not support OpenMP. This leads to errors in the installation of galpy such as:

```
ld: library not found for -lgomp
clang: error: linker command failed with exit code 1 (use -v to see invocation)
```

If you get these errors, you can use the commands given above to install without OpenMP, or specify to use gcc by specifying the CC and LDSHARED environment variables to use gcc.

### 1.2 Introduction

The most basic features of galpy are its ability to display rotation curves and perform orbit integration for arbitrary combinations of potentials. This section introduce the most basic features of galpy.potential and galpy. orbit.

### 1.2.1 Rotation curves

The following code example shows how to initialize a Miyamoto-Nagai disk potential and plot its rotation curve

```
>>> from galpy.potential import MiyamotoNagaiPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=1.)
>>> mp.plotRotcurve(Rrange=[0.01,10.],grid=1001)
```

The normalize=1. option normalizes the potential such that the radial force is a fraction normalize=1. of the radial force necessary to make the circular velocity 1 at $\mathrm{R}=1$.

Similarly we can initialize other potentials and plot the combined rotation curve

```
>>> from galpy.potential import NFWPotential, HernquistPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=.6)
>>> np= NFWPotential(a=4.5,normalize=.35)
>>> hp= HernquistPotential(a=0.6/8, normalize=0.05)
>>> from galpy.potential import plotRotcurve
>>> plotRotcurve([hp,mp,np],Rrange=[0.01,10.],grid=1001,yrange=[0.,1.2])
```

Note that the normalize values add up to 1 . such that the circular velocity will be 1 at $\mathrm{R}=1$. The resulting rotation curve is approximately flat. To show the rotation curves of the three components do

```
>>> mp.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
>>> hp.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
>>> np.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
```

You'll see the following


As a shortcut the [hp,mp,np] Milky-Way-like potential is defined as

```
>>> from galpy.potential import MWPotential
```

This is not the recommended Milky-Way-like potential in galpy. The (currently) recommended Milky-Way-like potential is MWPotential2014:

```
>>> from galpy.potential import MWPotential2014
```

MWPotential2014 has a more realistic bulge model and is actually fit to various dynamical constraints on the Milky Way (see here and the galpy paper).

### 1.2.2 Units in galpy

Above we normalized the potentials such that they give a circular velocity of 1 at $\mathrm{R}=1$. These are the standard galpy units (sometimes referred to as natural units in the documentation). galpy will work most robustly when using these natural units. When using galpy to model a real galaxy with, say, a circular velocity of $220 \mathrm{~km} / \mathrm{s}$ at $\mathrm{R}=8 \mathrm{kpc}$, all of the velocities should be scaled as $\mathrm{v}=\mathrm{V} /[220 \mathrm{~km} / \mathrm{s}]$ and all of the positions should be scaled as $\mathrm{x}=\mathrm{X} /[8 \mathrm{kpc}]$ when using galpy's natural units.

For convenience, a utility module bovy_conversion is included in galpy that helps in converting between physical units and natural units for various quantities. For example, in natural units the orbital time of a circular orbit at $\mathrm{R}=1$ is $2 \pi$; in physical units this corresponds to

```
>>> from galpy.util import bovy_conversion
>>> print 2.*numpy.pi*bovy_conversion.time_in_Gyr(220.,8.)
0.223405444283
```

or about 223 Myr. We can also express forces in various physical units. For example, for the Milky-Way-like potential defined in galpy, we have that the vertical force at 1.1 kpc is

```
>>> from galpy.potential import MWPotential2014, evaluatezforces
>>> -evaluatezforces(1.,1.1/8.,MWPotential2014)*bovy_conversion.force_in_pcMyr2(220.,
\hookrightarrow.)
2.0259181908629933
```

which we can also express as an equivalent surface-density by dividing by $2 \pi G$

```
>>> -evaluatezforces(1.,1.1/8.,MWPotential2014)*bovy_conversion.force_in__
\hookrightarrow2piGmsolpc2(220.,8.)
71.658016957792356
```

Because the vertical force at the solar circle in the Milky Way at 1.1 kpc above the plane is approximately $70\left(2 \pi G M_{\odot} \mathrm{pc}^{-2}\right)$ (e.g., 2013arXiv1309.0809B), this shows that our Milky-Way-like potential has a realistic disk (at least in this respect).
bovy_conversion further has functions to convert densities, masses, surface densities, and frequencies to physical units (actions are considered to be too obvious to be included); see here for a full list. As a final example, the local dark matter density in the Milky-Way-like potential is given by

```
>>> MWPotential2014[2].dens(1.,0.)*bovy_conversion.dens_in_msolpc3(220.,8.)
0.0075419566970079373
```

or

```
>>> MWPotential2014[2].dens(1.,0.)*bovy_conversion.dens_in_gevcc(220.,8.)
0.28643101789044584
```

or about $0.0075 M_{\odot} \mathrm{pc}^{-3} \approx 0.3 \mathrm{GeV} \mathrm{cm}^{-3}$, in line with current measurements (e.g., 2012ApJ...756...89B).
When galpy Orbits are initialized using a distance scale $r o=$ and a velocity scale vo=output quantities returned and plotted in physical coordinates. Specifically, positions are are returned in units of kpc , velocities in $\mathrm{km} / \mathrm{s}$, energies and the Jacobi integral in $(\mathrm{km} / \mathrm{s})^{\wedge} 2$, the angular momentum o. L() and actions in $\mathrm{km} / \mathrm{s} \mathrm{kpc}$, frequencies in $1 / \mathrm{Gyr}$, and times and periods in Gyr.

### 1.2.3 Orbit integration

We can also integrate orbits in all galpy potentials. Going back to a simple Miyamoto-Nagai potential, we initialize an orbit as follows

```
>>> from galpy.orbit import Orbit
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,amp=1., normalize=1.)
>>> o= Orbit(vxvv=[1.,0.1,1.1,0.,0.1])
```

Since we gave Orbit () a five-dimensional initial condition $[R, v R, v T, z, v z]$, we assume we are dealing with a three-dimensional axisymmetric potential in which we do not wish to track the azimuth. We then integrate the orbit for a set of times ts

```
>>> import numpy
>>> ts= numpy.linspace(0,100,10000)
>>> o.integrate(ts,mp,nethod='odeint')
```

Now we plot the resulting orbit as

```
>>> o.plot()
```

Which gives


The integrator used is not symplectic, so the energy error grows with time, but is small nonetheless

```
>>> o.plotE(normed=True)
```



When we use a symplectic leapfrog integrator, we see that the energy error remains constant

```
>>> O.integrate(ts,mp,method='leapfrog')
>>> o.plotE(xlabel=r'$t$',ylabel=r'$E(t)/E(0)$')
```



Because stars have typically only orbited the center of their galaxy tens of times, using symplectic integrators is mostly unnecessary (compared to planetary systems which orbits millions or billions of times). galpy contains fast integrators written in C , which can be accessed through the method= keyword (e.g., integrate (..., method='dopr54_c') is a fast high-order Dormand-Prince method).
When we integrate for much longer we see how the orbit fills up a torus (this could take a minute)

```
>>> ts= numpy.linspace(0,1000,10000)
>>> o.integrate(ts,mp,method='odeint')
>>> o.plot()
```



As before, we can also integrate orbits in combinations of potentials. Assuming mp, np, and hp were defined as above, we can
>>> ts= numpy.linspace $(0,100,10000)$
$\ggg$ o.integrate(ts, [mp,hp, np])
>>> o.plot()


Energy is again approximately conserved

```
>>> o.plotE(xlabel=r'$t$',ylabel=r'$E(t)/E(0)$')
```



### 1.2.4 Escape velocity curves

Just like we can plot the rotation curve for a potential or a combination of potentials, we can plot the escape velocity curve. For example, the escape velocity curve for the Miyamoto-Nagai disk defined above

```
>>> mp.plotEscapecurve(Rrange=[0.01,10.],grid=1001)
```


or of the combination of potentials defined above

```
>>> from galpy.potential import plotEscapecurve
>>> plotEscapecurve([mp,hp,np],Rrange=[0.01,10.],grid=1001)
```



For the Milky-Way-like potential MWPotential2014, the escape-velocity curve is

```
>>> plotEscapecurve(MWPotential2014,Rrange=[0.01,10.],grid=1001)
```



At the solar radius, the escape velocity is

```
>>> from galpy.potential import vesc
>>> vesc(MWPotential2014,1.)
2.3316389848832784
```

Or, for a local circular velocity of $220 \mathrm{~km} / \mathrm{s}$

```
>>> vesc(MWPotential2014,1.)*220.
512.96057667432126
```

similar to direct measurements of this (e.g., 2007MNRAS.379..755S and 2014A\%26A...562A..91P).

### 1.3 Potentials in galpy

galpy contains a large variety of potentials in galpy.potential that can be used for orbit integration, the calculation of action-angle coordinates, as part of steady-state distribution functions, and to study the properties of gravitational potentials. This section introduces some of these features.

### 1.3.1 Potentials and forces

Various 3D and 2D potentials are contained in galpy, list in the API page. Another way to list the latest overview of potentials included with galpy is to run

```
>>> import galpy.potential
>>> print [p for p in dir(galpy.potential) if 'Potential' in p]
['CosmphiDiskPotential',
    'DehnenBarPotential',
    'DoubleExponentialDiskPotential',
    'EllipticalDiskPotential',
    'FlattenedPowerPotential',
    'HernquistPotential',
....]
```

(list cut here for brevity). Section Rotation curves explains how to initialize potentials and how to display the rotation curve of single Potential instances or of combinations of such instances. Similarly, we can evaluate a Potential instance

```
>>> from galpy.potential import MiyamotoNagaiPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=1.)
>>> mp(1.,0.)
-1.2889062500000001
```

Most member functions of Potential instances have corresponding functions in the galpy.potential module that allow them to be evaluated for lists of multiple Potential instances. galpy.potential. MWPotential2014 is such a list of three Potential instances

```
>>> from galpy.potential import MWPotential2014
>>> print MWPotential2014
[<galpy.potential_src.PowerSphericalPotentialwCutoff.PowerSphericalPotentialwCutoff
\hookrightarrowinstance at 0x1089b23b0>, <galpy.potential_src.MiyamotoNagaiPotential.
MiyamotoNagaiPotential instance at 0x1089b2320>, <galpy.potential_src.
\hookrightarrowwoPowerSphericalPotential.NFWPotential instance at 0x1089b2248>]
```

and we can evaluate the potential by using the evaluatePotentials function

```
>>> from galpy.potential import evaluatePotentials
>>> evaluatePotentials(1.,0.,MWPotential2014)
-1.3733506513947895
```

We can plot the potential of axisymmetric potentials (or of non-axisymmetric potentials at phi=0) using the plot member function

```
>>> mp.plot()
```

which produces the following plot


Similarly, we can plot combinations of Potentials using plotPotentials, e.g.,
>>> from galpy.potential import plotPotentials
>>> plotPotentials(MWPotential2014,rmin=0.01)


These functions have arguments that can provide custom $R$ and $z$ ranges for the plot, the number of grid points, the number of contours, and many other parameters determining the appearance of these figures.
galpy also allows the forces corresponding to a gravitational potential to be calculated. Again for the Miyamoto-Nagai Potential instance from above

```
>>> mp.Rforce(1.,0.)
-1.0
```

This value of -1.0 is due to the normalization of the potential such that the circular velocity is 1 . at $\mathrm{R}=1$. Similarly, the vertical force is zero in the mid-plane

```
>>> mp.zforce(1.,0.)
-0.0
```

but not further from the mid-plane

```
>>> mp.zforce(1.,0.125)
-0.53488743705310848
```

As explained in Units in galpy, these forces are in standard galpy units, and we can convert them to physical units using methods in the galpy.util.bovy_conversion module. For example, assuming a physical circular velocity of $220 \mathrm{~km} / \mathrm{s}$ at $\mathrm{R}=8 \mathrm{kpc}$

```
>>> from galpy.util import bovy_conversion
>>> mp.zforce(1.,0.125) *bovy_conversion.force_in_kmsMyr(220.,8.)
-3.3095671288657584 #km/s/Myr
>>> mp.zforce(1.,0.125)*bovy_conversion.force_in_2piGmsolpc2(220.,8.)
-119.72021771473301 #2 \pi G Msol / pc^2
```

Again, there are functions in galpy.potential that allow for the evaluation of the forces for lists of Potential instances, such that

```
>>> from galpy.potential import evaluateRforces
>>> evaluateRforces(1.,0.,MWPotential2014)
-1.0
>>> from galpy.potential import evaluatezforces
>>> evaluatezforces(1.,0.125,MWPotential2014)*bovy_conversion.force_in_
\hookrightarrow2piGmsolpc2(220.,8.)
>>> -69.680720137571114 #2 \pi G MSOI / pc^2
```

We can evaluate the flattening of the potential as $\sqrt{\left|z F_{R} / R F_{Z}\right|}$ for a Potential instance as well as for a list of such instances

```
>>> mp.flattening(1.,0.125)
0.4549542914935209
>>> from galpy.potential import flattening
>>> flattening(MWPotential2014,1.,0.125)
0.61231675305658628
```


### 1.3.2 Densities

galpy can also calculate the densities corresponding to gravitational potentials. For many potentials, the densities are explicitly implemented, but if they are not, the density is calculated using the Poisson equation (second derivatives of the potential have to be implemented for this). For example, for the Miyamoto-Nagai potential, the density is explicitly implemented

```
>>> mp.dens(1.,0.)
1.1145444383277576
```

and we can also calculate this using the Poisson equation

```
>>> mp.dens(1.,0.,forcepoisson=True)
1.1145444383277574
```

which are the same to machine precision

```
>>> mp.dens(1.,0., forcepoisson=True)-mp.dens(1.,0.)
-2.2204460492503131e-16
```

Similarly, all of the potentials in galpy.potential.MWPotential2014 have explicitly-implemented densities, so we can do

```
>>> from galpy.potential import evaluateDensities
>>> evaluateDensities(1.,0.,MWPotential2014)
0.57508603122264867
```

In physical coordinates, this becomes

```
>>> evaluateDensities(1.,0.,MWPotential2014)*bovy_conversion.dens_in_msolpc3(220.,8.)
0.1010945632524705 #Msol / pc^3
```

We can also plot densities

```
>>> from galpy.potential import plotDensities
>>> plotDensities(MWPotential2014,rmin=0.1, zmax=0.25, zmin=-0.25,nrs=101,nzs=101)
```

which gives


Another example of this is for an exponential disk potential

```
>>> from galpy.potential import DoubleExponentialDiskPotential
>>> dp= DoubleExponentialDiskPotential(hr=1./4.,hz=1./20.,normalize=1.)
```

The density computed using the Poisson equation now requires multiple numerical integrations, so the agreement between the analytical density and that computed using the Poisson equation is slightly less good, but still better than a percent

```
>>> (dp.dens(1.,0., forcepoisson=True)-dp.dens(1.,0.))/dp.dens(1.,0.)
0.0032522956769123019
```

The density is

```
>>> dp.plotDensity(rmin=0.1, zmax=0.25,zmin=-0.25,nrs=101,nzs=101)
```


and the potential is
>>> dp.plot (rmin=0.1, zmin=-0.25, zmax=0.25)


Clearly, the potential is much less flattened than the density.

### 1.3.3 Close-to-circular orbits and orbital frequencies

We can also compute the properties of close-to-circular orbits. First of all, we can calculate the circular velocity and its derivative

```
>>> mp.vcirc(1.)
1.0
>>> mp.dvcircdR(1.)
-0.163777427566978
```

or, for lists of Potential instances

```
>>> from galpy.potential import vcirc
>>> vcirc(MWPotential2014,1.)
1.0
>>> from galpy.potential import dvcircdR
>>> dvcircdR(MWPotential2014,1.)
-0.10091361254334696
```

We can also calculate the various frequencies for close-to-circular orbits. For example, the rotational frequency

```
>>> mp.omegac(0.8)
1.2784598203204887
```

```
>>> from galpy.potential import omegac
>>> omegac(MWPotential2014,0.8)
1.2733514576122869
```

and the epicycle frequency

```
>>> mp.epifreq(0.8)
1.7774973530267848
>>> from galpy.potential import epifreq
>>> epifreq(MWPotential2014,0.8)
1.7452189766287691
```

as well as the vertical frequency

```
>>> mp.verticalfreq(1.0)
3.7859388972001828
>>> from galpy.potential import verticalfreq
>>> verticalfreq(MWPotential2014,1.)
2.7255405754769875
```

For close-to-circular orbits, we can also compute the radii of the Lindblad resonances. For example, for a frequency similar to that of the Milky Way's bar

```
>> mp.lindbladR(5./3.,m='corotation') #args are pattern speed and m of pattern
0.6027911166042229 #~ 5kpc
>> print mp.lindbladR(5./3.,m=2)
None
>> mp.lindbladR(5./3.,m=-2)
0.9906190683480501
```

The None here means that there is no inner Lindblad resonance, the $m=-2$ resonance is in the Solar neighborhood (see the section on the Hercules stream in this documentation).

### 1.3.4 Using interpolations of potentials

galpy contains a general Potential class interpRZPotential that can be used to generate interpolations of potentials that can be used in their stead to speed up calculations when the calculation of the original potential is computationally expensive (for example, for the DoubleExponentialDiskPotential). Full details on how to set this up are given here. Interpolated potentials can be used anywhere that general three-dimensional galpy potentials can be used. Some care must be taken with outside-the-interpolation-grid evaluations for functions that use C to speed up computations.

### 1.3.5 NEW: The potential of N-body simulations

galpy can setup and work with the frozen potential of an N-body simulation. This allows us to study the properties of such potentials in the same way as other potentials in galpy. We can also investigate the properties of orbits in these potentials and calculate action-angle coordinates, using the galpy framework. Currently, this functionality is limited to axisymmetrized versions of the N -body snapshots, although this capability could be somewhat straightforwardly expanded to full triaxial potentials. The use of this functionality requires pynbody to be installed; the potential of any snapshot that can be loaded with pynbody can be used within galpy.

As a first, simple example of this we look at the potential of a single simulation particle, which should correspond to galpy's KeplerPotential. We can create such a single-particle snapshot using pynbody by doing

```
>>> import pynbody
>>> s= pynbody.new(star=1)
>>> s['mass']= 1.
>>> s['eps']= 0.
```

and we get the potential of this snapshot in galpy by doing

```
>>> from galpy.potential import SnapshotRZPotential
>>> sp= SnapshotRZPotential(s,num_threads=1)
```

With these definitions, this snapshot potential should be the same as KeplerPotential with an amplitude of one, which we can test as follows

```
>>> from galpy.potential import KeplerPotential
>>> kp= KeplerPotential(amp=1.)
>>> print(sp(1.1,0.),kp(1.1,0.),sp(1.1,0.)-kp(1.1,0.))
(-0.90909090909090906, -0.9090909090909091, 0.0)
>>> print(sp.Rforce(1.1,0.),kp.Rforce(1.1,0.),sp.Rforce(1.1,0.)-kp.Rforce(1.1,0.))
(-0.82644628099173545, -0.8264462809917353, -1.1102230246251565e-16)
```

SnapshotRZPotential instances can be used wherever other galpy potentials can be used (note that the second derivatives have not been implemented, such that functions depending on those will not work). For example, we can plot the rotation curve

```
>>> sp.plotRotcurve()
```



Because evaluating the potential and forces of a snapshot is computationally expensive, most useful applications of frozen N-body potentials employ interpolated versions of the snapshot potential. These can be setup in galpy using an InterpSnapshotRZPotential class that is a subclass of the interpRZPotential described above and that can be used in the same manner. To illustrate its use we will make use of one of pynbody's example snapshots, g15784. This snapshot is used here to illustrate pynbody's use. Please follow the instructions there on how to download this snapshot.

Once you have downloaded the pynbody testdata, we can load this snapshot using

```
>>> s = pynbody.load('testdata/g15784.lr.01024.gz')
```

(please adjust the path according to where you downloaded the pynbody testdata). We get the main galaxy in this snapshot, center the simulation on it, and align the galaxy face-on using

```
>>> h = s.halos()
>>> h1 = h[1]
>>> pynbody.analysis.halo.center(h1,mode='hyb')
>>> pynbody.analysis.angmom.faceon(h1, cen=(0,0,0),mode='ssc')
```

we also convert the simulation to physical units, but set $G=1$ by doing the following

```
>>> s.physical_units()
>>> from galpy.util.bovy_conversion import _G
>>> g= pynbody.array.SimArray(_G/1000.)
```

```
>>> g.units= 'kpc Msol**-1 km**2 s**-2 G**-1'
>>> s._arrays['mass']= s._arrays['mass']*g
```

We can now load an interpolated version of this snapshot's potential into galpy using

```
>>> from galpy.potential import InterpSnapshotRZPotential
>>> spi= InterpSnapshotRZPotential(h1,rgrid=(numpy.log(0.01),numpy.log(20.),101),
\hookrightarrowlogR=True,zgrid=(0.,10.,101), interpPot=True, zsym=True)
```

where we further assume that the potential is symmetric around the mid-plane $(z=0)$. This instantiation will take about ten to fiteen minutes. This potential instance has physical units (and thus the rgrid= and zgrid= inputs are given in kpc if the simulation's distance unit is kpc ). For example, if we ask for the rotation curve, we get the following:
$\ggg$ spi.plotRotcurve (Rrange=[0.01,19.9],xlabel=r'\$R<br>, (\mathrm\{kpc\}) \$',ylabel=r'\$v_ $\left.\hookrightarrow C(R) \backslash,\left(\backslash m a t h r m\{k m \backslash, s\}^{\wedge}\{-1\}\right) \${ }^{\prime}\right)$


This can be compared to the rotation curve calculated by pynbody, see here.
Because galpy works best in a system of natural units as explained in Units in galpy, we will convert this instance to natural units using the circular velocity at $R=10 \mathrm{kpc}$, which is

```
>>> spi.vcirc(10.)
294.62723076942245
```

To convert to natural units we do

```
>>> spi.normalize(R0=10.)
```

We can then again plot the rotation curve, keeping in mind that the distance unit is now $R_{0}$

```
>>> spi.plotRotcurve(Rrange=[0.01,1.99])
```

which gives

in particular

```
>>> spi.vcirc(1.)
1.0000000000000002
```

We can also plot the potential

```
>>> spi.plot(rmin=0.01,rmax=1.9,nrs=51, zmin=-0.99, zmax=0.99,nzs=51)
```



Clearly, this simulation's potential is quite spherical, which is confirmed by looking at the flattening

```
>>> spi.flattening(1.,0.1)
0.86675711023391921
>>> spi.flattening(1.5,0.1)
0.94442750306256895
```

The epicyle and vertical frequencies can also be interpolated by setting the interpepifreq=True or interpverticalfreq=True keywords when instantiating the InterpSnapshotRZPotential object.

### 1.3.6 NEW: Conversion to NEMO potentials

NEMO is a set of tools for studying stellar dynamics. Some of its functionality overlaps with that of galpy, but many of its programs are very complementary to galpy. In particular, it has the ability to perform N -body simulations with a variety of poisson solvers, which is currently not supported by galpy (and likely will never be directly supported). To encourage interaction between galpy and NEMO it is quite useful to be able to convert potentials between these two frameworks, which is not completely trivial. In particular, NEMO contains Walter Dehnen's fast collisionless gyrfalcon code (see 2000ApJ...536L..39D and 2002JCoPh.179...27D) and the discussion here focuses on how to run N -body simulations using external potentials defined in galpy.

Some galpy potential instances support the functions nemo_accname and nemo_accpars that return the name of the NEMO potential corresponding to this galpy Potential and its parameters in NEMO units. These functions assume that you use NEMO with WD_units, that is, positions are specified in kpc, velocities in kpc/Gyr, times in Gyr , and $\mathrm{G}=1$. For the Miyamoto-Nagai potential above, you can get its name in the NEMO framework as

```
>>> mp.nemo_accname()
'MiyamotoNagai'
```

and its parameters as

```
>>> mp.nemo_accpars(220.,8.)
'0,592617.11132,4.0,0.3'
```

assuming that we scale velocities by $\mathrm{vo}=220 \mathrm{~km} / \mathrm{s}$ and positions by $\mathrm{ro}=8 \mathrm{kpc}$ in galpy. These two strings can then be given to the gyrfalcON accname= and accpars=keywords.

We can do the same for lists of potentials. For example, for MWP otential2014 we do

```
>>> from galpy.potential import nemo_accname, nemo_accpars
>>> nemo_accname(MWPotential2014)
'PowSphwCut+MiyamotoNagai+NFW'
>>> nemo_accpars(MWPotential2014,220.,8.)
'0,1001.79126907,1.8,1.9#0,306770.418682,3.0,0.28#0,16.0,162.958241887'
```

Therefore, these are the accname= and accpars= that one needs to provide to gyrfalcon to run a simulation in MWPotential2014.

Note that the NEMO potential PowSphwCut is not a standard NEMO potential. This potential can be found in the nemo/ directory of the galpy source code; this directory also contains a Makefile that can be used to compile the extra NEMO potential and install it in the correct NEMO directory (this requires one to have NEMO running, i.e., having sourced nemo_start).

You can use the PowSphwCut . cc file in the nemo/ directory as a template for adding additional potentials in galpy to the NEMO framework. To figure out how to convert the normalized galpy potential to an amplitude when scaling to physical coordinates (like kpc and kpc/Gyr), one needs to look at the scaling of the radial force with R. For example, from the definition of MiyamotoNagaiPotential, we see that the radial force scales as $R^{-2}$. For a general scaling $R^{-\alpha}$, the amplitude will scale as $V_{0}^{2} R_{0}^{\alpha-1}$ with the velocity $V_{0}$ and position $R_{0}$ of the $\mathrm{v}=1$ at $\mathrm{R}=1$ normalization. Therefore, for the MiyamotoNagaiPotential, the physical amplitude scales as $V_{0}^{2} R_{0}$. For the LogarithmicHaloPotential, the radial force scales as $R^{-1}$, so the amplitude scales as $V_{0}^{2}$.

Currently, only the MiyamotoNagaiPotential, NFWPotential, PowerSphericalPotentialwCutoff, PlummerPotential, MN3ExponentialDiskPotential, and the LogarithmicHaloPotential have this NEMO support. Combinations of the first three are also supported (e.g., MWPotential2014); they can also be combined with spherical LogarithmicHaloPotentials. Because of the definition of the logarithmic potential in NEMO, it cannot be flattened in $z$, so to use a flattened logarithmic potential, one has to flip $y$ and $z$ between galpy and NEMO (one can flatten in y).

### 1.3.7 Adding potentials to the galpy framework

Potentials in galpy can be used in many places such as orbit integration, distribution functions, or the calculation of action-angle variables, and in most cases any instance of a potential class that inherits from the general Potential class (or a list of such instances) can be given. For example, all orbit integration routines work with any list of instances of the general Potential class. Adding new potentials to galpy therefore allows them to be used everywhere in galpy where general Potential instances can be used. Adding a new class of potentials to galpy consists of the following series of steps (some of these are also given in the file README. dev in the galpy distribution):

1. Implement the new potential in a class that inherits from galpy.potential.Potential. The new class should have an __init__ method that sets up the necessary parameters for the class. An amplitude parameter $a m p=$ should be taken as an argument for this class and before performing any other setup, the galpy. potential.Potential.__init__(self,amp=amp) method should be called to setup the amplitude.

To add support for normalizing the potential to standard galpy units, one can call the galpy.potential. Potential.normalize function at the end of the __init__ function.
The new potential class should implement some of the following functions:

- _evaluate (self, $R, z, p h i=0, t=0$ ) which evaluates the potential itself (without the amp factor, which is added in the $\qquad$ call $\qquad$ method of the general Potential class).
- _Rforce (self,R,z,phi=0.,t=0.) which evaluates the radial force in cylindrical coordinates (-d potential / d R).
- _zforce (self,R,z,phi=0.,t=0.) which evaluates the vertical force in cylindrical coordinates ( -d potential / d z).
- _R2deriv (self, R, z, phi=0.,t=0.) which evaluates the second (cylindrical) radial derivative of the potential ( $\mathrm{d}^{\wedge} 2$ potential / $\mathrm{d}^{\wedge} \mathrm{R}^{\wedge} 2$ ).
- _z2deriv(self,R,z,phi=0.,t=0.) which evaluates the second (cylindrical) vertical derivative of the potential ( $d^{\wedge} 2$ potential / $\mathrm{d}^{\wedge} 2$ ).
- _Rzderiv (self, R, z, phi=0.,t=0.) which evaluates the mixed (cylindrical) radial and vertical derivative of the potential ( $\mathrm{d}^{\wedge} 2$ potential / d R d z ).
- _dens (self,R,z,phi=0.,t=0.) which evaluates the density. If not given, the density is computed using the Poisson equation from the first and second derivatives of the potential (if all are implemented).
- _mass (self, $R, z=0 ., t=0$.) which evaluates the mass. For spherical potentials this should give the mass enclosed within the spherical radius; for axisymmetric potentials this should return the mass up to $R$ and between $-Z$ and $Z$. If not given, the mass is computed by integrating the density (if it is implemented or can be calculated from the Poisson equation).
- _phiforce (self, R, z, phi=0., t=0.) : the azimuthal force in cylindrical coordinates (assumed zero if not implemented).
- _phi2deriv (self, R, z, phi=0., t=0.) : the second azimuthal derivative of the potential in cylindrical coordinates ( $\mathrm{d}^{\wedge} 2$ potential / d phi^ $\mathrm{\wedge}^{\wedge}$; assumed zero if not given).
- _Rphideriv(self,R,z,phi=0.,t=0.): the mixed radial and azimuthal derivative of the potential in cylindrical coordinates ( $\mathrm{d}^{\wedge} 2$ potential / d R d phi; assumed zero if not given).

If you want to be able to calculate the concentration for a potential, you also have to set self._scale to a scale parameter for your potential.

The code for galpy.potential.MiyamotoNagaiPotential gives a good template to follow for 3D axisymmetric potentials. Similarly, the code for galpy.potential. CosmphiDiskPotential provides a good template for 2 D , non-axisymmetric potentials.
After this step, the new potential will work in any part of galpy that uses pure python potentials. To get the potential to work with the C implementations of orbit integration or action-angle calculations, the potential also has to be implemented in C and the potential has to be passed from python to C .

The ___init__ method should be written in such a way that a relevant object can be initialized using Classname () (i.e., there have to be reasonable defaults given for all parameters, including the amplitude); doing this allows the nose tests for potentials to automatically check that your Potential's potential function, force functions, second derivatives, and density (through the Poisson equation) are correctly implemented (if they are implemented). The continuous-integration platform that builds the galpy codebase upon code pushes will then automatically test all of this, streamlining push requests of new potentials.
2. To add a C implementation of the potential, implement it in a c file under potential_src/potential_c_ext. Look at potential_src/potential_c_ext/ LogarithmicHaloPotential.c for the right format for 3D, axisymmetric potentials, or at
potential_src/potential_c_ext/LopsidedDiskPotential.c for 2D, non-axisymmetric potentials.

For orbit integration, the functions such as:

- double LogarithmicHaloPotentialRforce(double R, double Z, double phi,double t,struct potentialArg * potentialArgs)
- double LogarithmicHaloPotentialzforce(double R,double Z, double phi, double t,struct potentialArg * potentialArgs)
are most important. For some of the action-angle calculations
- double LogarithmicHaloPotentialEval(double R,double Z, double phi,double t,struct potentialArg * potentialArgs)
is most important (i.e., for those algorithms that evaluate the potential). The arguments of the potential are passed in a potentialArgs structure that contains args, which are the arguments that should be unpacked. Again, looking at some example code will make this clear. The potentialArgs structure is defined in potential_src/potential_c_ext/galpy_potentials.h.

3. Add the potential's function declarations to potential_src/potential_c_ext/galpy_potentials. h
4. (4. and 5. for planar orbit integration) Edit the code under orbit_src/orbit_c_ext/ integratePlanarOrbit. $c$ to set up your new potential (in the parse_leapFuncArgs function).
5. Edit the code in orbit_src/integratePlanarOrbit.py to set up your new potential (in the _parse_pot function).
6. Edit the code under orbit_src/orbit_c_ext/integrateFullorbit.c to set up your new potential (in the parse_leapFuncArgs_Full function).
7. Edit the code in orbit_src/integrateFullorbit.py to set up your new potential (in the _parse_pot function).
8. (for using the actionAngleStaeckel methods in $C$ ) Edit the code in actionAngle_src/ actionAngle_c_ext/actionAngle.c to parse the new potential (in the parse_actionAngleArgs function).
9. Finally, add self.hasC= True to the initialization of the potential in question (after the initialization of the super class, or otherwise it will be undone). If you have implemented the necessary second derivatives for integrating phase-space volumes, also add self.hasC_dxdv=True.

After following the relevant steps, the new potential class can be used in any galpy context in which C is used to speed up computations.

### 1.4 Two-dimensional disk distribution functions

galpy contains various disk distribution functions, both in two and three dimensions. This section introduces the twodimensional distribution functions, useful for studying the dynamics of stars that stay relatively close to the mid-plane of a galaxy. The vertical motions of these stars may be approximated as being entirely decoupled from the motion in the plane.

### 1.4.1 Types of disk distribution functions

galpy contains the following distribution functions for razor-thin disks: galpy.df. dehnendf and galpy.df. shudf. These are the distribution functions of Dehnen (1999AJ....118.1201D) and Shu (1969ApJ... 158..505S). Everything shown below for dehnendf can also be done for shudf.

These disk distribution functions are functions of the energy and the angular momentum alone. They can be evaluated for orbits, or for a given energy and angular momentum. At this point, only power-law rotation curves are supported. A dehnendf instance is initialized as follows

```
>>> from galpy.df import dehnendf
>>> dfc= dehnendf(beta=0.)
```

This initializes a dehnendf instance based on an exponential surface-mass profile with scale-length $1 / 3$ and an exponential radial-velocity-dispersion profile with scale-length 1 and a value of 0.2 at $\mathrm{R}=1$. Different parameters for these profiles can be provided as an initialization keyword. For example,

```
>>> dfc= dehnendf(beta=0.,profileParams=(1./4.,1.,0.2))
```

initializes the distribution function with a radial scale length of $1 / 4$ instead.
We can show that these distribution functions have an asymmetric drift built-in by evaluating the DF at $\mathrm{R}=1$. We first create a set of orbit-instances and then evaluate the DF at them

```
>>> from galpy.orbit import Orbit
>>> os= [Orbit([1.,0.,1.+-0.9+1.8/1000*ii]) for ii in range(1001)]
>>> dfro= [dfc(o) for o in os]
>>> plot([1.+-0.9+1.8/1000*ii for ii in range(1001)],dfro)
```



Or we can plot the two-dimensional density at $\mathrm{R}=1$.

```
>>> dfro= [[dfc(Orbit([1.,-0.7+1.4/200*jj,1.-0.6+1.2/200*ii])) for jj in
\hookrightarrowrange(201)] for ii in range(201)]
>>> dfro= numpy.array(dfro)
>>> from galpy.util.bovy_plot import bovy_dens2d
>>> bovy_dens2d(dfro,origin='lower',cmap='gist_yarg',contours=True,xrange=[-0.7,0.7],
\hookrightarrowyrange=[0.4,1.6],xlabel=r'$v_R$',ylabel=r'$v_T$')
```



### 1.4.2 Evaluating moments of the DF

galpy can evaluate various moments of the disk distribution functions. For example, we can calculate the mean velocities (for the DF with a scale length of $1 / 3$ above)

```
>>> dfc.meanvT(1.)
0.91715276979447324
>>> dfc.meanvR(1.)
0.0
```

and the velocity dispersions

```
>>> numpy.sqrt(dfc.sigmaR2(1.))
0.19321086259083936
>>> numpy.sqrt(dfc.sigmaT2(1.))
0.15084122011271159
```

and their ratio

```
>>> dfc.sigmaR2(1.)/dfc.sigmaT2(1.)
1.6406766813028968
```

In the limit of zero velocity dispersion (the epicycle approximation) this ratio should be equal to 2 , which we can check as follows

```
>>> dfccold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.02))
>>> dfccold.sigmaR2(1.)/dfccold.sigmaT2(1.)
1.9947493895454664
```

We can also calculate higher order moments

```
>>> dfc.skewvT(1.)
-0.48617143862047763
>>> dfc.kurtosisvT(1.)
0.13338978593181494
>>> dfc.kurtosisvR(1.)
-0.12159407676394096
```

We already saw above that the velocity dispersion at $\mathrm{R}=1$ is not exactly equal to the input velocity dispersion ( 0.19321086259083936 vs. 0.2 ). Similarly, the whole surface-density and velocity-dispersion profiles are not quite equal to the exponential input profiles. We can calculate the resulting surface-mass density profile using surfacemass, sigmaR2, and sigma2surfacemass. The latter calculates the product of the velocity dispersion squared and the surface-mass density. E.g.,

```
>>> dfc.surfacemass(1.)
0.050820867101511534
```

We can plot the surface-mass density as follows

```
>>> Rs= numpy.linspace(0.01,5.,151)
>>> out= [dfc.surfacemass(r) for r in Rs]
>>> plot(Rs, out)
```

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or
>>> plot(Rs, numpy.log(out))

which shows the exponential behavior expected for an exponential disk. We can compare this to the input surface-mass density

```
>>> input_out= [dfc.targetSurfacemass(r) for r in Rs]
>>> plot(Rs,numpy.log(input_out)-numpy.log(out))
```


which shows that there are significant differences between the desired surface-mass density and the actual surface-mass density. We can do the same for the velocity-dispersion profile

```
>>> out= [dfc.sigmaR2(r) for r in Rs]
>>> input_out= [dfc.targetSigma2(r) for r in Rs]
>>> plot(Rs,numpy.log(input_out)-numpy.log(out))
```



That the input surface-density and velocity-dispersion profiles are not the same as the output profiles, means that estimates of DF properties based on these profiles will not be quite correct. Obviously this is the case for the surfacedensity and velocity-dispersion profiles themselves, which have to be explicitly calculated by integration over the DF rather than by evaluating the input profiles. This also means that estimates of the asymmetric drift based on the input profiles will be wrong. We can calculate the asymmetric drift at $\mathrm{R}=1$ using the asymmetric drift equation derived from the Jeans equation (eq. 4.228 in Binney \& Tremaine 2008), using the input surface-density and velocity dispersion profiles

```
>>> dfc.asymmetricdrift(1.)
0.090000000000000024
```

which should be equal to the circular velocity minus the mean rotational velocity

```
>>> 1.-dfc.meanvT(1.)
0.082847230205526756
```

These are not the same in part because of the difference between the input and output surface-density and velocitydispersion profiles (and because the asymmetricdrift method assumes that the ratio of the velocity dispersions squared is two using the epicycle approximation; see above).

### 1.4.3 Using corrected disk distribution functions

As shown above, for a given surface-mass density and velocity dispersion profile, the two-dimensional disk distribution functions only do a poor job of reproducing the desired profiles. We can correct this by calculating a
set of corrections to the input profiles such that the output profiles more closely resemble the desired profiles (see 1999AJ....118.1201D). galpy supports the calculation of these corrections, and comes with some pre-calculated corrections (these can be found here). For example, the following initializes a dehnendf with corrections up to 20th order (the default)

```
>>> dfc= dehnendf(beta=0.,correct=True)
```

The following figure shows the difference between the actual surface-mass density profile and the desired profile for $1,2,3,4,5,10,15$, and 20 iterations

and the same for the velocity-dispersion profile

galpy will automatically save any new corrections that you calculate.
All of the methods for an uncorrected disk DF can be used for the corrected DFs as well. For example, the velocity dispersion is now

```
>>> numpy.sqrt(dfc.sigmaR2(1.))
0.19999985069451526
```

and the mean rotation velocity is

```
>>> dfc.meanvT(1.)
0.90355161181498711
```

and (correct) asymmetric drift

```
>>> 1.-dfc.meanvT(1.)
0.09644838818501289
```

That this still does not agree with the simple dfc.asymmetricdrift estimate is because of the latter's using the epicycle approximation for the ratio of the velocity dispersions.

### 1.4.4 Oort constants and functions

galpy also contains methods to calculate the Oort functions for two-dimensional disk distribution functions. These are known as the Oort constants when measured in the solar neighborhood. They are combinations of the mean velocities and derivatives thereof. galpy calculates these by direct integration over the DF and derivatives of the DF. Thus, we can calculate

```
>>> dfc= dehnendf(beta=0.)
>>> dfc.oortA(1.)
0.43190780889218749
>>> dfc.oortB(1.)
-0.48524496090228575
```

The $K$ and $C$ Oort constants are zero for axisymmetric DFs

```
>>> dfc.oortC(1.)
0.0
>>> dfc.oortK(1.)
0.0
```

In the epicycle approximation, for a flat rotation curve $A=-B=0.5$. The explicit calculates of $A$ and $B$ for warm DFs quantify how good (or bad) this approximation is

```
>>> dfc.oortA(1.) +dfc.oortB(1.)
-0.053337152010098254
```

For the cold DF from above the approximation is much better

```
>>> dfccold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.02))
>>> dfccold.oortA(1.), dfccold.oortB(1.)
(0.49917556666144003, -0.49992824742490816)
```


### 1.4.5 Sampling data from the DF

We can sample from the disk distribution functions using sample. sample can return either an energy-angularmomentum pair, or a full orbit initialization. We can sample 4000 orbits for example as (could take two minutes)

```
>>> o= dfc.sample(n=4000,returnOrbit=True,nphi=1)
```

We can then plot the histogram of the sampled radii and compare it to the input surface-mass density profile

```
>>> Rs= [e.R() for e in o]
>>> hists, bins, edges= hist(Rs,range=[0,2], normed=True,bins=30)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> plot(xs, xs*exp(-xs*3.)*9.,'r-')
```

E.g.,


We can also plot the spatial distribution of the sampled disk

```
>>> xs= [e.x() for e in o]
>>> ys= [e.y() for e in o]
>>> figure()
>>> plot(xs,ys,',')
```

E.g.,


We can also sample points in a specific radial range (might take a few minutes)

```
>>> o= dfc.sample(n=1000,returnOrbit=True,nphi=1,rrange=[0.8,1.2])
```

and we can plot the distribution of tangential velocities

```
>>> vTs= [e.vXVV[2] for e in o]
>>> hists, bins, edges= hist(vTs,range=[.5,1.5],normed=True,bins=30)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> dfro= [dfc(Orbit([1.,0.,x]))/9./numpy.exp(-3.) for x in xs]
>>> plot(xs,dfro,'r-')
```



The agreement between the sampled distribution and the theoretical curve is not as good because the sampled distribution has a finite radial range. If we sample 10,000 points in rrange $=[0.95,1.05]$ the agreement is better (this takes a long time):


We can also directly sample velocities at a given radius rather than in a range of radii. Doing this for a correct DF gives

```
>>> dfc= dehnendf(beta=0., correct=True)
>>> vrvt= dfc.sampleVRVT(1.,n=10000)
>>> hists, bins, edges= hist(vrvt[:,1],range=[.5,1.5],normed=True,bins=101)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> dfro= [dfc(Orbit([1.,0.,x])) for x in xs]
>>> plot(xs,dfro/numpy.sum(dfro)/(xs[1]-xs[0]),'r-')
```


galpy further has support for sampling along a given line of sight in the disk, which is useful for interpreting surveys consisting of a finite number of pointings. For example, we can sampled distances along a given line of sight

```
>>> ds= dfc.sampledSurfacemassLOS(30./180.*numpy.pi,n=10000)
```

which is very fast. We can histogram these

```
>>> hists, bins, edges= hist(ds,range=[0., 3.5], normed=True,bins=101)
```

and compare it to the predicted distribution, which we can calculate as

```
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> fd= numpy.array([dfc.surfacemassLOS(d,30.) for d in xs])
>>> plot(xs,fd/numpy.sum(fd)/(xs[1]-xs[0]),'r-')
```

which shows very good agreement with the sampled distances

galpy can further sample full 4D phase-space coordinates along a given line of sight through dfc. sampleLOS.

### 1.4.6 Non-axisymmetric, time-dependent disk distribution functions

galpy also supports the evaluation of non-axisymmetric, time-dependent two-dimensional DFs. These specific DFs are constructed by assuming an initial axisymmetric steady state, described by a DF of the family discussed above, that is then acted upon by a non-axisymmetric, time-dependent perturbation. The DF at a given time and phase-space position is evaluated by integrating the orbit backwards in time in the non-axisymmetric potential until the time of the initial DF is reached. From Liouville's theorem, which states that phase-space volume is conserved along the orbit, we then know that we can evaluate the non-axisymmetric DF today as the initial DF at the initial point on the orbit. This procedure was first used by Dehnen (2000).
This is implemented in galpy as galpy.df.evolveddiskdf. Such a DF is setup by specifying the initial DF, the non-axisymmetric potential, and the time of the initial state. For example, we can look at the effect of an elliptical perturbation to the potential like that described by Kuijken \& Tremaine. To do this, we set up an elliptical perturbation to a logarithmic potential that is grown slowly to minimize non-adiabatic effects

```
>>> from galpy.potential import LogarithmicHaloPotential, EllipticalDiskPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> ep= EllipticalDiskPotential(twophio=0.05,phib=0.,p=0.,tform=-150.,tsteady=125.)
```

This perturbation starts to be grown at $t$ form $=-150$ over a time period of $t s t e a d y=125$ time units. We will consider the effect of this perturbation on a very cold disk (velocity dispersion $\sigma_{R}=0.0125 v_{c}$ ) and a warm disk $\left(\sigma_{R}=0.15 v_{c}\right)$. We set up these two initial DFs

```
>>> idfcold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.0125))
>>> idfwarm= dehnendf(beta=0.,profileParams=(1./3.,1.,0.15))
```

and then set up the evolveddiskdf

```
>>> from galpy.df import evolveddiskdf
>>> edfcold= evolveddiskdf(idfcold,[lp,ep],to=-150.)
>>> edfwarm= evolveddiskdf(idfwarm,[lp,ep],to=-150.)
```

where we specify that the initial state is at $t o=-150$.
We can now use these evolveddiskdf instances in much the same way as diskdf instances. One difference is that there is much more support for evaluating the DF on a grid (to help speed up the rather slow computations involved). Thus, we can evaluate the mean radial velocity at $\mathrm{R}=0.9$, phi=22.5 degree, and $\mathrm{t}=0$ by using a grid

```
>>> mvrcold, gridcold= edfcold.meanvR(0.9,phi=22.5,deg=True,t=0.,grid=True,
\hookrightarrowreturnGrid=True,gridpoints=51,nsigma=6.)
>>> mvrwarm, gridwarm= edfcold.meanvR(0.9,phi=22.5,deg=True,t=0.,grid=True,
\hookrightarrowreturnGrid=True,gridpoints=51)
>>> print mvrcold, mvrwarm
-0.0358753028951 -0.0294763627935
```

The cold response agrees well with the analytical calculation, which predicts that this is $-0.05 / \sqrt{2}$ :

```
>>> print mvrcold+0.05/sqrt(2.)
-0.000519963835811
```

The warm response is slightly smaller in amplitude

```
>>> print mvrwarm/mvrcold
0.821633837619
```

although the numerical uncertainty in mvrwarm is large, because the grid is not sufficiently fine.
We can then re-use this grid in calculations of other moments of the DF, e.g.,

```
>>> print edfcold.meanvT(0.9,phi=22.5,deg=True,t=0.,grid=gridcold)
0.965058551359
>>> print edfwarm.meanvT(0.9,phi=22.5,deg=True,t=0.,grid=gridwarm)
0.915397094614
```

which returns the mean rotational velocity, and

```
>>> print edfcold.vertexdev(0.9,phi=22.5,deg=True,t=0.,grid=gridcold)
3.21160878582
>>> print edfwarm.vertexdev(0.9,phi=22.5,deg=True,t=0.,grid=gridwarm)
4.23510254333
```

which gives the vertex deviation. The reason we have to calculate the grid out to 6 ns igma for the cold response is that the response is much bigger than the velocity dispersion of the population. This velocity dispersion is used to automatically to set the grid edges, but sometimes has to be adjusted to contain the full DF.
evolveddiskdf can also calculate the Oort functions, by directly calculating the spatial derivatives of the DF. These can also be calculated on a grid, such that we can do

```
>>> oortacold, gridcold, gridrcold, gridphicold= edfcold.oortA(0.9,phi=22.5,deg=True,
\hookrightarrow=0.,returnGrids=True,gridpoints=51, derivGridpoints=51,grid=True,derivphiGrid=True,
\hookrightarrowderivRGrid=True,nsigma=6.)
>>> oortawarm, gridwarm, gridrwarm, gridphiwarm= edfwarm.oortA(0.9,phi=22.5,deg=True,
\hookrightarrowt=0.,returnGrids=True,gridpoints=51, derivGridpoints=51,grid=True, derivphiGrid=True,
@derivRGrid=True)
>>> print oortacold, oortawarm
0.575494559999 0.526389833249
```

It is clear that these are quite different. The cold calculation is again close to the analytical prediction, which says that $A=A_{\text {axi }}+0.05 /(2 \sqrt{2})$ where $A_{\text {axi }}=1 /(2 \times 0.9)$ in this case:

```
>>> print oortacold-(0.5/0.9+0.05/2./sqrt(2.))
0.0022613349141670236
```

These grids can then be re-used for the other Oort functions, for example,

```
>>> print edfcold.oortB(0.9,phi=22.5,deg=True,t=0.,grid=gridcold,
\hookrightarrowderivphiGrid=gridphicold, derivRGrid=gridrcold)
-0.574674310521
>>> print edfwarm.oortB(0.9,phi=22.5,deg=True,t=0.,grid=gridwarm,
\hookrightarrowderivphiGrid=gridphiwarm, derivRGrid=gridrwarm)
-0.555546911144
```

and similar for oortC and oortk. These warm results should again be considered for illustration only, as the grid is not sufficiently fine to have a small numerical error.
The grids that have been calculated can also be plotted to show the full velocity DF. For example,

```
>>> gridcold.plot()
```

gives

which demonstrates that the DF is basically the initial DF that has been displaced (by a significant amount compared to the velocity dispersion). The warm velocityd distribution is given by

```
>>> gridwarm.plot()
```

which returns


The shift of the smooth DF here is much smaller than the velocity dispersion.

### 1.4.7 Example: The Hercules stream in the Solar neighborhood as a result of the Galactic bar

We can combine the orbit integration capabilities of galpy with the provided distribution functions and see the effect of the Galactic bar on stellar velocities. By backward integrating orbits starting at the Solar position in a potential that includes the Galactic bar we can evaluate what the velocity distribution is that we should see today if the Galactic bar stirred up a steady-state disk. For this we initialize a flat rotation curve potential and Dehnen's bar potential

```
>>> from galpy.potential import LogarithmicHaloPotential, DehnenBarPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> dp= DehnenBarPotential()
```

The Dehnen bar potential is initialized to start bar formation four bar periods before the present day and to have completely formed the bar two bar periods ago. We can integrate back to the time before bar-formation:

```
>>> ts= numpy.linspace(0,dp.tform(),1000)
```

where dp.tform () is the time of bar-formation (in the usual time-coordinates).
We initialize orbits on a grid in velocity space and integrate them

```
>>> ins=[[Orbit([1.,-0.7+1.4/100*jj,1.-0.6+1.2/100*ii,0.]) for jj in range(101)] for,
\hookrightarrowii in range(101)]
>>> int=[[o.integrate(ts,[lp,dp]) for o in j] for j in ins]
```

We can then evaluate the weight of these orbits by assuming that the disk was in a steady-state before bar-formation with a Dehnen distribution function. We evaluate the Dehnen distribution function at dp.tform () for each of the orbits

```
>>> dfc= dehnendf(beta=0.,correct=True)
>>> out= [[dfc(o(dp.tform())) for o in j] for j in ins]
>>> out= numpy.array(out)
```

This gives

```
>>> from galpy.util.bovy_plot import bovy_dens2d
>>> bovy_dens2d(out,origin='lower', cmap='gist_yarg',contours=True,xrange=[-0.7,0.7],
\hookrightarrowyrange=[0.4,1.6],xlabel=r'$v_R$',ylabel=r'$v_T$')
```



Now that galpy contains the evolveddiskdf described above, this whole calculation is encapsulated in this module and can be done much more easily as

```
>>> edf= evolveddiskdf(dfc,[lp,dp],to=dp.tform())
>>> mvr, grid= edf.meanvR(1.,grid=True,gridpoints=101,returnGrid=True)
```

The gridded DF can be accessed as grid. df , which we can plot as before

```
>>> bovy_dens2d(grid.df.T,origin='lower',cmap='gist_yarg',contours=True,xrange=[grid.
\hookrightarrowvRgrid[0],grid.vRgrid[-1]],yrange=[grid.vTgrid[0],grid.vTgrid[-1]],xlabel=r''$v_R$',
\hookrightarrowylabel=r'$v_T$')
```



For more information see 2000AJ....119..800D and 2010ApJ...725.1676B. Note that the $x$-axis in the Figure above is defined as minus the x -axis in these papers.

### 1.5 A closer look at orbit integration

### 1.5.1 Orbit initialization

## Standard initialization

Orbits can be initialized in various coordinate frames. The simplest initialization gives the initial conditions directly in the Galactocentric cylindrical coordinate frame (or in the rectangular coordinate frame in one dimension). Orbit () automatically figures out the dimensionality of the space from the initial conditions in this case. In three dimensions initial conditions are given either as $v \times v v=[R, v R, v T, z, v z, p h i]$ or one can choose not to specify the azimuth of the orbit and initialize with $v x v v=[R, v R, v T, z, v z]$. Since potentials in galpy are easily initialized to have a circular velocity of one at a radius equal to one, initial coordinates are best given as a fraction of the radius at which
one specifies the circular velocity, and initial velocities are best expressed as fractions of this circular velocity. For example,

```
>>> o= Orbit(vxvv=[1.,0.1,1.1,0.,0.1,0.])
```

initializes a fully three-dimensional orbit, while
$\ggg 0=$ Orbit(vXVV=[1.,0.1,1.1,0.,0.1])
initializes an orbit in which the azimuth is not tracked, as might be useful for axisymmetric potentials.
In two dimensions, we can similarly specify fully two-dimensional orbits $O=O r b i t(v x v v=[R, v R, v T, p h i]$ ) or choose not to track the azimuth and initialize with $O=$ Orbit (vxvv=[R, vR, vT]).

In one dimension we simply initialize with $o=$ Orbit (vxvv=[x,vx]).

## Initialization with physical scales

Orbits are normally used in galpy's natural coordinates. When Orbits are initialized using a distance scale ro= and a velocity scale $v \circ=$, then many Orbit methods return quantities in physical coordinates. Specifically, physical distance and velocity scales are specified as

```
>>> op= Orbit(vxvv=[1.,0.1,1.1,0.,0.1,0.],ro=8.,vo=220.)
```

All output quantities will then be automatically be specified in physical units: kpc for positions, $\mathrm{km} / \mathrm{s}$ for velocities, $(\mathrm{km} / \mathrm{s})^{\wedge} 2$ for energies and the Jacobi integral, $\mathrm{km} / \mathrm{s} \mathrm{kpc}$ for the angular momentum o.L() and actions, $1 / \mathrm{Gyr}$ for frequencies, and Gyr for times and periods. See below for examples of this.
Physical units are only used for outputs: internally natural units are still used and inputs have to also be specified in natural units (for example, integration times or the time at which an output is requested must be specified in natural units). If for any output you do not want the output in physical units, you can specify this by supplying the keyword argument use_physical=False.

## Initialization from observed coordinates

For orbit integration and characterization of observed stars or clusters, initial conditions can also be specified directly as observed quantities when radec=True is set. In this case a full three-dimensional orbit is initialized as $o=$ Orbit (vxvv=[RA, Dec, distance, pmRA, pmDec,Vlos], radec=True) where RA and Dec are expressed in degrees, the distance is expressed in kpc, proper motions are expressed in mas/yr (pmra $=$ pmra' * $\cos [\mathrm{Dec}]$ ), and Vlos is the heliocentric line-of-sight velocity given in $\mathrm{km} / \mathrm{s}$. The observed epoch is currently assumed to be J2000.00. These observed coordinates are translated to the Galactocentric cylindrical coordinate frame by assuming a Solar motion that can be specified as either solarmotion=hogg (default; 2005ApJ...629..268H), solarmotion=dehnen (1998MNRAS.298..387D) or solarmotion=shoenrich (2010MNRAS.403.1829S). A circular velocity can be specified as $\mathrm{v} O=220 \mathrm{in} \mathrm{km} / \mathrm{s}$ and a value for the distance between the Galactic center and the Sun can be given as ro=8. 0 in kpc (e.g., 2012ApJ...759..131B). While the inputs are given in physical units, the orbit is initialized assuming a circular velocity of one at the distance of the Sun (that is, the orbit's position and velocity is scaled to galpy's natural units after converting to the Galactocentric coordinate frame, using the specified ro= and $\mathrm{vO}=$ ). The parameters of the coordinate transformations are stored internally, such that they are automatically used for relevant outputs (for example, when the RA of an orbit is requested). An example of all of this is:

```
>>> o= Orbit(vxvv=[20.,30.,2.,-10.,20.,50.],radec=True,ro=8.,vo=220.)
```

However, the internally stored position/velocity vector is

```
>>> print o.vxvv
[1.1476649101960512, 0.20128601278731811, 1.8303776114906387, -0.13107066602923434, 0.
\hookrightarrow58171049004255293, 0.14071341020496472]
```

and is therefore in natural units.
Similarly, one can also initialize orbits from Galactic coordinates using o= Orbit(vxvv=[glon,glat, distance, pmll, pmbb, Vlos], lb=True), where glon and glat are Galactic longitude and latitude expressed in degrees, and the proper motions are again given in mas/yr ((pmll = pmll' * cos[glat] ):

```
>>> o= Orbit(vxvv=[20.,30.,2.,-10.,20.,50.],lb=True,ro=8.,vo=220.)
>>> print o.vxvv
[0.79998509943955398, 0.075939950035477488, 0.52838231795389867,0.12812499999999999, -
\hookrightarrow0.89052135379600328, 0.092696334097541536]
```

When radec $=$ True or $l b=$ True is set, velocities can also be specified in Galactic coordinates if UVW=True is set. The input is then $V X V V=[R A, D e c$, distance, $U, V, W]$, where the velocities are expressed in $\mathrm{km} / \mathrm{s}$. U is, as usual, defined as $-v R$ (minus $v R$ ).

When orbits are initialized using radec $=$ True or $l b=$ True, physical scales $r o=$ and $v o=$ are automatically specified (because they have defaults of $r o=8$ and $v o=220$ ). Therefore, all output quantities will be specified in physical units (see above). If you do want to get outputs in galpy's natural coordinates, you can turn this behavior off by doing

```
>>> o.turn_physical_off()
```

All outputs will then be specified in galpy's natural coordinates.

### 1.5.2 Orbit integration

After an orbit is initialized, we can integrate it for a set of times $t s$, given as a numpy array. For example, in a simple logarithmic potential we can do the following

```
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> o= Orbit(vxvv=[1.,0.1,1.1,0.,0.1,0.])
>>> import numpy
>>> ts= numpy.linspace(0,100,10000)
>>> o.integrate(ts,lp)
```

to integrate the orbit from $t=0$ to $t=100$, saving the orbit at 10000 instances.
If we initialize the Orbit using a distance scale $r o=$ and a velocity scale vo=, then Orbit plots and outputs will use physical coordinates (currently, times, positions, and velocities)

```
>>> op=Orbit(vXVV=[1.,0.1,1.1,0.,0.1,0.],ro=8.,vo=220.) #Use Vc=220 km/s at R= 8 kpcu
\hookrightarrowas the normalization
>>> op.integrate(ts,lp) #times are still specified in natural coordinates
```


### 1.5.3 Displaying the orbit

After integrating the orbit, it can be displayed by using the plot () function. The quantities that are plotted when plot () is called depend on the dimensionality of the orbit: in 3D the ( $\mathrm{R}, \mathrm{z}$ ) projection of the orbit is shown; in 2D either $(\mathrm{X}, \mathrm{Y})$ is plotted if the azimuth is tracked and $(\mathrm{R}, \mathrm{vR})$ is shown otherwise; in $1 \mathrm{D}(\mathrm{x}, \mathrm{vx})$ is shown. E.g., for the example given above,

```
>>> o.plot()
```

gives


If we do the same for the Orbit that has physical distance and velocity scales associated with it, we get the following

```
>>> op.plot()
```



If we call op.plot (use_physical=False), the quantities will be displayed in natural galpy coordinates.
Other projections of the orbit can be displayed by specifying the quantities to plot. E.g.,

```
>>> o.plot(d1='x',d2='y')
```

gives the projection onto the plane of the orbit:

while
>>> o.plot (d1='R', d2='vR')
gives the projection onto $(\mathrm{R}, \mathrm{vR})$ :


We can also plot the orbit in other coordinate systems such as Galactic longitude and latitude

```
>>> o.plot('k.',d1='ll',d2='bb')
```

which shows



See the documentation of the o.plot function and the o.ra(), o.ll(), etc. functions on how to provide the necessary parameters for the coordinate transformations.

### 1.5.4 Orbit characterization

The properties of the orbit can also be found using galpy. For example, we can calculate the peri- and apocenter radii of an orbit, its eccentricity, and the maximal height above the plane of the orbit

```
>>> o.rap(), o.rperi(), o.e(), o.zmax()
(1.2581455175173673,0.97981663263371377,0.12436710999105324,0.11388132751079502)
```

We can also calculate the energy of the orbit, either in the potential that the orbit was integrated in, or in another potential:

```
>>> O.E(), O.E (pot=mp)
(0.6150000000000001, -0.67390625000000015)
```

where mp is the Miyamoto-Nagai potential of Introduction: Rotation curves.
For the Orbit op that was initialized above with a distance scale $r o=$ and a velocity scale $v o=$, these outputs are all in physical units

```
>>> op.rap(), op.rperi(), op.e(), op.zmax()
(10.065158988860341,7.8385312810643057,0.12436696983841462,0.91105035688072711) #kpc
>>> op.E(), op.E(pot=mp)
(29766.000000000004, -32617.062500000007) #(km/s)^2
```

We can also show the energy as a function of time (to check energy conservation)

```
>>> o.plotE(normed=True)
```

gives


We can specify another quantity to plot the energy against by specifying $\mathrm{d} 1=$. We can also show the vertical energy, for example, as a function of R

```
>>> o.plotEz(dl='R',normed=True)
```



Often, a better approximation to an integral of the motion is given by Ez/sqrt(density[R]). We refer to this quantity as EzJz and we can plot its behavior

```
>>> o.plotEzJz(dl='R', normed=True)
```



### 1.5.5 Accessing the raw orbit

The value of $R, v R, v T, z, v z, x, v x, y, v y, p h i$, and vphi at any time can be obtained by calling the corresponding function with as argument the time (the same holds for other coordinates ra, dec, pmra, pmdec, vra, vdec, ll, bb, pmll, pmbb, vll, vbb, vlos, dist, helioX, helioY, helioZ, U, V, and W). If no time is given the initial condition is returned, and if a time is requested at which the orbit was not saved spline interpolation is used to return the value. Examples include

```
>>> 0.R(1.)
1.1545076874679474
>>> 0.phi(99.)
88.105603035901169
>>> o.ra(2.,obs=[8.,0.,0.],ro=8.)
array([ 285.76403985])
>>> o.helioX(5.)
array([ 1.24888927])
>>> 0.pmll(10.,obs=[8.,0.,0.,0.,245.,0.],ro=8.,vo=230.)
array([-6.45263888])
```

For the Orbit op that was initialized above with a distance scale $r o=$ and a velocity scale $v o=$, the first of these would be

```
>>> op.R(1.)
9.2360614837829225 #kpc
```

which we can also access in natural coordinates as

```
>>> op.R(1.,use_physical=False)
1.1545076854728653
```

We can also specify a different distance or velocity scale on the fly, e.g.,

```
>>> op.R(1.,ro=4.) #different velocity scale would be vo=
4.6180307418914612
```

We can also initialize an Orbit instance using the phase-space position of another Orbit instance evaulated at time t. For example,

```
>>> newOrbit= o(10.)
```

will initialize a new Orbit instance with as initial condition the phase-space position of orbit o at time=10..
The whole orbit can also be obtained using the function getOrbit

```
>>> o.getOrbit()
```

which returns a matrix of phase-space points with dimensions [ntimes,ndim].

### 1.5.6 Fast orbit integration

The standard orbit integration is done purely in python using standard scipy integrators. When fast orbit integration is needed for batch integration of a large number of orbits, a set of orbit integration routines are written in C that can be accessed for most potentials, as long as they have C implementations, which can be checked by using the attribute hasC

```
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,amp=1., normalize=1.)
>>> mp.hasC
True
```

Fast C integrators can be accessed through the method= keyword of the orbit.integrate method. Currently available integrators are

- rk4_c
- rk6_c
- dopr54_c
which are Runge-Kutta and Dormand-Prince methods. There are also a number of symplectic integrators available
- leapfrog_c
- symplec4_c
- symplec6_c

The higher order symplectic integrators are described in Yoshida (1993).
For most applications I recommend dopr54_c. For example, compare

```
>>> o= Orbit(vxvv=[1.,0.1,1.1,0.,0.1])
>>> timeit(o.integrate(ts,mp))
1 loops, best of 3: 553 ms per loop
>>> timeit(o.integrate(ts,mp,method='dopr54_c'))
```

```
galpyWarning: Using C implementation to integrate orbits
10 loops, best of 3: 25.6 ms per loop
```

As this example shows, galpy will issue a warning that $C$ is being used. Speed-ups by a factor of 20 are typical.

### 1.5.7 Integration of the phase-space volume

galpy further supports the integration of the phase-space volume through the method integrate_dxdv, although this is currently only implemented for two-dimensional orbits (planarOrbit). As an example, we can check Liouville's theorem explicitly. We initialize the orbit

```
>>> o= Orbit(vxvv=[1.,0.1,1.1,0.])
```

and then integrate small deviations in each of the four phase-space directions

```
>>> ts= numpy.linspace(0.,28.,1001) #~1 Gyr at the Solar circle
>>> o.integrate_dxdv([1.,0.,0.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dx= O.getOrbit_dxdv()[-1,:] # evolution of dxdv[0] along the orbit
>>> o.integrate_dxdv([0.,1.,0.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dy= o.getOrbit_dxdv()[-1,:]
>>> o.integrate_dxdv([0.,0.,1.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dvx= o.getOrbit_dxdv() [-1,:]
>>> o.integrate_dxdv([0.,0.,0.,1.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dvy= o.getOrbit_dxdv()[-1,:]
```

We can then compute the determinant of the Jacobian of the mapping defined by the orbit integration from time zero to the final time

```
>>> tjac= numpy.linalg.det(numpy.array([dx,dy,dvx,dvy]))
```

This determinant should be equal to one

```
>>> print tjac
0.999999991189
>>> numpy.fabs(tjac-1.) < 10.**-8.
True
```

The calls to integrate_dxdv above set the keywords rectIn= and rectOut= to True, as the default input and output uses phase-space volumes defined as ( $\mathrm{dR}, \mathrm{dvR}, \mathrm{dvT}, \mathrm{dphi}$ ) in cylindrical coordinates. When rect In or rectOut is set, the in- or output is in rectangular coordinates ( $[\mathrm{x}, \mathrm{y}, \mathrm{vx}, \mathrm{vy}]$ in two dimensions).

Implementing the phase-space integration for three-dimensional Fullorbit instances is straightforward and is part of the longer term development plan for galpy. Let the main developer know if you would like this functionality, or better yet, implement it yourself in a fork of the code and send a pull request!

### 1.5.8 Example: The eccentricity distribution of the Milky Way's thick disk

A straightforward application of galpy's orbit initialization and integration capabilities is to derive the eccentricity distribution of a set of thick disk stars. We start by downloading the sample of SDSS SEGUE (2009AJ. .. 137.4377Y) thick disk stars compiled by Dierickx et al. (2010arXiv1009.1616D) at
http://www.mpia-hd.mpg.de/homes/rix/Data/Dierickx-etal-tab2.txt

After reading in the data (RA,Dec,distance,pmRA,pmDec,vlos; see above) as a vector vxvv with dimensions [6,ndata] we (a) define the potential in which we want to integrate the orbits, and (b) integrate each orbit and save its eccentricity (running this for all 30,000-ish stars will take about half an hour)

```
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> ts= nu.linspace(0.,20.,10000)
>>> mye= nu.zeros(ndata)
>>> for ii in range(len(e)):
... O= Orbit(vxvv[ii,:],radec=True,vo=220.,ro=8.) #Initialize
... O.integrate(ts,lp) #Integrate
... mye[ii]= o.e() #Calculate eccentricity
```

We then find the following eccentricity distribution


The eccentricity calculated by galpy compare well with those calculated by Dierickx et al., except for a few objects


The script that calculates and plots everything can be downloaded here.

### 1.6 Action-angle coordinates

galpy can calculate actions and angles for a large variety of potentials (any time-independent potential in principle). These are implemented in a separate module galpy.actionAngle, and the preferred method for accessing them is through the routines in this module. There is also some support for accessing the actionAngle routines as methods of the Orbit class.

Action-angle coordinates can be calculated for the following potentials/approximations:

- Isochrone potential
- Spherical potentials
- Adiabatic approximation
- Staeckel approximation
- A general orbit-integration-based technique

There are classes corresponding to these different potentials/approximations and actions, frequencies, and angles can typically be calculated using these three methods:

- __call__: returns the actions
- actionsFreqs: returns the actions and the frequencies
- actionsFreqsAngles: returns the actions, frequencies, and angles

These are not all implemented for each of the cases above yet.
The adiabatic and Staeckel approximation have also been implemented in C and using grid-based interpolation, for extremely fast action-angle calculations (see below).

### 1.6.1 Action-angle coordinates for the isochrone potential

The isochrone potential is the only potential for which all of the actions, frequencies, and angles can be calculated analytically. We can do this in galpy by doing

```
>>> from galpy.potential import IsochronePotential
>>> from galpy.actionAngle import actionAngleIsochrone
>>> ip= IsochronePotential(b=1.,normalize=1.)
>>> aAI= actionAngleIsochrone(ip=ip)
```

aAI is now an instance that can be used to calculate action-angle variables for the specific isochrone potential ip. Calling this instance returns ( $J_{R}, L_{Z}, J_{Z}$ )

```
>>> aAI(1.,0.1,1.1,0.1,0.) #inputs R,vR,vT,z,vz
(array([ 0.00713759]), array([ 1.1]), array([ 0.00553155]))
```

or for a more eccentric orbit

```
>>> aAI (1.,0.5,1.3,0.2,0.1)
(array([ 0.13769498]), array([ 1.3]), array([ 0.02574507]))
```

Note that we can also specify phi, but this is not necessary

```
>> aAI (1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.13769498]), array([ 1.3]), array([ 0.02574507]))
```

We can likewise calculate the frequencies as well

```
>>> aAI.actionsFreqs(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.13769498]),
array([ 1.3]),
array([ 0.02574507]),
array([ 1.29136096]),
array([ 0.79093738]),
array([ 0.79093738]))
```

The output is $\left(J_{R}, L_{Z}, J_{Z}, \Omega_{R}, \Omega_{\phi}, \Omega_{Z}\right)$. For any spherical potential, $\Omega_{\phi}=\operatorname{sgn}\left(L_{Z}\right) \Omega_{Z}$, such that the last two frequencies are the same.

We obtain the angles as well by calling

```
>>> aAI.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.13769498]),
array([ 1.3]),
array([ 0.02574507]),
array([ 1.29136096]),
array([ 0.79093738]),
array([ 0.79093738]),
```

```
array([ 0.57101518]),
array([ 5.96238847]),
array([ 1.24999949]))
```

The output here is $\left(J_{R}, L_{Z}, J_{Z}, \Omega_{R}, \Omega_{\phi}, \Omega_{Z}, \theta_{R}, \theta_{\phi}, \theta_{Z}\right)$.
To check that these are good action-angle variables, we can calculate them along an orbit

```
>>> from galpy.orbit import Orbit
>>> o= Orbit([1.,0.5,1.3,0.2,0.1,0.])
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,ip)
>>> jfa= aAI.actionsFreqsAngles(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),o.phi(ts))
```

which works because we can provide arrays for the $R$ etc. inputs.
We can then check that the actions are constant over the orbit

```
>>> plot(ts,numpy.log10(numpy.fabs((jfa[0]-numpy.mean(jfa[0])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[1]-numpy.mean(jfa[1])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[2]-numpy.mean(jfa[2])))))
```

which gives


The actions are all conserved. The angles increase linearly with time

```
>>> plot(ts,jfa[6],'b.')
>>> plot(ts,jfa[7],'g.')
>>> plot(ts,jfa[8],'r.')
```



### 1.6.2 Action-angle coordinates for spherical potentials

Action-angle coordinates for any spherical potential can be calculated using a few orbit integrations. These are implemented in galpy in the actionAngleSpherical module. For example, we can do

```
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> from galpy.actionAngle import actionAngleSpherical
>>> aAS= actionAngleSpherical(pot=lp)
```

For the same eccentric orbit as above we find

```
>> aAS(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.22022112]), array([ 1.3]), array([ 0.02574507]))
>>> aAS.actionsFreqs(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.22022112]),
array([ 1.3]),
array([ 0.02574507]),
array([ 0.87630459]),
```

(continued from previous page)

```
array([ 0.60872881]),
array([ 0.60872881]))
>>> aAS.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.22022112]),
array([ 1.3]),
array([ 0.02574507]),
array([ 0.87630459]),
array([ 0.60872881]),
array([ 0.60872881]),
array([ 0.40443857]),
array([ 5.85965048]),
array([ 1.1472615]))
```

We can again check that the actions are conserved along the orbit and that the angles increase linearly with time:

```
>>> o.integrate(ts,lp)
>>> jfa= aAS.actionsFreqsAngles(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),o.phi(ts),
\hookrightarrowixed_quad=True)
```

where we use fixed_quad=True for a faster evaluation of the required one-dimensional integrals using Gaussian quadrature. We then plot the action fluctuations

```
>>> plot(ts,numpy.log10(numpy.fabs((jfa[0]-numpy.mean(jfa[0])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[1]-numpy.mean(jfa[1])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[2]-numpy.mean(jfa[2])))))
```

which gives

showing that the actions are all conserved. The angles again increase linearly with time
>>> plot(ts,jfa[6],'b.')
>>> plot (ts,jfa[7],'g.')
>>> plot (ts,jfa[8],'r.')


We can check the spherical action-angle calculations against the analytical calculations for the isochrone potential. Starting again from the isochrone potential used in the previous section

```
>>> ip= IsochronePotential(b=1.,normalize=1.)
>>> aAI= actionAngleIsochrone(ip=ip)
>>> aAS= actionAngleSpherical(pot=ip)
```

we can compare the actions, frequencies, and angles computed using both

```
>>> aAI.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.13769498]),
array([ 1.3]),
array([ 0.02574507]),
array([ 1.29136096]),
array([ 0.79093738]),
array([ 0.79093738]),
array([ 0.57101518]),
array([ 5.96238847]),
array([ 1.24999949]))
>>> aAS.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
(array([ 0.13769498]),
array([ 1.3]),
array([ 0.02574507]),
array([ 1.29136096]),
array([ 0.79093738]),
```

(continues on next page)

```
array([ 0.79093738]),
array([ 0.57101518]),
array([ 5.96238838]),
array([ 1.2499994]))
```

or more explicitly comparing the two

```
>> [r-s for r,s in zip(aAI.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.),aAS.
\hookrightarrowactionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.))]
[array([ 6.66133815e-16]),
array([ 0.]),
array([ 0.]),
array([ -4.53851845e-10]),
array([ 4.74775219e-10]),
array([ 4.74775219e-10]),
array([ -1.65965242e-10]),
array([ 9.04759645e-08]),
array([ 9.04759649e-08])]
```


### 1.6.3 Action-angle coordinates using the adiabatic approximation

For non-spherical, axisymmetric potentials galpy contains multiple methods for calculating approximate action-angle coordinates. The simplest of those is the adiabatic approximation, which works well for disk orbits that do not go too far from the plane, as it assumes that the vertical motion is decoupled from that in the plane (e.g., 2010MNRAS.401.2318B).

Setup is similar as for other actionAngle objects

```
>>> from galpy.potential import MWPotential2014
>>> from galpy.actionAngle import actionAngleAdiabatic
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014)
```

and evaluation then proceeds similarly as before

```
>>> aAA(1.,0.1,1.1,0.,0.05)
(0.01351896260559274, 1.1, 0.0004690133479435352)
```

We can again check that the actions are conserved along the orbit

```
>>> from galpy.orbit import Orbit
>>> ts=numpy.linspace(0.,100.,1001)
>>> o= Orbit([1.,0.1,1.1,0.,0.05])
>>> O.integrate(ts,MWPotential2014)
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
```

This takes a while. The adiabatic approximation is also implemented in C, which leads to great speed-ups. Here is how to use it

```
>>> timeit(aAA(1.,0.1,1.1,0.,0.05))
10 loops, best of 3: 73.7 ms per loop
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
>>> timeit(aAA(1.,0.1,1.1,0.,0.05))
1000 loops, best of 3: 1.3 ms per loop
```

or about a 50 times speed-up. For arrays the speed-up is even more impressive

```
>>> s= numpy.ones(100)
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
10 loops, best of 3: 37.8 ms per loop
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014) #back to no C
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
1 loops, best of 3: 7.71 s per loop
```

or a speed-up of 200! Back to the previous example, you can run it with $c=$ True to speed up the computation

```
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
```

We can plot the radial- and vertical-action fluctuation as a function of time

```
>>> plot(ts, numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts,numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2]))/numpy.mean(js[2]))))
```

which gives


The radial action is conserved to about half a percent, the vertical action to two percent.
Another way to speed up the calculation of actions using the adiabatic approximation is to tabulate the actions on a grid in (approximate) integrals of the motion and evaluating new actions by interpolating on this grid. How this is done in practice is described in detail in the galpy paper. To setup this grid-based interpolation method, which is contained in actionAngleAdiabaticGrid, do

```
>>> from galpy.actionAngle import actionAngleAdiabaticGrid
>>> aAG= actionAngleAdiabaticGrid(pot=MWPotential2014,nR=31,nEz=31,nEr=51,nLz=51,
C=True)
```

where $c=T r u e ~ s p e c i f i e s ~ t h a t ~ w e ~ u s e ~ t h e ~ C ~ i m p l e m e n t a t i o n ~ o f ~ a c t i o n A n g l e A d i a b a t i c ~ f o r ~ s p e e d . ~ W e ~ c a n ~ n o w ~$ evaluate in the same was as before, for example

```
>>> aAA(1.,0.1,1.1,0.,0.05), aAG(1.,0.1,1.1,0.,0.05)
((array([ 0.01352523]), array([ 1.1]), array([ 0.00046909])),
    (0.013527010324238781, 1.1, 0.00047747359874375148))
```

which agree very well. To look at the timings, we first switch back to not using C and then list all of the relevant timings:

```
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=False)
# Not using C, direct calculation
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
1 loops, best of 3: 9.05 s per loop
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
# Using C, direct calculation
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
10 loops, best of 3: 39.7 ms per loop
# Grid-based calculation
>>> timeit(aAG(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
1000 loops, best of 3: 1.09 ms per loop
```

Thus, in this example (and more generally) the grid-based calculation is significantly faster than even the direct implementation in C. The overall speed up between the direct Python version and the grid-based version is larger than 8,000 ; the speed up between the direct C version and the grid-based version is 36 . For larger arrays of input phase-space positions, the latter speed up can increase to 150 . For simpler, fully analytical potentials the speed up will be slightly less, but for MWPotential2014 and other more complicated potentials (such as those involving a double-exponential disk), the overhead of setting up the grid is worth it when evaluating more than a few thousand actions.

The adiabatic approximation works well for orbits that stay close to the plane. The orbit we have been considering so far only reaches a height two percent of $R_{0}$, or about 150 pc for $R_{0}=8 \mathrm{kpc}$.

```
>>> 0. zmax ()*8.
0.17903686455491979
```

For orbits that reach distances of a kpc and more from the plane, the adiabatic approximation does not work as well. For example,

```
>>> o= Orbit([1.,0.1,1.1,0.,0.25])
>>> o.integrate(ts,MWPotential2014)
>>> 0. zmax ()*8.
1.3506059038621048
```

and we can again calculate the actions along the orbit

```
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
>>> plot(ts,numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts, numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2]))/numpy.mean(js[2]))))
```

which gives


The radial action is now only conserved to about ten percent and the vertical action to approximately five percent.

Warning: Frequencies and angles using the adiabatic approximation are not implemented at this time.

### 1.6.4 Action-angle coordinates using the Staeckel approximation

A better approximation than the adiabatic one is to locally approximate the potential as a Staeckel potential, for which actions, frequencies, and angles can be calculated through numerical integration. galpy contains an implementation of the algorithm of Binney (2012; 2012MNRAS.426.1324B), which accomplishes the Staeckel approximation for disklike (i.e., oblate) potentials without explicitly fitting a Staeckel potential. For all intents and purposes the adiabatic approximation is made obsolete by this new method, which is as fast and more precise. The only advantage of the adiabatic approximation over the Staeckel approximation is that the Staeckel approximation requires the user to specify a focal length $\Delta$ to be used in the Staeckel approximation. However, this focal length can be easily estimated from the second derivatives of the potential (see Sanders 2012; 2012MNRAS.426..128S).

Starting from the second orbit example in the adiabatic section above, we first estimate a good focal length of the MWPotential2014 to use in the Staeckel approximation. We do this by averaging (through the median) estimates at positions around the orbit (which we integrated in the example above)

```
>>> from galpy.actionAngle import estimateDeltaStaeckel
>>> estimateDeltaStaeckel(o.R(ts),o.z(ts),pot=MWPotential2014)
0.40272708556203662
```

We will use $\Delta=0.4$ in what follows. We set up the actionAngleStaeckel object

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False) #c=True is the 
\hookrightarrowdefault
```

and calculate the actions

```
>>> aAS(o.R(),0.vR(),0.vT(),O.z(),O.vz())
(0.019212848866725911, 1.1000000000000001, 0.015274597971510892)
```

The adiabatic approximation from above gives

```
>>> aAA(0.R(),0.vR(),0.vT(),O.z(),O.vz())
(array([ 0.01686478]), array([ 1.1]), array([ 0.01590001]))
```

The actionAngleStaeckel calculations are sped up in two ways. First, the action integrals can be calculated using Gaussian quadrature by specifying fixed_quad=True

```
>>> aAS(0.R(),0.vR(),0.vT(),o.z(),o.vz(),fixed_quad=True)
(0.01922167296633687, 1.1000000000000001, 0.015276825017286706)
```

which in itself leads to a ten times speed up

```
>>> timeit(aAS(O.R(),O.vR(),O.vT(),O.z(),O.vz(),fixed_quad=False))
10 loops, best of 3: 129 ms per loop
>>> timeit(aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz(),fixed_quad=True))
100 loops, best of 3: 10.3 ms per loop
```

Second, the actionAngleStaeckel calculations have also been implemented in C , which leads to even greater speed-ups, especially for arrays

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014, delta=0.4,c=True)
>>> s= numpy.ones(100)
>>> timeit(aAS (1.*s,0.1*s,1.1*s,0.*s,0.05*s))
10 loops, best of 3: 35.1 ms per loop
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False) #back to no C
>>> timeit(aAS (1.*s,0.1*s,1.1*s,0.*s,0.05*s,fixed_quad=True))
1 loops, best of 3: 496 ms per loop
```

or a fifteen times speed up. The speed up is not that large because the bulge model in MWPotential2014 requires expensive special functions to be evaluated. Computations could be sped up ten times more when using a simpler bulge model.

Similar to actionAngleAdiabaticGrid, we can also tabulate the actions on a grid of (approximate) integrals of the motion and interpolate over this look-up table when evaluating new actions. The details of how this look-up table is setup and used are again fully explained in the galpy paper. To use this grid-based Staeckel approximation, contained in actionAngleStaeckelGrid, do

```
>>> from galpy.actionAngle import actionAngleStaeckelGrid
>>> aASG= actionAngleStaeckelGrid(pot=MWPotential2014,delta=0.4,nE=51,npsi=51,nLz=61,
C=True)
```

 this is a fully three-dimensional grid, setting up the grid takes longer than it does for the adiabatic method (which only uses two two-dimensional grids). We can then evaluate actions as before

```
>>> aAS(o.R(),O.vR(),O.vT(),0.z(),O.vz()), aASG(o.R(),O.vR(),O.vT(),O.z(),O.vz())
((0.019212848866725911, 1.1000000000000001, 0.015274597971510892),
    (0.019221119033345408, 1.1000000000000001, 0.015022528662310393))
```

These actions agree very well. We can compare the timings of these methods as above

```
>>> timeit(aAS(1.*s,0.1*S,1.1*s,0.*s,0.05*s,fixed_quad=True))
1 loops, best of 3: 576 ms per loop # Not using C, direct calculation
>>> aAS= actionAngleStaeckel(pot=MWPotential2014, delta=0.4,c=True)
>>> timeit(aAS(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
100 loops, best of 3: 17.8 ms per loop # Using C, direct calculation
>>> timeit(aASG(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
100 loops, best of 3: 3.45 ms per loop # Grid-based calculation
```

This demonstrates that the grid-based interpolation again leeds to a significant speed up, even over the C implementation of the direct calculation. This speed up becomes more significant for larger array input, although it saturates at about 25 times (at least for MWPotential2014).
We can now go back to checking that the actions are conserved along the orbit (going back to the $\mathrm{C}=\mathrm{Fal}$ se version of actionAngleStaeckel)

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False)
>>> js= aAS(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),fixed_quad=True)
>>> plot(ts, numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts,numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2]))/numpy.mean(js[2]))))
```

which gives


The radial action is now conserved to better than a percent and the vertical action to only a fraction of a percent. Clearly, this is much better than the five to ten percent errors found for the adiabatic approximation above.
For the Staeckel approximation we can also calculate frequencies and angles through the actionsFreqs and actionsFreqsAngles methods.

Warning: Frequencies and angles using the Staeckel approximation are only implemented in C. So use $\mathrm{c}=$ True in the setup of the actionAngleStaeckel object.

Warning: Angles using the Staeckel approximation in galpy are such that (a) the radial angle starts at zero at pericenter and increases then going toward apocenter; (b) the vertical angle starts at zero at $z=0$ and increases toward positive zmax. The latter is a different convention from that in Binney (2012), but is consistent with that in actionAngleIsochrone and actionAngleSpherical.

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=True)
>>> O= Orbit([1.,0.1,1.1,0.,0.25,0.]) #need to specify phi for angles
>>> aAS.actionsFreqsAngles(o.R(),o.vR(),o.vT(),o.z(),o.vz(),o.phi())
(array([ 0.01922167]),
array([ 1.1]),
array([ 0.01527683]),
array([ 1.11317796]),
```

```
array([ 0.82538032])
array([ 1.34126138])
array([ 0.37758087])
array([ 6.17833493])
array([ 6.13368239]))
```

and we can check that the angles increase linearly along the orbit

```
>>> o.integrate(ts,MWPotential2014)
>>> jfa= aAS.actionsFreqsAngles(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),o.phi(ts))
>>> plot(ts,jfa[6],'b.')
>>> plot(ts,jfa[7],'g.')
>>> plot(ts,jfa[8],'r.')
```



Or

[^0]

### 1.6.5 Action-angle coordinates using an orbit-integration-based approximation

The adiabatic and Staeckel approximations used above are good for stars on close-to-circular orbits, but they break down for more eccentric orbits (specifically, orbits for which the radial and/or vertical action is of a similar magnitude as the angular momentum). This is because the approximations made to the potential in these methods (that it is separable in $R$ and $z$ for the adiabatic approximation and that it is close to a Staeckel potential for the Staeckel approximation) break down for such orbits. Unfortunately, these methods cannot be refined to provide better approximations for eccentric orbits.
galpy contains a new method for calculating actions, frequencies, and angles that is completely general for any static potential. It can calculate the actions to any desired precision for any orbit in such potentials. The method works by employing an auxiliary isochrone potential and calculates action-angle variables by arithmetic operations on the actions and angles calculated in the auxiliary potential along an orbit (integrated in the true potential). Full details can be found in Appendix A of Bovy (2014).

We setup this method for a logarithmic potential as follows

```
>>> from galpy.actionAngle import actionAngleIsochroneApprox
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.,q=0.9)
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=0.8)
```

$\mathrm{b}=0.8$ here sets the scale parameter of the auxiliary isochrone potential; this potential can also be specified as an

IsochronePotential instance through $i p=$ ). We can now calculate the actions for an orbit similar to that of the GD-1 stream

```
>>> obs= numpy.array([1.56148083,0.35081535,-1.15481504,0.88719443,-0.47713334,0.
\hookrightarrow12019596]) #orbit similar to GD-1
>>> aAIA(*obs)
(array([ 0.16605011]), array([-1.80322155]), array([ 0.50704439]))
```

An essential requirement of this method is that the angles calculated in the auxiliary potential go through the full range $[0,2 \pi]$. If this is not the case, galpy will raise a warning

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=10.8)
>>> aAIA(*obs)
galpyWarning: Full radial angle range not covered for at least one object; actions,
\hookrightarroware likely not reliable
(array([ 0.08985167]), array([-1.80322155]), array([ 0.50849276]))
```

Therefore, some care should be taken to choosing a good auxiliary potential. galpy contains a method to estimate a decent scale parameter for the auxiliary scale parameter, which works similar to estimateDeltaStaeckel above except that it also gives a minimum and maximum b if multiple $R$ and $z$ are given

```
>>> from galpy.actionAngle import estimateBIsochrone
>>> from galpy.orbit import Orbit
>>> o= Orbit(obs)
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,lp)
>>> estimateBIsochrone(o.R(ts),0.z(ts),pot=lp)
(0.78065062339131952, 1.2265541473461612, 1.4899326335155412) #bmin,bmedian,bmax over,
\hookrightarrowthe orbit
```

Experience shows that a scale parameter somewhere in the range returned by this function makes sure that the angles go through the full $[0,2 \pi]$ range. However, even if the angles go through the full range, the closer the angles increase to linear, the better the converenge of the algorithm is (and especially, the more accurate the calculation of the frequencies and angles is, see below). For example, for the scale parameter at the upper and of the range

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=1.5)
>>> aAIA(*obs)
(array([ 0.01120145]), array([-1.80322155]), array([ 0.50788893]))
```

which does not agree with the previous calculation. We can inspect how the angles increase and how the actions converge by using the aAIA. plot function. For example, we can plot the radial versus the vertical angle in the auxiliary potential

```
>>> aAIA.plot(*obs,type='araz')
```

which gives

and this clearly shows that the angles increase very non-linearly, because the auxiliary isochrone potential used is too far from the real potential. This causes the actions to converge only very slowly. For example, for the radial action we can plot the converge as a function of integration time

```
>>> aAIA.plot(*obs,type='jr')
```

which gives


This Figure clearly shows that the radial action has not converged yet. We need to integrate much longer in this auxiliary potential to obtain convergence and because the angles increase so non-linearly, we also need to integrate the orbit much more finely:

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=1.5,tintJ=1000,ntintJ=800000)
>>> aAIA(*obs)
(array([ 0.01711635]), array([-1.80322155]), array([ 0.51008058]))
>>> aAIA.plot(*obs,type='jr')
```

which shows slow convergence


Finding a better auxiliary potential makes convergence much faster and also allows the frequencies and the angles to be calculated by removing the small wiggles in the auxiliary angles vs. time (in the angle plot above, the wiggles are much larger, such that removing them is hard). The auxiliary potential used above had $b=0.8$, which shows very quick converenge and good behavior of the angles

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=0.8)
>>> aAIA.plot(*obs,type='jr')
```

gives

and
>>> aAIA.plot(*obs,type='araz')
gives


We can remove the periodic behavior from the angles, which clearly shows that they increase close-to-linear with time

```
>>> aAIA.plot(*obs,type='araz',deperiod=True)
```



We can then calculate the frequencies and the angles for this orbit as

```
>>> aAIA.actionsFreqsAngles(*obs)
(array([ 0.16392384]),
array([-1.80322155]),
array([ 0.50999882]),
array([ 0.55808933]),
array([-0.38475753]),
array([ 0.42199713]),
array([ 0.18739688]),
array([ 0.3131815]),
array([ 2.18425661]))
```

This function takes as an argument maxn= the maximum $n$ for which to remove sinusoidal wiggles. So we can raise this, for example to 4 from 3

```
>>> aAIA.actionsFreqsAngles(*obs,maxn=4)
(array([ 0.16392384]),
array([-1.80322155]),
array([ 0.50999882]),
array([ 0.55808776]),
array([-0.38475733]),
array([ 0.4219968]),
```

```
array([ 0.18732009]),
array([ 0.31318534]),
array([ 2.18421296]))
```

Clearly, there is very little change, as most of the wiggles are of low $n$.

Warning: While the orbit-based actionAngle technique in principle works for triaxial potentials, angles and frequencies for non-axisymmetric potentials are not implemented yet.

This technique also works for triaxial potentials, but using those requires the code to also use the azimuthal angle variable in the auxiliary potential (this is unnecessary in axisymmetric potentials as the $z$ component of the angular momentum is conserved). We can calculate actions for triaxial potentials by specifying that nonaxi=True:

```
>>> aAIA(*obs,nonaxi=True)
(array([ 0.16605011]), array([-1.80322155]), array([ 0.50704439]))
```

galpy currently does not contain any triaxial potentials, so we cannot illustrate this here with any real triaxial potentials.

### 1.6.6 Accessing action-angle coordinates for Orbit instances

While the recommended way to access the actionAngle routines is through the methods in the galpy. actionAngle modules, action-angle coordinates can also be cacluated for galpy. orbit. Orbit instances. This is illustrated here briefly. We initialize an Orbit instance

```
>>> from galpy.orbit import Orbit
>>> from galpy.potential import MWPotential2014
>>> o= Orbit([1.,0.1,1.1,0.,0.25,0.])
```

and we can then calculate the actions (default is to use the adiabatic approximation)

```
>>> o.jr(MWPotential2014), o.jp(MWPotential2014), o.jz(MWPotential2014)
(0.01685643005901713, 1.1, 0.015897730620467752)
```

-.jp here gives the azimuthal action (which is the $z$ component of the angular momentum for axisymmetric potentials). We can also use the other methods described above, but note that these require extra parameters related to the approximation to be specified (see above):

```
>>> O.jr(MWPotential2014,type='staeckel',delta=0.4), o.jp(MWPotential2014,type=
\hookrightarrow'staeckel',delta=0.4), o.jz(MWPotential2014,type='staeckel',delta=0.4)
(array([ 0.01922167]), array([ 1.1]), array([ 0.01527683]))
>>> o.jr(MWPotential2014,type='isochroneApprox',b=0.8), o.jp(MWPotential2014,type=
\hookrightarrow'isochroneApprox',b=0.8), o.jz(MWPotential2014,type='isochroneApprox',b=0.8)
(array([ 0.01906609]), array([ 1.1]), array([ 0.01528049]))
```

These two methods give very precise actions for this orbit (both are converged to about $1 \%$ ) and they agree very well

```
>>> (o.jr(MWPotential2014,type='staeckel',delta=0.4)-o.jr(MWPotential2014,type=
\hookrightarrow'isochroneApprox', b=0.8))/o.jr(MWPotential2014,type='isochroneApprox',b=0.8)
array([ 0.00816012])
>>> (o.jz(MWPotential2014,type='staeckel',delta=0.4)-o.jz(MWPotential2014,type=
\hookrightarrow'isochroneApprox', b=0.8))/o.jz(MWPotential2014,type='isochroneApprox',b=0.8)
array([-0.00024])
```

Warning: Once an action, frequency, or angle is calculated for a given type of calculation (e.g., staeckel), the parameters for that type are fixed in the Orbit instance. Call o.resetaA() to reset the action-angle instance used when using different parameters (i.e., different delta=for staeckel or different $b=$ for isochroneApprox.

We can also calculate the frequencies and the angles. This requires using the Staeckel or Isochrone approximations, because frequencies and angles are currently not supported for the adiabatic approximation. For example, the radial frequency

```
>>> 0.Or(MWPotential2014,type='staeckel',delta=0.4)
1.1131779637307115
>>> O.Or(MWPotential2014,type='isochroneApprox',b=0.8)
1.1134635974560649
```

and the radial angle

```
>>> 0.wr(MWPotential2014,type='staeckel',delta=0.4)
0.37758086786371969
>>> o.wr(MWPotential2014,type='isochroneApprox',b=0.8)
0.38159809018175395
```

which again agree to $1 \%$. We can also calculate the other frequencies, angles, as well as periods using the functions $\circ . O p, \circ . \circ z, \circ . w p, \circ \cdot w z, \circ . T r, \circ . T p, \circ . T z$.

### 1.6.7 Example: Evidence for a Lindblad resonance in the Solar neighborhood

We can use galpy to calculate action-angle coordinates for a set of stars in the Solar neighborhood and look for unexplained features. For this we download the data from the Geneva-Copenhagen Survey (2009A\&A...501..941H; data available at viZier). Since the velocities in this catalog are given as U,V, and W, we use the radec and UVW keywords to initialize the orbits from the raw data. For each object ii

```
>>> o= Orbit(vxvv[ii,:],radec=True,uvw=True,vo=220.,ro=8.)
```

We then calculate the actions and angles for each object in a flat rotation curve potential

```
>>> lp= LogarithmicHaloPotential(normalize=1.)
```

>>> myjr[ii]= o.jr(lp)
etc.
Plotting the radial action versus the angular momentum

```
>>> plot.bovy_plot(myjp,myjr,'k.',ms=2.,xlabel=r'$J_{\phi}$',ylabel=r'$J_R$',
<range=[0.7,1.3],yrange=[0.,0.05])
```

shows a feature in the distribution


If instead we use a power-law rotation curve with power-law index 1

```
>>> pp= PowerSphericalPotential(normalize=1.,alpha=-2.)
```

>>> myjr[ii]= o.jr(pp)

We find that the distribution is stretched, but the feature remains


Code for this example can be found here (note that this code uses a particular download of the GCS data set; if you use your own version, you will need to modify the part of the code that reads the data). For more information see 2010MNRAS.409..145S.

### 1.6.8 NEW: Example: actions in an N -body simulation

To illustrate how we can use galpy to calculate actions in a snapshot of an N -body simulation, we again look at the g15784 snapshot in the pynbody test suite, discussed in The potential of $N$-body simulations. Please look at that section for information on how to setup the potential of this snapshot in galpy. One change is that we should set enable_c=True in the instantiation of the InterpSnapshotRZPotential object

```
>>> spi= InterpSnapshotRZPotential(h1,rgrid=(numpy.log(0.01),numpy.log(20.),101),
\hookrightarrowlogR=True,zgrid=(0.,10.,101), interpPot=True, zsym=True,enable_c=True)
>>> spi.normalize(R0=10.)
```

where we again normalize the potential to use galpy's natural units.
We first load a pristine copy of the simulation (because the normalization above leads to some inconsistent behavior in pynbody)

```
>>> sc = pynbody.load('Repos/pynbody-testdata/g15784.lr.01024.gz'); hc = sc.halos();
\hookrightarrowhc1= hc[1]; pynbody.analysis.halo.center(hc1,mode='hyb'); pynbody.analysis.angmom.
\hookrightarrowfaceon(hc1, cen=(0,0,0),mode='ssc'); sc.physical_units()
```

and then select particles near $R=8 \mathrm{kpc}$ by doing

```
>>> sn= pynbody.filt.BandPass('rxy','7 kpc','9 kpc')
>>> R,vR,vT,z,vz = [numpy.ascontiguousarray(hc1.s[sn][x]) for x in ('rxy','vr','vt','z
\hookrightarrow','vz')]
```

These have physical units, so we normalize them (the velocity normalization is the circular velocity at $R=10 \mathrm{kpc}$, see here).

```
>>> ro, vo= 10., 294.62723076942245
>>> R/= ro
>>> z/= ro
>>> vR/= vo
>>> vT/= vo
>>> vz/= vo
```

We will calculate actions using actionAngleStaeckel above. We can first integrate a random orbit in this potential

```
>>> from galpy.orbit import Orbit
>>> numpy.random.seed(1)
>>> ii= numpy.random.permutation(len(R)) [0]
>>> o= Orbit([R[ii],vR[ii],vT[ii],z[ii],vz[ii]])
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,spi)
```

This orbit looks like this

```
>>> o.plot()
```



We can now calculate the actions by doing

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=spi,delta=0.45,c=True)
>>> jr,lz,jz= aAS(R,vR,vT,z,vz)
```

These actions are also in natural units; you can obtain physical units by multiplying with ro*vo. We can now plot these actions

```
>>> from galpy.util import bovy_plot
>>> bovy_plot.scatterplot(lz,jr,'k.',xlabel=r'$J_\phi$',ylabel=r'$J_R$',xrange=[0.,1.
\hookrightarrow3],yrange=[0., . 6])
```

which gives


Note the similarity between this figure and the GCS figure above. The curve shape is due to the selection (low angular momentum stars can only enter the selected radial ring if they are very elliptical and therefore have large radial action) and the density gradient in angular momentum is due to the falling surface density of the disk. We can also look at the distribution of radial and vertical actions.

```
>>> bovy_plot.bovy_plot(jr,jz,'k,',xlabel=r'$J_R$',ylabel=r'$J_z$',xrange=[0.,.4],
yrange=[0.,0.2],onedhists=True)
```



With the other methods in the actionAngle module we can also calculate frequencies and angles.

### 1.7 Three-dimensional disk distribution functions

galpy contains a fully three-dimensional disk distribution: galpy.df. quasiisothermaldf, which is an approximately isothermal distribution function expressed in terms of action-angle variables (see 2010MNRAS.401.2318B and 2011MNRAS.413.1889B). Recent research shows that this distribution function provides a good model for the DF of mono-abundance sub-populations (MAPs) of the Milky Way disk (see 2013MNRAS.434..652T and 2013ApJ...779..115B). This distribution function family requires action-angle coordinates to evaluate the DF, so galpy.df.quasiisothermaldf makes heavy use of the routines in galpy.actionAngle (in particular those in galpy.actionAngleAdiabatic and galpy.actionAngle.actionAngleStaeckel).

### 1.7.1 Setting up the DF and basic properties

The quasi-isothermal DF is defined by a gravitational potential and a set of parameters describing the radial surfacedensity profile and the radial and vertical velocity dispersion as a function of radius. In addition, we have to provide an instance of a galpy. actionAngle class to calculate the actions for a given position and velocity. For example, for a galpy.potential.MWPotential2014 potential using the adiabatic approximation for the actions, we import and define the following

```
>>> from galpy.potential import MWPotential2014
>>> from galpy.actionAngle import actionAngleAdiabatic
>>> from galpy.df import quasiisothermaldf
>>> aA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
```

and then setup the quasiisothermaldf instance

```
>>> qdf= quasiisothermaldf(1./3.,0.2,0.1,1.,1.,pot=MWPotential2014,aA=aA,
\hookrightarrowutcounter=True)
```

which sets up a DF instance with a radial scale length of $R_{0} / 3$, a local radial and vertical velocity dispersion of $0.2 V_{c}\left(R_{0}\right)$ and $0.1 V_{c}\left(R_{0}\right)$, respectively, and a radial scale lengths of the velocity dispersions of $R_{0}$. cutcounter=True specifies that counter-rotating stars are explicitly excluded (normally these are just exponentially suppressed). As for the two-dimensional disk DFs, these parameters are merely input (or target) parameters; the true density and velocity dispersion profiles calculated by evaluating the relevant moments of the DF (see below) are not exactly exponential and have scale lengths and local normalizations that deviate slightly from these input parameters. We can estimate the DF's actual radial scale length near $R_{0}$ as

```
>>> qdf.estimate_hr(1.)
0.32908034635647182
```

which is quite close to the input value of $1 / 3$. Similarly, we can estimate the scale lengths of the dispersions

```
>>> qdf.estimate_hsr(1.)
1.1913935820372923
>>> qdf.estimate_hsz(1.)
1.0506918075359255
```

The vertical profile is fully specified by the velocity dispersions and radial density / dispersion profiles under the assumption of dynamical equilibrium. We can estimate the scale height of this DF at a given radius and height as follows

```
>>> qdf.estimate_hz(1.,0.125)
0.021389597757156088
```

Near the mid-plane this vertical scale height becomes very large because the vertical profile flattens, e.g.,

```
>>> qdf.estimate_hz(1.,0.125/100.)
1.006386030587223
```

or even

```
>>> qdf.estimate_hz(1.,0.)
187649.98447377066
```

which is basically infinity.

### 1.7.2 Evaluating moments

We can evaluate various moments of the DF giving the density, mean velocities, and velocity dispersions. For example, the mean radial velocity is again everywhere zero because the potential and the DF are axisymmetric

```
>>> qdf.meanvR(1.,0.)
0.0
```

Likewise, the mean vertical velocity is everywhere zero

```
>>> qdf.meanvz(1.,0.)
0.0
```

The mean rotational velocity has a more interesting dependence on position. Near the plane, this is the same as that calculated for a similar two-dimensional disk DF (see Evaluating moments of the DF)

```
>>> qdf.meanvT(1.,0.)
0.91988346380781227
```

However, this value decreases as one moves further from the plane. The quasiisothermaldf allows us to calculate the average rotational velocity as a function of height above the plane. For example,

```
>>> zs= numpy.linspace(0.,0.25,21)
>>> mvts= numpy.array([qdf.meanvT(1.,z) for z in zs])
```

which gives

```
>>> plot(zs,mvts)
```



We can also calculate the second moments of the DF. We can check whether the radial and velocity dispersions at $R_{0}$ are close to their input values

```
>>> numpy.sqrt(qdf.sigmaR2(1.,0.))
0.20807112565801389
```

(continued from previous page)

```
>>> numpy.sqrt(qdf.sigmaz2(1.,0.))
```

0.090453510526130904
and they are pretty close. We can also calculate the mixed $R$ and $z$ moment, for example,

```
>>> qdf.sigmaRz(1.,0.125)
0.0
```

or expressed as an angle (the tilt of the velocity ellipsoid)

```
>>> qdf.tilt(1.,0.125)
0.0
```

This tilt is zero because we are using the adiabatic approximation. As this approximation assumes that the motions in the plane are decoupled from the vertical motions of stars, the mixed moment is zero. However, this approximation is invalid for stars that go far above the plane. By using the Staeckel approximation to calculate the actions, we can model this coupling better. Setting up a quasiisothermaldf instance with the Staeckel approximation

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.45,c=True)
>>> qdfS= quasiisothermaldf(1./3.,0.2,0.1,1.,1.,pot=MWPotential2014,aA=aAS,
\hookrightarrowutcounter=True)
```

we can similarly calculate the tilt

```
>>> qdfS.tilt(1.,0.125)
5.9096430410862419
```

or about 5 degrees. As a function of height, we find

```
>>> tilts= numpy.array([qdfS.tilt(1.,z) for z in zs])
>>> plot(zs,tilts)
```

which gives


We can also calculate the density and surface density (the zero-th velocity moments). For example, the vertical density

```
>>> densz= numpy.array([qdf.density(1.,z) for z in zs])
```

and

```
>>> denszS= numpy.array([qdfS.density(1.,z) for z in zs])
```

We can compare the vertical profiles calculated using the adiabatic and Staeckel action-angle approximations

```
>>> semilogy(zs,densz/densz[0])
>>> semilogy(zs,denszS/denszS[0])
```

which gives


Similarly, we can calculate the radial profile of the surface density

```
>>> rs= numpy.linspace(0.5,1.5,21)
>>> surfr= numpy.array([qdf.surfacemass_z(r) for r in rs])
>>> surfrS= numpy.array([qdfS.surfacemass_z(r) for r in rs])
```

and compare them with each other and an exponential with scale length $1 / 3$

```
>>> semilogy(rs,surfr/surfr[10])
>>> semilogy(rs,surfrS/surfrS[10])
>>> semilogy(rs,numpy.exp(-(rs-1.)/(1./3.)))
```

which gives


The two radial profiles are almost indistinguishable and are very close, if somewhat shallower, than the pure exponential profile.
General velocity moments, including all higher order moments, are implemented in quasiisothermaldf. vmomentdensity.

### 1.7.3 Evaluating and sampling the full probability distribution function

We can evaluate the distribution itself by calling the object, e.g.,

```
>>> qdf(1.,0.1,1.1,0.1,0.) #input: R,vR,vT,z,vz
array([ 16.86790643])
```

or as a function of rotational velocity, for example in the mid-plane

```
>>> vts= numpy.linspace(0.,1.5,101)
>>> pvt= numpy.array([qdfS(1.,0.,vt,0.,0.) for vt in vts])
>>> plot(vts,pvt/numpy.sum(pvt)/(vts[1]-vts[0]))
```

which gives


This is, however, not the true distribution of rotational velocities at $R=0$ and $z=0$, because it is conditioned on zero radial and vertical velocities. We can calculate the distribution of rotational velocities marginalized over the radial and vertical velocities as

```
>>> qdfS.pvT(1.,1.,0.) #input vT,R,z
14.677231196899195
```

or as a function of rotational velocity

```
>>> pvt= numpy.array([qdfS.pvT(vt,1.,0.) for vt in vts])
```

overplotting this over the previous distribution gives

```
>>> plot(vts,pvt/numpy.sum(pvt)/(vts[1]-vts[0]))
```


which is slightly different from the conditioned distribution. Similarly, we can calculate marginalized velocity probabilities ` $\mathrm{pvR}, \mathrm{pvz}, \mathrm{pvRvT}, \mathrm{pvRvz}$, and pvTvz . These are all multiplied with the density, such that marginalizing these over the remaining velocity component results in the density.

We can sample velocities at a given location using quasiisothermaldf. sampleV (there is currently no support for sampling locations from the density profile, although that is rather trivial):

```
>>> vs= qdfS.sampleV(1.,0.,n=10000)
>>> hist(vs[:,1], normed=True,histtype='step',bins=101,range=[0.,1.5])
```

gives

which shows very good agreement with the green (marginalized over $v R$ and $v z$ ) curve (as it should).

## CHAPTER 2

### 2.1 Dynamical modeling of tidal streams

galpy contains tools to model the dynamics of tidal streams, making extensive use of action-angle variables. As an example, we can model the dynamics of the following tidal stream (that of Bovy 2014; 2014ApJ...795...95B). This movie shows the disruption of a cluster on a GD-1-like orbit around the Milky Way:

The blue line is the orbit of the progenitor cluster and the black points are cluster members. The disruption is shown in an approximate orbital plane and the movie is comoving with the progenitor cluster.

Streams can be represented by simple dynamical models in action-angle coordinates. In action-angle coordinates, stream members are stripped from the progenitor cluster onto orbits specified by a set of actions $\left(J_{R}, J_{\phi}, J_{Z}\right)$, which remain constant after the stars have been stripped. This is shown in the following movie, which shows the generation of the stream in action space

The color-coding gives the angular momentum $J_{\phi}$ and the black dot shows the progenitor orbit. These actions were calculated using galpy.actionAngle.actionAngleIsochroneApprox. The points move slightly because of small errors in the action calculation (these are correlated, so the cloud of points moves coherently because of calculation errors). The same movie that also shows the actions of stars in the cluster can be found here. This shows that the actions of stars in the cluster are not conserved (because the self-gravity of the cluster is important), but that the actions of stream members freeze once they are stripped. The angle difference between stars in a stream and the progenitor increases linearly with time, which is shown in the following movie:
where the radial and vertical angle difference with respect to the progenitor (co-moving at $\left(\theta_{R}, \theta_{\phi}, \theta_{Z}\right)=(\pi, \pi, \pi)$ ) is shown for each snapshot (the color-coding gives $\theta_{\phi}$ ).

One last movie provides further insight in how a stream evolves over time. The following movie shows the evolution of the stream in the two dimensional plane of frequency and angle along the stream (that is, both are projections of the three dimensional frequencies or angles onto the angle direction along the stream). The points are color-coded by the time at which they were removed from the progenitor cluster.

It is clear that disruption happens in bursts (at pericenter passages) and that the initial frequency distribution at the time of removal does not change (much) with time. However, stars removed at larger frequency difference move away from the cluster faster, such that the end of the stream is primarily made up of stars with large frequency differences
with respect to the progenitor. This leads to a gradient in the typical orbit in the stream, and the stream is on average not on a single orbit.

### 2.1.1 Modeling streams in galpy

In galpy we can model streams using the tools in galpy.df.streamdf. We setup a streamdf instance by specifying the host gravitational potential pot=, an actionAngle method (typically galpy.actionAngle. actionAngleIsochroneApprox), a galpy.orbit. Orbit instance with the position of the progenitor, a parameter related to the velocity dispersion of the progenitor, and the time since disruption began. We first import all of the necessary modules

```
>>> from galpy.df import streamdf
>>> from galpy.orbit import Orbit
>>> from galpy.potential import LogarithmicHaloPotential
>>> from galpy.actionAngle import actionAngleIsochroneApprox
>>> from galpy.util import bovy_conversion #for unit conversions
```

setup the potential and actionAngle instances

```
>>> lp= LogarithmicHaloPotential(normalize=1.,q=0.9)
>>> aAI= actionAngleIsochroneApprox(pot=lp,b=0.8)
```

define a progenitor Orbit instance

```
>>> obs= Orbit([1.56148083,0.35081535,-1.15481504,0.88719443,-0.47713334,0.12019596])
```

and instantiate the streamdf model

```
>>> sigv= 0.365 #km/s
>>> sdf= streamdf(sigv/220.,progenitor=obs,pot=lp,aA=aAI,leading=True,nTrackChunks=11,
\hookrightarrowtdisrupt=4.5/bovy_conversion.time_in_Gyr(220.,8.))
```

for a leading stream. This runs in about half a minute on a 2011 Macbook Air.
Bovy (2014) discusses how the calculation of the track needs to be iterated for potentials where there is a large offset between the track and a single orbit. One can increase the default number of iterations by specifying nTrackIterations = in the streamdf initialization (the default is set based on the angle between the track's frequency vector and the progenitor orbit's frequency vector; you can access the number of iterations used as sdf. nTrackIterations). To check whether the track is calculated accurately, one can use the following

```
>>> sdf.plotCompareTrackAAModel()
```

which in this case gives


This displays the stream model's track in frequency offset (y axis) versus angle offset ( x axis) as the solid line; this is the track that the model should have if it is calculated correctly. The points are the frequency and angle offset calculated from the calculated track's ( $\mathbf{x}, \mathbf{v}$ ). For a properly computed track these should line up, as they do in this figure. If they do not line up, increasing nTrackIterations is necessary.
We can calculate some simple properties of the stream, such as the ratio of the largest and second-to-largest eigenvalue of the Hessian $\partial \boldsymbol{\Omega} / \partial \mathbf{J}$

```
>>> sdf.freqEigvalRatio(isotropic=True)
34.450028399901434
```

or the model's ratio of the largest and second-to-largest eigenvalue of the model frequency variance matrix

```
>>> sdf.freqEigvalRatio()
29.625538344985291
```

The fact that this ratio is so large means that an approximately one dimensional stream will form.
Similarly, we can calculate the angle between the frequency vector of the progenitor and of the model mean frequency vector

```
>>> sdf.misalignment()
-0.49526013844831596
```

which returns this angle in degrees. We can also calculate the angle between the frequency vector of the progenitor and the principal eigenvector of $\partial \boldsymbol{\Omega} / \partial \mathbf{J}$

```
>>> sdf.misalignment(isotropic=True)
1.2825116841963993
```

(the reason these are obtained by specifying isotropic=True is that these would be the ratio of the eigenvalues or the angle if we assumed that the disrupted materials action distribution were isotropic).

### 2.1.2 Calculating the average stream location (track)

We can display the stream track in various coordinate systems as follows

```
>>> sdf.plotTrack(dl='r',d2='z',interp=True, color='k',spread=2,overplot=False,lw=2.,
\hookrightarrowscaleToPhysical=True)
```

which gives

which shows the track in Galactocentric $R$ and $Z$ coordinates as well as an estimate of the spread around the track as the dash-dotted line. We can overplot the points along the track along which the $(\mathbf{x}, \mathbf{v}) \rightarrow(\boldsymbol{\Omega}, \boldsymbol{\theta})$ transformation and the track position is explicitly calculated, by turning off the interpolation

```
>>> sdf.plotTrack(d1='r',d2='z',interp=False,color='k',spread=0,overplot=True,ls='none
\hookrightarrow',marker='O',scaleToPhysical=True)
```

which gives


We can also overplot the orbit of the progenitor
>>> sdf.plotProgenitor( $\mathrm{d} 1=$ 'r', $\mathrm{d} 2==^{\prime} \mathrm{z}^{\prime}$, color='r', overplot=True, ls='--', $\hookrightarrow$ scaleToPhysical=True)
to give


We can do the same in other coordinate systems, for example $X$ and $Z$ (as in Figure 1 of Bovy 2014)

```
>>> sdf.plotTrack(d1='x',d2='z',interp=True, color='k',spread=2,overplot=False,lw=2.,
@scaleToPhysical=True)
>>> sdf.plotTrack(dl='x',d2='z',interp=False,color='k',spread=0,overplot=True,ls='none
\hookrightarrow',marker='o',scaleToPhysical=True)
>>> sdf.plotProgenitor(dl='x',d2='z', color='r',overplot=True,ls='--',
\hookrightarrowscaleToPhysical=True)
>>> xlim(12.,14.5); ylim(-3.5,7.6)
```

which gives

or we can calculate the track in observable coordinates, e.g.,

```
>>> sdf.plotTrack(d1='ll',d2='dist',interp=True,color='k',spread=2,overplot=False,
\hookrightarrowlw=2.)
>>> sdf.plotTrack(dl='ll',d2='dist',interp=False,color='k',spread=0,overplot=True,ls=
\hookrightarrow'none',marker='○')
>>> sdf.plotProgenitor(d1='ll',d2='dist',color='r',overplot=True,ls='--')
>>> xlim(155.,255.); ylim(7.5,14.8)
```

which displays


Coordinate transformations to physical coordinates are done using parameters set when initializing the sdf instance. See the help for ?streamdf for a complete list of initialization parameters.

### 2.1.3 Mock stream data generation

We can also easily generate mock data from the stream model. This uses streamdf. sample. For example,
$\ggg \operatorname{RvR}=$ sdf.sample( $\mathrm{n}=1000$ )
which returns the sampled points as a set $\left(R, v_{R}, v_{T}, Z, v_{Z}, \phi\right)$ in natural galpy coordinates. We can plot these and compare them to the track location

```
>>> sdf.plotTrack(dl='r', d2='z',interp=True,color='b',spread=2,overplot=False,lw=2.,
@scaleToPhysical=True)
>>> plot(RvR[0]*8.,RvR[3]*8.,'k.',ms=2.) #multiply by the physical distance scale
>>> xlim(12.,16.5); ylim(2.,7.6)
```

which gives


Similarly, we can generate mock data in observable coordinates

```
>>> lb= sdf.sample(n=1000,lb=True)
```

and plot it

```
>>> sdf.plotTrack(d1='ll',d2='dist',interp=True,color='b',spread=2,overplot=False,
\hookrightarrowlw=2.)
>>> plot(lb[0],lb[2],'k.',ms=2.)
>>> xlim(155.,235.); ylim(7.5,10.8)
```

which displays


We can also just generate mock stream data in frequency-angle coordinates

```
>>> mockaA= sdf.sample(n=1000,returnaAdt=True)
```

which returns a tuple with three components: an array with shape $[3, \mathrm{~N}]$ of frequency vectors $\left(\Omega_{R}, \Omega_{\phi}, \Omega_{Z}\right)$, an array with shape $[3, \mathrm{~N}]$ of angle vectors $\left(\theta_{R}, \theta_{\phi}, \theta_{Z}\right)$ and $t_{s}$, the stripping time. We can plot the vertical versus the radial frequency

```
>>> plot(mockaA[0][0],mockaA[0][2],'k.',ms=2.)
```


or we can plot the magnitude of the angle offset as a function of stripping time

```
>>> plot(mockaA[2], numpy.sqrt(numpy.sum((mockaA[1]-numpy.tile(sdf._progenitor_angle,
\hookrightarrow(1000,1)).T)**2., axis=0)),'k.',ms=2.)
```



### 2.1.4 Evaluating and marginalizing the full PDF

We can also evaluate the stream PDF, the probability of a $(\mathbf{x}, \mathbf{v})$ phase-space position in the stream. We can evaluate the PDF, for example, at the location of the progenitor

```
>>> sdf(obs.R(),obs.vR(),obs.vT(),obs.z(),obs.vz(),obs.phi())
array([-33.16985861])
```

which returns the natural log of the PDF. If we go to slightly higher in $Z$ and slightly smaller in $R$, the PDF becomes zero

```
>>> sdf(obs.R()-0.1,obs.vR(),obs.vT(),obs.z()+0.1,obs.vz(),obs.phi())
array([-inf])
```

because this phase-space position cannot be reached by a leading stream star. We can also marginalize the PDF over unobserved directions. For example, similar to Figure 10 in Bovy (2014), we can evaluate the PDF $p(X \mid Z)$ near a point on the track, say near $Z=2 \mathrm{kpc}(=0.25$ in natural units. We first find the approximate Gaussian PDF near this point, calculated from the stream track and dispersion (see above)

```
>>> meanp, varp= sdf.gaussApprox([None,None,2./8.,None,None,None])
```

where the input is a array with entries $[\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{vX}, \mathrm{vY}, \mathrm{vZ}]$ and we substitute None for directions that we want to establish the approximate PDF for. So the above expression returns an approximation to $p\left(X, Y, v_{X}, v_{Y}, v_{Z} \mid Z\right)$. This
approximation allows us to get a sense of where the PDF peaks and what its width is

```
>>> meanp[0]*8.
14.267559400127833
>>> numpy.sqrt(varp[0,0])*8.
0.04152968631186698
```

We can now evaluate the $\operatorname{PDF} p(X \mid Z)$ as a function of $X$ near the peak

```
>>> xs= numpy.linspace(-3.*numpy.sqrt(varp[0,0]),3.*numpy.sqrt(varp[0,0]),21)+meanp[0]
>>> logps= numpy.array([sdf.callMarg([x,None,2./8.,None,None,None]) for x in xs])
>>> ps= numpy.exp(logps)
```

and we normalize the PDF

```
>>> ps/= numpy.sum(ps)*(xs[1]-xs[0])*8.
```

and plot it together with the Gaussian approximation

```
>>> plot(xs*8.,ps)
>>> plot(xs*8.,1./numpy.sqrt(2.*numpy.pi)/numpy.sqrt(varp[0,0])/8.*numpy.exp(-0.5*(xs-
\hookrightarrowmeanp[0])**2./varp[0,0]))
```

which gives


Sometimes it is hard to automatically determine the closest point on the calculated track if only one phase-space
coordinate is given. For example, this happens when evaluating $p(Z \mid X)$ for $X>13 \mathrm{kpc}$ here, where there are two branches of the track in $Z$ (see the figure of the track above). In that case, we can determine the closest track point on one of the branches by hand and then provide this closest point as the basis of PDF calculations. The following example shows how this is done for the upper $Z$ branch at $X=13.5 \mathrm{kpc}$, which is near $Z=5 \mathrm{kpc}$ (Figure 10 in Bovy 2014).

```
>>> cindx= sdf.find_closest_trackpoint(13.5/8.,None,5.32/8.,None,None,None,xy=True)
```

gives the index of the closest point on the calculated track. This index can then be given as an argument for the PDF functions:

```
>>> meanp, varp= meanp, varp= sdf.gaussApprox([13.5/8.,None, None,None,None,None],
Cindx=cindx)
```

computes the approximate $p\left(Y, Z, v_{X}, v_{Y}, v_{Z} \mid X\right)$ near the upper $Z$ branch. In $Z$, this PDF has mean and dispersion

```
>>> meanp[1]*8.
5.4005530328542077
>>> numpy.sqrt(varp[1,1])*8.
0.05796023309510244
```

We can then evaluate $p(Z \mid X)$ for the upper branch as

```
>>> zs= numpy.linspace(-3.*numpy.sqrt(varp[1,1]),3.*numpy.sqrt(varp[1,1]),21)+meanp[1]
>>> logps= numpy.array([sdf.callMarg([13.5/8.,None,z,None,None,None],cindx=cindx) for
\hookrightarrowz in zs])
>>> ps= numpy.exp(logps)
>>> ps/= numpy.sum(ps)*(zs[1]-zs[0])*8.
```

and we can again plot this and the approximation

```
>>> plot(zs*8.,ps)
>>> plot(zs*8.,1./numpy.sqrt(2.*numpy.pi)/numpy.sqrt(varp[1,1])/8.*numpy.exp(-0.5*(zs-
\hookrightarrowmeanp[1])**2./varp[1,1]))
```

which gives


The approximate PDF in this case is very close to the correct PDF. When supplying the closest track point, care needs to be taken that this really is the closest track point. Otherwise the approximate PDF will not be quite correct.

## CHAPTER 3

## Library reference

### 3.1 Orbit

See Orbit initialization for a detailed explanation on how to set up Orbit instances.

### 3.1.1 Class

galpy.orbit.Orbit

### 3.1.2 Methods

galpy.orbit.Orbit.__add__
galpy.orbit.Orbit.__call__
galpy.orbit.Orbit.bb
galpy.orbit.Orbit.dec
galpy.orbit.Orbit.dist
galpy.orbit.Orbit.E
galpy.orbit.Orbit.e
galpy.orbit.Orbit.ER
galpy.orbit.Orbit.Ez
galpy.orbit.Orbit.fit

```
galpy.orbit.Orbit.flip
galpy.orbit.Orbit.integrate
galpy.orbit.Orbit.integrate_dxdv
Currently only supported for planarOrbit instances.
galpy.orbit.Orbit.getOrbit
galpy.orbit.Orbit.getOrbit_dxdv
integrate_dxdv is currently only supported for planarOrbit instances. getOrbit_dxdv is therefore also
only supported for those types of Orbit.
galpy.orbit.Orbit.helioX
galpy.orbit.Orbit.helioY
galpy.orbit.Orbit.helioZ
galpy.orbit.Orbit.Jacobi
galpy.orbit.Orbit.jp
galpy.orbit.Orbit.jr
galpy.orbit.Orbit.jz
galpy.orbit.Orbit.II
galpy.orbit.Orbit.L
galpy.orbit.Orbit.Op
galpy.orbit.Orbit.Or
galpy.orbit.Orbit.Oz
galpy.orbit.Orbit.phi
galpy.orbit.Orbit.plot
galpy.orbit.Orbit.plot3d
galpy.orbit.Orbit.plotE
galpy.orbit.Orbit.plotER
galpy.orbit.Orbit.plotEz
```

```
galpy.orbit.Orbit.plotEzJz
galpy.orbit.Orbit.plotphi
galpy.orbit.Orbit.plotR
galpy.orbit.Orbit.plotvR
galpy.orbit.Orbit.plotvT
galpy.orbit.Orbit.plotvx
galpy.orbit.Orbit.plotvy
galpy.orbit.Orbit.plotvz
galpy.orbit.Orbit.plotx
galpy.orbit.Orbit.ploty
galpy.orbit.Orbit.plotz
galpy.orbit.Orbit.pmbb
galpy.orbit.Orbit.pmdec
galpy.orbit.Orbit.pmll
galpy.orbit.Orbit.pmra
galpy.orbit.Orbit.R
galpy.orbit.Orbit.ra
galpy.orbit.Orbit.rap
galpy.orbit.Orbit.resetaA
galpy.orbit.Orbit.rperi
galpy.orbit.Orbit.setphi
galpy.orbit.Orbit.time
galpy.orbit.Orbit.toLinear
galpy.orbit.Orbit.toPlanar
galpy.orbit.Orbit.Tp
galpy.orbit.Orbit.Tr
```

```
galpy.orbit.Orbit.TrTp
galpy.orbit.Orbit.turn_physical_off
galpy.orbit.Orbit.Tz
galpy.orbit.Orbit.U
galpy.orbit.Orbit.V
galpy.orbit.Orbit.vbb
galpy.orbit.Orbit.vdec
galpy.orbit.Orbit.vll
galpy.orbit.Orbit.vlos
galpy.orbit.Orbit.vphi
galpy.orbit.Orbit.vR
galpy.orbit.Orbit.vra
galpy.orbit.Orbit.vT
galpy.orbit.Orbit.vx
galpy.orbit.Orbit.vy
galpy.orbit.Orbit.vz
galpy.orbit.Orbit.W
galpy.orbit.Orbit.wp
galpy.orbit.Orbit.wr
galpy.orbit.Orbit.wz
galpy.orbit.Orbit.x
galpy.orbit.Orbit.y
galpy.orbit.Orbit.z
galpy.orbit.Orbit.zmax
```


### 3.2 Potential

### 3.2.1 3D potentials

## General instance routines

Use as Potential-instance.method(...)
galpy.potential.Potential. call

Potential. $\qquad$ $(R, z, p h i=0.0, t=0.0, d R=0, d p h i=0)$

NAME: __call__
PURPOSE: evaluate the potential at (R,z,phi,t)
INPUT: R - Cylindrical Galactocentric radius
z - vertical height
phi - azimuth (optional)
t-time (optional)
$d R=d p h i=$, if set to non-zero integers, return the $d R$, dphi't derivative instead
OUTPUT: $\operatorname{Phi}(\mathrm{R}, \mathrm{z}, \mathrm{t})$
HISTORY: 2010-04-16 - Written - Bovy (NYU)
galpy.potential.Potential.dens

Potential.dens ( $R, z, p h i=0.0, t=0.0$, forcepoisson $=$ False )
NAME:
dens
PURPOSE:
evaluate the density rho(R,z,t)
INPUT:
R-Cylindrical Galactocentric radius
z - vertical height
phi - azimuth (optional)
t-time (optional)
KEYWORDS:
forcepoisson= if True, calculate the density through the Poisson equation, even if an explicit expression for the density exists

OUTPUT: rho ( $\mathrm{R}, \mathrm{z}, \mathrm{phi}, \mathrm{t}$ )

HISTORY:
2010-08-08 - Written - Bovy (NYU)

## galpy.potential.Potential.dvcircdR

## Potential.dvcircdR ( $R$ )

NAME:
dvcircdR
PURPOSE:
calculate the derivative of the circular velocity at R wrt R in this potential
INPUT:
R-Galactocentric radius
OUTPUT:
derivative of the circular rotation velocity wrt R
HISTORY:
2013-01-08 - Written - Bovy (IAS)
galpy.potential.Potential.epifreq

Potential.epifreq $(R)$
NAME:
epifreq
PURPOSE:
calculate the epicycle frequency at R in this potential
INPUT:
R-Galactocentric radius
OUTPUT: epicycle frequency
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.flattening

Potential.flattening $(R, z)$
NAME:
flattening
PURPOSE:
calculate the potential flattening, defined as $\operatorname{sqrt}\left(\left|\mathbf{z} / \mathbf{R} \mathbf{F} \_\mathbf{R} / \mathbf{F} \_\mathbf{z}\right|\right)$
INPUT:
R - Galactocentric radius
z - height
OUTPUT:
flattening
HISTORY:
2012-09-13 - Written - Bovy (IAS)
galpy.potential.Potential.lindbladR

Potential.lindbladR(OmegaP, $m=2$, **kwargs)
NAME:
lindbladR
PURPOSE:
calculate the radius of a Lindblad resonance
INPUT:
OmegaP - pattern speed
$\mathbf{m}=$ order of the resonance (as in $\mathbf{m}(\mathbf{O - O p})=$ kappa (negative $\mathbf{m}$ for outer) use $m=$ 'corotation' for corotation +scipy.optimize.brentq xtol,rtol, maxiter kwargs

## OUTPUT:

 radius of Linblad resonance, None if there is no resonanceHISTORY: 2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.mass

Potential.mass ( $R, z=$ None, $t=0.0$, forceint $=$ False )
NAME:
mass
PURPOSE:
evaluate the mass enclosed
INPUT:
R-Cylindrical Galactocentric radius
$\mathrm{z}=$ (None) vertical height
t-time (optional)

## KEYWORDS:

forceint= if True, calculate the mass through integration of the density, even if an explicit expression for the mass exists

## OUTPUT:

1) for spherical potentials: $\mathrm{M}(<\mathrm{R})$ [or if z is None], when the mass is implemented explicitly, the mass enclosed within $r=\operatorname{sqrt}\left(R^{\wedge} 2+z^{\wedge} 2\right)$ is returned when not $z$ is None; forceint will integrate between $-z$ and z , so the two are inconsistent (If you care to have this changed, raise an issue on github)
2) for axisymmetric potentials: $M(<R,<|Z|)$

HISTORY: 2014-01-29 - Written - Bovy (IAS)
galpy.potential.Potential.nemo_accname

Potential.nemo_accname()
NAME: nemo_accname

PURPOSE: return the accname potential name for use of this potential with NEMO

INPUT:
(none)
OUTPUT:
Acceleration name
HISTORY:
2014-12-18 - Written - Bovy (IAS)
galpy.potential.Potential.nemo_accpars

Potential.nemo_accpars (vo, ro)
NAME:
nemo_accpars
PURPOSE:
return the accpars potential parameters for use of this potential with NEMO
INPUT:
vo - velocity unit in km/s
ro - length unit in kpc
OUTPUT: accpars string

HISTORY:
2014-12-18 - Written - Bovy (IAS)
galpy.potential.Potential.omegac

Potential.omegac ( $R$ )
NAME: omegac

PURPOSE: calculate the circular angular speed at R in this potential

INPUT:
R-Galactocentric radius
OUTPUT:
circular angular speed
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.phiforce

Potential.phiforce $(R, z, p h i=0.0, t=0.0)$
NAME:
phiforce
PURPOSE:
evaluate the azimuthal force F_phi (R,z,phi,t)
INPUT:
R-Cylindrical Galactocentric radius
z - vertical height phi - azimuth (rad)
t-time (optional)
OUTPUT:
F_phi (R,z,phi,t)
HISTORY:
2010-07-10 - Written - Bovy (NYU)
galpy.potential.Potential.phi2deriv

Potential.phi2deriv ( $R, Z, p h i=0.0, t=0.0$ )
NAME:
phi2deriv
PURPOSE:
evaluate the second azimuthal derivative
INPUT:
R - Galactocentric radius
Z - vertical height phi - Galactocentric azimuth t-time

OUTPUT: d2Phi/dphi2

## HISTORY:

2013-09-24 - Written - Bovy (IAS)

## galpy.potential.Potential.plot

Potential.plot ( $t=0.0$, rmin=0.0, rmax $=1.5$, nrs $=21$, zmin $=-0.5$, zmax=0.5, nzs=21, effective $=$ False, $L z=$ None, xrange $=$ None, yrange $=$ None, justcontours $=$ False, ncontours $=21$, savefilename=None)
NAME:
plot
PURPOSE:
plot the potential

## INPUT:

$\mathrm{t}=$ time tp plot potential at
rmin $=$ minimum R at which to calculate
$r m a x=$ maximum $R$
nrs= grid in R
$\mathrm{zmin}=$ minimum z
zmax $=$ maximum z
nzs $=\operatorname{grid}$ in z
effective $=($ False $)$ if True, plot the effective potential Phi $+\mathrm{Lz}^{\wedge} 2 / 2 / \mathrm{R}^{\wedge} 2$
$\mathrm{Lz}=$ (None) angular momentum to use for the effective potential when effective=True
ncontours - number of contours
justcontours $=$ (False) if True, just plot contours
savefilename - save to or restore from this savefile (pickle)
xrange, yrange $=$ can be specified independently from rmin,zmin, etc.
OUTPUT:
plot to output device
HISTORY:
2010-07-09 - Written - Bovy (NYU)
2014-04-08 - Added effective= - Bovy (IAS)
galpy.potential.Potential.plotDensity

Potential.plotDensity (rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, nzs=21, ncontours=21, savefilename $=$ None, aspect $=$ None, log $=$ False, justcontours $=$ False )

NAME: plotDensity
PURPOSE: plot the density of this potential
INPUT:
$r m i n=$ minimum $R$
rmax $=$ maximum R
$\mathrm{nrs}=$ grid in R
$\mathrm{zmin}=\operatorname{minimum~} \mathrm{z}$
zmax $=$ maximum z
$n z s=\operatorname{grid}$ in z
ncontours= number of contours
justcontours $=($ False $)$ if True, just plot contours
savefilename $=$ save to or restore from this savefile (pickle)
$\log =$ if True, plot the log density
OUTPUT: plot to output device
HISTORY: 2014-01-05 - Written - Bovy (IAS)

## galpy.potential.Potential.plotEscapecurve

Potential.plotEscapecurve (*args, **kwargs)
NAME:
plotEscapecurve
PURPOSE:
plot the escape velocity curve for this potential (in the $\mathrm{z}=0$ plane for non-spherical potentials)
INPUT:
Rrange - range
grid= number of points to plot
savefilename $=$ save to or restore from this savefile (pickle)
+bovy_plot(*args,**kwargs)
OUTPUT:
plot to output device
HISTORY:
2010-08-08 - Written - Bovy (NYU)
galpy.potential.Potential.plotRotcurve

Potential.plotRotcurve (*args, **kwargs)
NAME:
plotRotcurve
PURPOSE:
plot the rotation curve for this potential (in the $\mathrm{z}=0$ plane for non-spherical potentials)
INPUT:

Rrange - range
grid= number of points to plot
savefilename=- save to or restore from this savefile (pickle)

+ bovy_plot(*args,**kwargs)
OUTPUT:
plot to output device
HISTORY:
2010-07-10 - Written - Bovy (NYU)


## galpy.potential.Potential.R2deriv

Potential.R2deriv ( $R, Z, p h i=0.0, t=0.0$ )
NAME:
R2deriv
PURPOSE:
evaluate the second radial derivative
INPUT:
R-Galactocentric radius
Z - vertical height
phi - Galactocentric azimuth
t-time
OUTPUT:
d2phi/dR2
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.Rzderiv

Potential.Rzderiv ( $R, Z, p h i=0.0, t=0.0$ )
NAME:
Rzderiv
PURPOSE:
evaluate the mixed $\mathrm{R}, \mathrm{z}$ derivative
INPUT:
R-Galactocentric radius
Z - vertical height
phi - Galactocentric azimuth
t-time

## OUTPUT:

d2phi/dz/dR
HISTORY:
2013-08-26 - Written - Bovy (IAS)

## galpy.potential.Potential.Rforce

Potential.Rforce ( $R, z, p h i=0.0, t=0.0$ )
NAME:
Rforce
PURPOSE:
evaluate radial force $\mathrm{F}_{-} \mathrm{R}(\mathrm{R}, \mathrm{z})$
INPUT:
R - Cylindrical Galactocentric radius
z - vertical height
phi - azimuth (optional)
t-time (optional)
OUTPUT:
F_R (R,z,phi,t)
HISTORY:
2010-04-16 - Written - Bovy (NYU)
galpy.potential.Potential.rl

Potential.rl(lz)
NAME:
rl
PURPOSE:
calculate the radius of a circular orbit of Lz
INPUT:
lz - Angular momentum
OUTPUT:
radius
HISTORY:
2012-07-30 - Written - Bovy (IAS @MPIA)
NOTE:
seems to take about $\sim 0.5 \mathrm{~ms}$ for a Miyamoto-Nagai potential; $\sim 0.75 \mathrm{~ms}$ for a MWPotential

## galpy.planar.Potential.toPlanar

## Potential.toPlanar()

NAME: toPlanar
PURPOSE: convert a 3D potential into a planar potential in the mid-plane
INPUT: (none)
OUTPUT: planarPotential
HISTORY
galpy.potential.Potential.toVertical

Potential.toVertical ( $R$ )
NAME: toVertical
PURPOSE: convert a 3D potential into a linear (vertical) potential at R
INPUT: R - Galactocentric radius at which to create the vertical potential
OUTPUT: linear (vertical) potential
HISTORY
galpy.potential.Potential.vcirc
Potential.vcirc ( $R$ )
NAME:
vcirc
PURPOSE: calculate the circular velocity at R in this potential
INPUT:
R-Galactocentric radius
OUTPUT:
circular rotation velocity
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.verticalfreq

Potential.verticalfreq ( $R$ )
NAME:
verticalfreq
PURPOSE:
calculate the vertical frequency at R in this potential

INPUT:
R-Galactocentric radius
OUTPUT:
vertical frequency
HISTORY:
2012-07-25 - Written - Bovy (IAS @MPIA)
galpy.potential.Potential.vesc

Potential.vesc $(R)$
NAME:
vesc
PURPOSE:
calculate the escape velocity at R for this potential
INPUT:
R-Galactocentric radius
OUTPUT:
escape velocity
HISTORY: 2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.vterm

Potential.vterm (l, deg=True)
NAME:
vterm
PURPOSE: calculate the terminal velocity at 1 in this potential

INPUT:
1-Galactic longitude [deg/rad] deg= if True (default), 1 in deg

OUTPUT: terminal velocity

HISTORY: 2013-05-31 - Written - Bovy (IAS)

## galpy.potential.Potential.z2deriv

## Potential.z2deriv $(R, Z, p h i=0.0, t=0.0)$

NAME: z2deriv

PURPOSE:
evaluate the second vertical derivative
INPUT:
R-Galactocentric radius
Z - vertical height phi - Galactocentric azimuth t-time

OUTPUT: d2phi/dz2

HISTORY: 2012-07-25 - Written - Bovy (IAS @MPIA)
galpy.potential.Potential.zforce

Potential. $\mathbf{z f o r c e}(R, z, p h i=0.0, t=0.0)$
NAME: zforce

PURPOSE: evaluate the vertical force $\mathrm{F}_{\mathbf{\prime}} \mathrm{z}(\mathrm{R}, \mathrm{z}, \mathrm{t})$

INPUT:
R - Cylindrical Galactocentric radius z - vertical height phi - azimuth (optional) t-time (optional)

OUTPUT: F_z (R,z,phi,t)

HISTORY: 2010-04-16 - Written - Bovy (NYU)

In addition to these, the NFWP otential also has methods to calculate virial quantities

## galpy.potential.Potential.conc

Potential.conc (vo, ro, $H=70.0$, Om=0.3, overdens $=200.0$, wrtcrit=False)
NAME:
conc
PURPOSE:
return the concentration

## INPUT:

vo - velocity unit in $\mathrm{km} / \mathrm{s}$
ro - length unit in kpc
$\mathrm{H}=$ (default: 70) Hubble constant in $\mathrm{km} / \mathrm{s} / \mathrm{Mpc}$
Om= (default: 0.3) Omega matter overdens $=(200)$ overdensity which defines the virial radius wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density

OUTPUT:
concentration (scale/rvir)
HISTORY:
2014-04-03 - Written - Bovy (IAS)
galpy.potential.Potential.mvir

Potential.mvir (vo, ro, $H=70.0$, Om=0.3, overdens $=200.0$, wrtcrit=False, forceint=False)
NAME:
mvir
PURPOSE:
calculate the virial mass

## INPUT:

vo - velocity unit in $\mathrm{km} / \mathrm{s}$
ro - length unit in kpc
$\mathrm{H}=$ (default: 70) Hubble constant in $\mathrm{km} / \mathrm{s} / \mathrm{Mpc}$
Om= (default: 0.3) Omega matter
overdens $=(200)$ overdensity which defines the virial radius
wrtcrit $=$ (False) if True, the overdensity is wrt the critical density rather than the mean matter density
KEYWORDS:
forceint = if True, calculate the mass through integration of the density, even if an explicit expression for the mass exists

OUTPUT:
M(<rvir)

## HISTORY:

2014-09-12 - Written - Bovy (IAS)
galpy.potential.NFWPotential.rvir

NFWPotential.rvir (vo, ro, $H=70.0$, Om=0.3, overdens=200.0, wrtcrit=False)
NAME:
rvir

## PURPOSE:

calculate the virial radius for this density distribution
INPUT:
vo - velocity unit in $\mathrm{km} / \mathrm{s}$
ro - length unit in kpc
$\mathrm{H}=$ (default: 70) Hubble constant in $\mathrm{km} / \mathrm{s} / \mathrm{Mpc}$
Om= (default: 0.3) Omega matter overdens $=(200)$ overdensity which defines the virial radius wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density

OUTPUT:
virial radius in natural units
HISTORY:
2014-01-29 - Written - Bovy (IAS)

## General 3D potential routines

Use as method (...)

## galpy.potential.dvcircdR

```
galpy.potential.dvcircdR(Pot,R)
NAME:
dvcircdR
PURPOSE:
```

calculate the derivative of the circular velocity wrt R at R in potential Pot
INPUT:
Pot - Potential instance or list of such instances
R - Galactocentric radius
OUTPUT
derivative of the circular rotation velocity wrt R
HISTORY:

2013-01-08 - Written - Bovy (IAS)
galpy.potential.epifreq
galpy.potential.epifreq (Pot, $R$ )
NAME:
epifreq
PURPOSE:
calculate the epicycle frequency at R in the potential Pot
INPUT:
Pot - Potential instance or list thereof
R-Galactocentric radius
OUTPUT:
epicycle frequency
HISTORY:
2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluateDensities
galpy.potential.evaluateDensities $(R, z$, Pot, phi=0.0, $t=0.0$, forcepoisson $=$ False $)$
NAME:
evaluateDensities
PURPOSE:
convenience function to evaluate a possible sum of densities
INPUT:
R - cylindrical Galactocentric distance
z - distance above the plane
Pot - potential or list of potentials
phi - azimuth
t-time
forcepoisson= if True, calculate the density through the Poisson equation, even if an explicit expression for the density exists

OUTPUT:
rho(R,z)
HISTORY:
2010-08-08 - Written - Bovy (NYU)
2013-12-28 - Added forcepoisson - Bovy (IAS)

## galpy.potential.evaluatephiforces

galpy.potential.evaluatephiforces $(R, z$, Pot, $p h i=0.0, t=0.0)$
NAME:
evaluatephiforces
PURPOSE:
convenience function to evaluate a possible sum of potentials
INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t - time (optional)

OUTPUT:
F_phi(R,z,phi,t)
HISTORY:
2010-04-16 - Written - Bovy (NYU)
galpy.potential.evaluatePotentials
galpy.potential.evaluatePotentials $(R, z$, Pot, phi=0.0, $t=0.0, d R=0, d p h i=0)$
NAME: evaluatePotentials
PURPOSE: convenience function to evaluate a possible sum of potentials
INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - potential or list of potentials
phi - azimuth
t - time
$d R=d p h i=$, if set to non-zero integers, return the $d R$, dphi't derivative instead
OUTPUT: Phi(R,z)
HISTORY: 2010-04-16 - Written - Bovy (NYU)
galpy.potential.evaluateR2derivs
galpy.potential.evaluateR2derivs $(R, z$, Pot, $p h i=0.0, t=0.0)$
NAME: evaluateR2derivs
PURPOSE: convenience function to evaluate a possible sum of potentials

INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t- time (optional)
OUTPUT: d2Phi/d2R(R,z,phi,t)
HISTORY: 2012-07-25 - Written - Bovy (IAS)

## galpy.potential.evaluateRzderivs

galpy.potential.evaluateRzderivs ( $R, z$, Pot, phi=0.0, $t=0.0$ )
NAME: evaluateRzderivs
PURPOSE: convenience function to evaluate a possible sum of potentials
INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t - time (optional)
OUTPUT: $\mathrm{d} 2 \mathrm{Phi} / \mathrm{dz} / \mathrm{dR}(\mathrm{R}, \mathrm{z}, \mathrm{phi}, \mathrm{t})$
HISTORY: 2013-08-28 - Written - Bovy (IAS)
galpy.potential.evaluateRforces
galpy.potential.evaluateRforces ( $R, z$, Pot, phi $=0.0, t=0.0$ )
NAME: evaluateRforce
PURPOSE: convenience function to evaluate a possible sum of potentials
INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t - time (optional)
OUTPUT: F_R(R,z,phi,t)
HISTORY: 2010-04-16 - Written - Bovy (NYU)

## galpy.potential.evaluatez2derivs

## galpy.potential.evaluatez2derivs $(R, z$, Pot, $p h i=0.0, t=0.0)$

NAME: evaluatez2derivs
PURPOSE: convenience function to evaluate a possible sum of potentials
INPUT: R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t-time (optional)
OUTPUT: d2Phi/d2z(R,z,phi,t)
HISTORY: 2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluatezforces
galpy.potential.evaluatezforces ( $R, z$, Pot, phi=0.0, $t=0.0$ )
NAME:
evaluatezforces
PURPOSE:
convenience function to evaluate a possible sum of potentials
INPUT:
R - cylindrical Galactocentric distance
z - distance above the plane
Pot - a potential or list of potentials
phi - azimuth (optional)
t - time (optional)
OUTPUT:
F_z(R,z,phi,t)
HISTORY:
2010-04-16 - Written - Bovy (NYU)
galpy.potential.flattening
galpy.potential.flattening (Pot, $R, z$ )
NAME:
flattening
PURPOSE:
calculate the potential flattening, defined as $\operatorname{sqrt}\left(\left|\mathbf{z} / \mathbf{R} \mathbf{F} \_\mathbf{R} / \mathbf{F} \_\mathbf{z}\right|\right)$
INPUT:

Pot - Potential instance or list thereof
R - Galactocentric radius
z - height
OUTPUT:
flattening
HISTORY:
2012-09-13 - Written - Bovy (IAS)

## galpy.potential.lindbladR

galpy.potential.lindbladR(Pot, OmegaP, $m=2$, **kwargs)
NAME:
lindbladR
PURPOSE:
calculate the radius of a Lindblad resonance
INPUT:
Pot - Potential instance or list of such instances
OmegaP - pattern speed
$\mathbf{m}=$ order of the resonance (as in $\mathbf{m}(\mathbf{O - O p})=$ kappa (negative $\mathbf{m}$ for outer) use $m=$ 'corotation' for corotation
+scipy.optimize.brentq xtol,rtol,maxiter kwargs
OUTPUT:
radius of Linblad resonance, None if there is no resonance
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.nemo_accname
galpy.potential.nemo_accname (Pot)
NAME:
nemo_accname
PURPOSE:
return the accname potential name for use of this potential or list of potentials with NEMO
INPUT:
Pot - Potential instance or list of such instances
OUTPUT:
Acceleration name in the correct format to give to accname $=$
HISTORY:

2014-12-18 - Written - Bovy (IAS)
galpy.potential.nemo_accpars
galpy.potential.nemo_accpars(Pot, vo, ro)
NAME:
nemo_accpars
PURPOSE:
return the accpars potential parameters for use of this potential or list of potentials with NEMO
INPUT:
Pot - Potential instance or list of such instances
vo - velocity unit in $\mathrm{km} / \mathrm{s}$
ro - length unit in kpc
OUTPUT:
accpars string in the corrct format to give to accpars
HISTORY:
2014-12-18 - Written - Bovy (IAS)

## galpy.potential.omegac

galpy.potential.omegac ( $\operatorname{Pot}, R$ )
NAME:
omegac
PURPOSE:
calculate the circular angular speed velocity at R in potential Pot
INPUT:
Pot - Potential instance or list of such instances
R-Galactocentric radius
OUTPUT:
circular angular speed
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.plotDensities
galpy.potential.plotDensities (Pot, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, $n z s=21$, ncontours $=21$, savefilename $=$ None, aspect $=$ None, log=False, justcontours=False)
NAME:
plotDensities

## PURPOSE:

plot the density a set of potentials
INPUT:
Pot - Potential or list of Potential instances
rmin= minimum $R$
rmax $=$ maximum R
nrs $=$ grid in R
zmin= $\operatorname{minimum~z}$
zmax $=\operatorname{maximum~} \mathrm{z}$
nzs= grid in z
ncontours= number of contours
justcontours= (False) if True, just plot contours
savefilename= save to or restore from this savefile (pickle)
$\log =$ if True, plot the $\log$ density
OUTPUT: plot to output device
HISTORY: 2013-07-05 - Written - Bovy (IAS)
galpy.potential.plotEscapecurve
galpy.potential.plotEscapecurve (Pot, *args, **kwargs)
NAME:
plotEscapecurve
PURPOSE:
plot the escape velocity curve for this potential (in the $\mathrm{z}=0$ plane for non-spherical potentials)
INPUT:
Pot - Potential or list of Potential instances
Rrange= Range in R to consider
grid= grid in R
savefilename= save to or restore from this savefile (pickle)
+bovy_plot.bovy_plot args and kwargs
OUTPUT:
plot to output device
HISTORY:
2010-08-08 - Written - Bovy (NYU)

## galpy.potential.plotPotentials

galpy.potential.plotPotentials(Pot, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, $n z s=21$, ncontours $=21$, savefilename $=$ None, aspect $=$ None, justcontours=False)
NAME:
plotPotentials
PURPOSE:
plot a set of potentials
INPUT:
Pot - Potential or list of Potential instances
rmin $=$ minimum R
rmax $=$ maximum R
nrs= grid in R
zmin $=\operatorname{minimum~} \mathrm{z}$
zmax $=$ maximum z
$n z s=\operatorname{grid} \operatorname{in} \mathrm{z}$
ncontours= number of contours
justcontours $=($ False $)$ if True, just plot contours
savefilename $=$ save to or restore from this savefile (pickle)
OUTPUT:
plot to output device
HISTORY:
2010-07-09 - Written - Bovy (NYU)
galpy.potential.plotRotcurve
galpy.potential.plotRotcurve (Pot, *args, **kwargs)
NAME:
plotRotcurve
PURPOSE:
plot the rotation curve for this potential (in the $\mathrm{z}=0$ plane for non-spherical potentials)
INPUT:
Pot - Potential or list of Potential instances
Rrange - Range in R to consider
grid $=\operatorname{grid}$ in R
savefilename $=$ save to or restore from this savefile (pickle)
+bovy_plot.bovy_plot args and kwargs
OUTPUT:
plot to output device
HISTORY:
2010-07-10 - Written - Bovy (NYU)
galpy.potential.rl
galpy.potential.rl(Pot,lz)
NAME:
rl
PURPOSE:
calculate the radius of a circular orbit of Lz
INPUT:
Pot - Potential instance or list thereof
lz - Angular momentum
OUTPUT:
radius
HISTORY:
2012-07-30 - Written - Bovy (IAS @MPIA)
NOTE:
seems to take about $\sim 0.5 \mathrm{~ms}$ for a Miyamoto-Nagai potential; $\sim 0.75 \mathrm{~ms}$ for a MWPotential
galpy.potential.vcirc
galpy.potential.vcirc (Pot, $R$ )
NAME:
vcirc
PURPOSE:
calculate the circular velocity at R in potential Pot
INPUT:
Pot - Potential instance or list of such instances
R - Galactocentric radius
OUTPUT: circular rotation velocity

HISTORY: 2011-10-09 - Written - Bovy (IAS)

## galpy.potential.verticalfreq

galpy.potential. verticalfreq (Pot, $R$ )
NAME:
verticalfreq
PURPOSE:
calculate the vertical frequency at R in the potential Pot
INPUT:
Pot - Potential instance or list thereof
R-Galactocentric radius
OUTPUT:
vertical frequency
HISTORY:
2012-07-25 - Written - Bovy (IAS @MPIA)
galpy.potential.vesc
galpy.potential. vesc (Pot, $R$ )
NAME:
vesc
PURPOSE:
calculate the escape velocity at R for potential Pot
INPUT:
Pot - Potential instances or list thereof
R - Galactocentric radius
OUTPUT:
escape velocity
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.vterm
galpy.potential.vterm(Pot, l, deg=True)
NAME:
vterm
PURPOSE:
calculate the terminal velocity at 1 in this potential
INPUT:

Pot - Potential instance
1- Galactic longitude [deg/rad]
deg= if True (default), 1 in deg
OUTPUT:
terminal velocity
HISTORY:
2013-05-31 - Written - Bovy (IAS)

## Specific potentials

Burkert potential
class galpy.potential.BurkertPotential (amp=1.0, $a=2.0$, normalize $=$ False )
BurkertPotential.py: Potential with a Burkert density

$$
\rho(r)=\frac{a m p}{(1+r / a)\left(1+[r / a]^{2}\right)}
$$

__init__( $a m p=1.0, a=2.0$, normalize $=$ False $)$
NAME:
$\qquad$
PURPOSE:
initialize a Burkert-density potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
$\mathrm{a}=$ scale radius
normalize - if True, normalize such that $\mathbf{v c}(\mathbf{1 . , 0 .})=\mathbf{1}$., or, if given as a number, such that the force is this fraction of the force necessary to make $\mathrm{vc}(1 ., 0)=$.1 .

OUTPUT:
(none)
HISTORY:
2013-04-10 - Written - Bovy (IAS)

Double exponential disk potential
class galpy.potential.DoubleExponentialDiskPotential (amp=1.0, $h r=0.3333333333333333$, $h z=0.0625, \quad$ maxiter $=20$, tol $=0.001, \quad$ normalize $=$ False, new=True, $\quad k \operatorname{maxFac}=2.0$, glorder=10)
Class that implements the double exponential disk potential

$$
\rho(R, z)=\operatorname{amp} \exp \left(-R / h_{R}-|z| / h_{z}\right)
$$

__init__ $(a m p=1.0, h r=0.3333333333333333, h z=0.0625$, maxiter $=20$, tol $=0.001$, normalize $=$ False, new $=$ True, kmaxFac $=2.0$, glorder $=10$ )
NAME:
__init__
PURPOSE:
initialize a double-exponential disk potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
hr - disk scale-length
hz - scale-height
tol - relative accuracy of potential-evaluations
maxiter - scipy.integrate keyword
normalize - if True, normalize such that $\mathrm{vc}(1 ., 0)=$.1 ., or, if given as a number, such that the force is this fraction of the force necessary to make $\mathrm{vc}(1 ., 0)=$.1 .
OUTPUT:
DoubleExponentialDiskPotential object
HISTORY:
2010-04-16 - Written - Bovy (NYU)
2013-01-01 - Re-implemented using faster integration techniques - Bovy (IAS)

## Double power-law density spherical potential

class galpy.potential.TwoPowerSphericalPotential (amp=1.0, $\quad a=5.0, \quad a l p h a=1.5$,
beta $=3.5$, normalize $=$ False )
Class that implements spherical potentials that are derived from two-power density models

$$
\rho(r)=\frac{\operatorname{amp}}{4 \pi a^{3}} \frac{1}{(r / a)^{\alpha}(1+r / a)^{\beta-\alpha}}
$$

_init__( $a m p=1.0, a=5.0$, alpha $=1.5$, beta $=3.5$, normalize $=$ False )
NAME:
__init__
PURPOSE:
initialize a two-power-density potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
a - "scale" (in terms of Ro)
alpha - inner power
beta - outer power
normalize - if True, normalize such that $\mathrm{vc}(1 ., 0)=$.1 ., or, if given as a number, such that the force is this fraction of the force necessary to make $\mathrm{vc}(1 ., 0)=$.1 .
OUTPUT:
(none)
HISTORY:
2010-07-09 - Started - Bovy (NYU)

## Jaffe potential

```
class galpy.potential.JaffePotential(amp=1.0, a=1.0, normalize=False)
```

Class that implements the Jaffe potential

$$
\rho(r)=\frac{\operatorname{amp}}{4 \pi a^{3}} \frac{1}{(r / a)^{2}(1+r / a)^{2}}
$$

__init__( $a m p=1.0, a=1.0$, normalize $=$ False $)$
NAME:
$\qquad$
PURPOSE:
Initialize a Jaffe potential
INPUT:
amp - amplitude to be applied to the potential
a - "scale" (in terms of Ro)
normalize - if True, normalize such that $\operatorname{vc}(1 ., 0)=$.1 ., or, if given as a number, such that the force is this fraction of the force necessary to make $\mathrm{vc}(1 ., 0)=$.1 .

OUTPUT:
(none)
HISTORY: 2010-07-09 - Written - Bovy (NYU)

## Flattened Power-law potential

Flattening is in the potential as in Evans (1994) rather than in the density

```
class galpy.potential.FlattenedPowerPotential(amp=1.0, alpha=0.5, q=0.9, core=1e-08,
``` normalize=False)
Class that implements a power-law potential that is flattened in the potential (NOT the density)
\[
\Phi(R, z)=-\frac{\mathrm{amp}}{\alpha\left(R^{2}+(z / q)^{2}+\operatorname{core}^{2}\right)^{\alpha / 2}}
\]
and the same as LogarithmicHaloPotential for \(\alpha=0\)
See Figure 1 in Evans (1994) for combinations of alpha and q that correspond to positive densities
__init__(amp \(=1.0\), alpha \(=0.5, q=0.9\), core \(=1 e-08\), normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a flattened power-law potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
alpha - power
q - flattening
core - core radius
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:
(none)
HISTORY:
2013-01-09 - Written - Bovy (IAS)

\section*{Hernquist potential}
class galpy.potential. HernquistPotential (amp=1.0, \(a=1.0\), normalize \(=\) False )
Class that implements the Hernquist potential
\[
\rho(r)=\frac{\operatorname{amp}}{4 \pi a^{3}} \frac{1}{(r / a)(1+r / a)^{3}}
\]

\section*{__init__( \(a m p=1.0, a=1.0\), normalize \(=\) False \()\)}

NAME:
\(\qquad\)
PURPOSE:
Initialize a Hernquist potential
INPUT:
amp - amplitude to be applied to the potential
a - "scale" (in terms of Ro)
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

OUTPUT:
(none)

\section*{HISTORY:}

2010-07-09 - Written - Bovy (NYU)

\section*{Interpolated axisymmetric potential}

The interpRZPotential class provides a general interface to generate interpolated instances of general threedimensional, axisymmetric potentials or lists of such potentials. This interpolated potential can be used in any function where other three-dimensional galpy potentials can be used. This includes functions that use \(C\) to speed up calculations, if the interpRZPotential instance was set up with enable_c=True. Initialize as
```

>>> from galpy import potential
>>> ip= potential.interpRZPotential(potential.MWPotential,interpPot=True)

```
which sets up an interpolation of the potential itself only. The potential and all different forces and functions (dens,'vcirc", epifreq, verticalfreq, dvcircdR) are interpolated separately and one needs to specify that these need to be interpolated separately (so, for example, one needs to set interpRforce=True to interpolate the radial force, or interpvcirc=True to interpolate the circular velocity).
When points outside the grid are requested within the python code, the instance will fall back on the original (noninterpolated) potential. However, when the potential is used purely in C , like during orbit integration in C or during action-angle evaluations in \(C\), there is no way for the potential to fall back onto the original potential and nonsense or NaNs will be returned. Therefore, when using interpRZPotential in C, one must make sure that the whole relevant part of the \((R, z)\) plane is covered. One more time:

Warning: When an interpolated potential is used purely in C , like during orbit integration in C or during ac-tion-angle evaluations in C , there is no way for the potential to fall back onto the original potential and nonsense or NaNs will be returned. Therefore, when using interpRZPotential in C, one must make sure that the whole relevant part of the \((R, z)\) plane is covered.
class galpy.potential.interpRZPotential(RZPot=None, rgrid=(-4.605170185988091, 2.995732273553991, 101), zgrid=(0.0, 1.0, 101), \(\log R=\) True, interpPot=False, interpRforce=False, interpzforce \(=\) False, interpDens \(=\) False, interpvcirc=False, interpdvcircdr=False, interpepifreq=False, interpverticalfreq=False, use_c=False, enable_c=False, zsym=True, numcores \(=\) None)
Class that interpolates a given potential on a grid for fast orbit integration
__init__(RZPot=None, rgrid=(-4.605170185988091, 2.995732273553991, 101), zgrid=(0.0, 1.0, 101), \(\operatorname{logR=True,~interpPot=False,~interpRforce=False,~interpzforce=False,~interp-~}\) Dens \(=\) False, interpvcirc \(=\) False, interpdvcircdr \(=\) False, interpepifreq \(=\) False, interpverticalfreq=False, use_c=False, enable_c=False, zsym=True, numcores=None)
NAME:
\(\qquad\)
PURPOSE:
Initialize an interpRZPotential instance
INPUT:
RZPot - RZPotential to be interpolated
rgrid -R grid to be given to linspace as in \(\mathrm{rs}=\) linspace \((*\) rgrid \()\)
zgrid - z grid to be given to linspace as in \(\mathrm{zs}=\) linspace \((* \mathrm{zgrid})\)
\(\log \mathrm{R}\) - if True, rgrid is in the \(\log\) of R so logrs= linspace(*rgrid)
interpPot, interpRforce, interpzforce, interpDens, interpvcirc, interpepifreq, interpverticalfreq, interpdvcircdr= if True, interpolate these functions
use_c= use C to speed up the calculation of the grid
enable_c= enable use of C for interpolations
zsym \(=\) if True (default), the potential is assumed to be symmetric around \(\mathrm{z}=0\) (so you can use, e.g., zgrid=(0.,1.,101)).
numcores \(=\) if set to an integer, use this many cores (only used for vcirc, dvcircdR, epifreq, and verticalfreq; NOT NECESSARILY FASTER, TIME TO MAKE SURE)

\section*{OUTPUT:}
instance

\section*{HISTORY:}

2010-07-21 - Written - Bovy (NYU)
2013-01-24 - Started with new implementation - Bovy (IAS)

\section*{Interpolated SnapshotRZ potential}

This class is built on the interpRZPotential class; see the documentation of that class here for additional information on how to setup objects of the InterpSnapshotRZPotential class.
```

class galpy.potential.InterpSnapshotRZPotential(s, rgrid=(-4.605170185988091,
2.995732273553991, 101), zgrid=(0.0,
1.0, 101), interpepifreq=False,
interpverticalfreq=False, interp-
Pot=True, enable_c=True, logR=True,
zsym=True, numcores=None, naz-
imuths=4,use_pkdgrav=False)

```

Interpolated axisymmetrized potential extracted from a simulation output (see interpRZPotential and SnapshotRZPotential)
__init__(s, rgrid=(-4.605170185988091, 2.995732273553991, 101), zgrid=(0.0, 1.0, 101), interpepifreq=False, interpverticalfreq=False, interpPot=True, enable_c=True, \(\operatorname{logR}=\) True, zsym=True, numcores=None, nazimuths \(=4\), use_pkdgrav=False)
NAME:
\(\qquad\)
PURPOSE:
Initialize an InterpSnapshotRZPotential instance
INPUT:
s - a simulation snapshot loaded with pynbody
rgrid -R grid to be given to linspace as in \(\mathrm{rs}=\) linspace(*rgrid)
zgrid - z grid to be given to linspace as in \(\mathrm{zs}=\) linspace ( \({ }^{*} \mathrm{zgrid}\) )
\(\log \mathrm{R}\) - if True, rgrid is in the \(\log\) of R so logrs= linspace(*rgrid)
interpPot, interpepifreq, interpverticalfreq= if True, interpolate these functions (interpPot=True also interpolates the R and zforce)
enable_c= enable use of C for interpolations
zsym= if True (default), the potential is assumed to be symmetric around \(\mathrm{z}=0\) (so you can use, e.g., zgrid=(0.,1.,101)).
numcores \(=\) if set to an integer, use this many cores
nazimuths \(=(4)\) number of azimuths to average over
use_pkdgrav= (False) use PKDGRAV to calculate the snapshot's potential and forces (CURRENTLY NOT IMPLEMENTED)

OUTPUT: instance

HISTORY:
2013 - Written - Rok Roskar (ETH)
2014-11-24 - Edited for merging into main galpy - Bovy (IAS)

\section*{Isochrone potential}
```

class galpy.potential.IsochronePotential (amp=1.0, b=1.0, normalize=False)

```

Class that implements the Isochrone potential
\[
\Phi(r)=-\frac{a m p}{b+\sqrt{b^{2}+r^{2}}}
\]
__init__( \(a m p=1.0, b=1.0\), normalize \(=\) False \()\)
NAME:
__init__
PURPOSE:
initialize an isochrone potential
INPUT:
amp \(=\) amplitude to be applied to the potential (default: 1)
\(\mathrm{b}=\) scale radius of the isochrone potential
normalize \(=\) if True, normalize such that \(\operatorname{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

OUTPUT:
(none)
HISTORY:
2013-09-08 - Written - Bovy (IAS)

\section*{Kepler potential}
class galpy.potential.KeplerPotential (amp=1.0, normalize=False)
Class that implements the Kepler potential
\[
\Phi(r)=-\frac{a m p}{r}
\]
_init__(amp=1.0, normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a Kepler potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
alpha - inner power
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:
(none)
HISTORY:
2010-07-10 - Written - Bovy (NYU)

\section*{Kuzmin-Kutuzov Staeckel potential}
class galpy.potential.KuzminKutuzovStaeckelPotential(amp=1.0, ac=5.0, Delta=1.0, normalize=False )
Class that implements the Kuzmin-Kutuzov Staeckel potential
\[
\Phi(R, z)=-\frac{a m p}{\sqrt{\lambda}+\sqrt{\nu}}
\]
__init__( \(a m p=1.0, a c=5.0\), Delta \(=1.0\), normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a Kuzmin-Kutuzov Staeckel potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
ac - axis ratio of the coordinate surfaces; \((\mathrm{a} / \mathrm{c})=\operatorname{sqrt}(-\mathrm{alpha}) / \operatorname{sqrt}(-\) gamma \()(\) default: 5.)
Delta - focal distance that defines the spheroidal coordinate system (default: 1.); Delta=sqrt(gamma-alpha) normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:
(none)
HISTORY:
2015-02-15 - Written - Trick (MPIA)

\section*{Logarithmic halo potential}
class galpy.potential.LogarithmicHaloPotential (amp=1.0, core \(=1 e-08, q=1.0\), normalize \(=\) False )
Class that implements the logarithmic halo potential
\[
\Phi(R, z)=\frac{\mathrm{amp}}{2} \ln \left(R^{2}+(z / q)^{2}+\text { core }^{2}\right)
\]
__init__( \(a m p=1.0\), core \(=1 e-08, q=1.0\), normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a Logarithmic Halo potential

\section*{INPUT:}
amp - amplitude to be applied to the potential (default: 1)
core - core radius at which the logarithm is cut
\(\mathrm{q}-\) potential flattening \((\mathrm{z} / \mathrm{q})^{* *} 2\).
normalize - if True, normalize such that \(\operatorname{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

OUTPUT:
(none)
HISTORY:
2010-04-02 - Started - Bovy (NYU)

\section*{Miyamoto-Nagai potential}
class galpy.potential.MiyamotoNagaiPotential (amp=1.0, \(a=1.0, b=0.1\), normalize \(=\) False )
Class that implements the Miyamoto-Nagai potential
\[
\Phi(R, z)=-\frac{\mathrm{amp}}{\sqrt{R^{2}+\left(a+\sqrt{z^{2}+b^{2}}\right)^{2}}}
\]
__init__( \(a m p=1.0, a=1.0, b=0.1\), normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a Miyamoto-Nagai potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
a - "disk scale" (in terms of Ro)
b-"disk height" (in terms of Ro)
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:
(none)
HISTORY:
2010-07-09 - Started - Bovy (NYU)

\section*{Three Miyamoto-Nagai disk approximation to an exponential disk}
class galpy.potential.MN3ExponentialDiskPotential (amp=1.0,
\(h r=0.3333333333333333\),
\(h z=0.0625\), sech=False, posdens \(=\) False, normalize \(=\) False )
class that implements the three Miyamoto-Nagai approximation to a radially-exponential disk potential of Smith et al. 2015
\[
\rho(R, z)=\operatorname{amp} \exp \left(-R / h_{R}-|z| / h_{z}\right)
\]
or
\[
\rho(R, z)=\operatorname{amp} \exp \left(-R / h_{R}\right) \operatorname{sech}^{2}\left(-|z| / h_{z}\right)
\]
depending on whether sech=True or not. This density is approximated using three Miyamoto-Nagai disks __init__(amp=1.0, hr=0.3333333333333333, hz=0.0625, sech=False, posdens=False, normalize=False)
NAME:
\(\qquad\)
PURPOSE:
initialize a 3MN approximation to an exponential disk potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
hr - disk scale-length
hz - scale-height
sech \(=(\) False \()\) if True, hz is the scale height of a sech vertical profile (default is exponential vertical profile)
posdens \(=(\) False \()\) if True, allow only positive density solutions (Table 2 in Smith et al. rather than Table 1)
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:
MN3ExponentialDiskPotential object
HISTORY:
2015-02-07 - Written - Bovy (IAS)

\section*{Moving object potential}
class galpy.potential.MovingObjectPotential (orbit, amp=1.0, GM=0.06, softening=None, softening_model='plummer', softening_length=0.01)
Class that implements the potential coming from a moving object
\[
\Phi(R, z, \phi, t)=-\operatorname{amp} G M S(d)
\]
where \(d\) is the distance between \((R, z, \phi)\) and the moving object at time \(t\) and \(S(\cdot)\) is a softening kernel. In the case of Plummer softening, this kernel is
\[
S(d)=\frac{1}{\sqrt{d^{2}+\text { softening_length }^{2}}}
\]

Plummer is currently the only implemented softening.
__init__(orbit, amp=1.0, GM=0.06, softening=None, softening_model='plummer', softening_length=0.01)
NAME:
\(\qquad\)
PURPOSE:
initialize a MovingObjectPotential
INPUT:
orbit - the Orbit of the object (Orbit object)
amp \(=-\) amplitude to be applied to the potential (default: 1)
GM - 'mass' of the object (degenerate with amp)
Softening: either provide
a) softening= with a ForceSoftening-type object
b) softening_model= type of softening to use ('plummer')
softening_length= (optional)

\section*{OUTPUT:}
(none)
HISTORY:
2011-04-10 - Started - Bovy (NYU)

\section*{NFW potential}
class galpy.potential.NFWPotential \((a m p=1.0, \quad a=1.0, \quad\) normalize \(=\) False,\(\quad\) conc=None, mvir=None, \(\quad v o=220.0, \quad r o=8.0, \quad H=70.0, \quad\) Om=0.3, overdens \(=200.0\), wrtcrit=False)
Class that implements the NFW potential
\[
\rho(r)=\frac{\mathrm{amp}}{4 \pi a^{3}} \frac{1}{(r / a)(1+r / a)^{2}}
\]
__init__( \(a m p=1.0, a=1.0\), normalize \(=\) False, conc=None, mvir \(=\) None, vo=220.0, ro=8.0, \(H=70.0\), Om=0.3, overdens \(=200.0\), wrtcrit \(=\) False )
NAME:
\(\qquad\)
PURPOSE:
Initialize a NFW potential
INPUT:
amp - amplitude to be applied to the potential
a - "scale" (in terms of Ro)
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

Alternatively, NFW potentials can be initialized using
conc \(=\) concentration
mvir= virial mass in \(10^{\wedge} 12\) Msolar
in which case you also need to supply the following keywords
\(v o=(220\).\() velocity unit in \mathrm{km} / \mathrm{s}\)
ro \(=\) (8.) length unit in kpc
\(\mathrm{H}=\) (default: 70) Hubble constant in \(\mathrm{km} / \mathrm{s} / \mathrm{Mpc}\)
Om= (default: 0.3) Omega matter
overdens \(=(200)\) overdensity which defines the virial radius
wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density

OUTPUT:
(none)
HISTORY:
2010-07-09 - Written - Bovy (NYU)
2014-04-03 - Initialization w/ concentration and mass - Bovy (IAS)

\section*{Plummer potential}
class galpy.potential.PlummerPotential (amp=1.0, \(b=0.8\), normalize \(=\) False )
Class that implements the Plummer potential
\[
\Phi(R, z)=-\frac{a m p}{\sqrt{R^{2}+z^{2}+b^{2}}}
\]
__init__(amp \(=1.0, b=0.8\), normalize \(=\) False \()\)
NAME:
\(\qquad\)
PURPOSE:
initialize a Plummer potential

\section*{INPUT:}
amp - amplitude to be applied to the potential (default: 1)
b-scale parameter
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

OUTPUT:
(none)
HISTORY:
2015-06-15 - Written - Bovy (IAS)

\section*{Power-law density spherical potential}
class galpy.potential.PowerSphericalPotential (amp=1.0, alpha=1.0, normalize=False)
Class that implements spherical potentials that are derived from power-law density models
\[
\rho(r)=\operatorname{amp} \frac{3-\alpha}{4 \pi} r^{-\alpha}
\]
__init__( \(a m p=1.0\), alpha \(=1.0\), normalize \(=\) False \()\)
NAME:
__init__
PURPOSE:
initialize a power-law-density potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
alpha - inner power
normalize - if True, normalize such that \(\mathrm{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .

\section*{OUTPUT:}
(none)
HISTORY:
2010-07-10 - Written - Bovy (NYU)

Power-law density spherical potential with an exponential cut-off
class galpy.potential.PowerSphericalPotentialwCutoff(amp=1.0, alpha=1.0, \(r c=1.0\), normalize=False )
Class that implements spherical potentials that are derived from power-law density models
\[
\rho(r)=\frac{\mathrm{amp}}{r^{\alpha}} \exp \left(-(r / r c)^{2}\right)
\]
```

    init__(amp=1.0, alpha=1.0,rc=1.0, normalize=False)
    ```

NAME:
\(\qquad\)
PURPOSE:
initialize a power-law-density potential
INPUT:
amp= amplitude to be applied to the potential (default: 1)
alpha= inner power
\(\mathrm{rc}=\) cut-off radius
normalize \(=\) if True, normalize such that \(\operatorname{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\operatorname{vc}(1 ., 0)=\).1 .
OUTPUT:
(none)

\section*{HISTORY:}

2013-06-28 - Written - Bovy (IAS)

\section*{Razor-thin exponential disk potential}
class galpy.potential.RazorThinExponentialDiskPotential(amp=1.0, \(\quad r o=1.0\), \(h r=0.3333333333333333\), maxiter \(=20, \quad\) tol \(=0.001\), normalize \(=\) False, new \(=\) True, glorder \(=100\) )
Class that implements the razor-thin exponential disk potential
\[
\rho(R, z)=\operatorname{amp} \exp \left(-R / h_{R}\right) \delta(z)
\]
__init__(amp=1.0, ro=1.0, hr=0.3333333333333333, maxiter \(=20\), tol \(=0.001\), normalize \(=\) False, new=True, glorder=100)
NAME:
\(\qquad\)
PURPOSE:
initialize a razor-thin-exponential disk potential
INPUT:
amp - amplitude to be applied to the potential (default: 1)
hr - disk scale-length in terms of ro
tol - relative accuracy of potential-evaluations
maxiter - scipy.integrate keyword
normalize - if True, normalize such that \(\operatorname{vc}(1 ., 0)=\).1 ., or, if given as a number, such that the force is this fraction of the force necessary to make \(\mathrm{vc}(1 ., 0)=\).1 .
OUTPUT:

\section*{RazorThinExponentialDiskPotential object}

\section*{HISTORY:}

2012-12-27 - Written - Bovy (IAS)

\section*{Axisymmetrized N -body snapshot potential}
```

class galpy.potential.SnapshotRZPotential(s, num_threads=None, nazimuths=4)

```

Class that implements an axisymmetrized version of the potential of an N -body snapshot (requires pynbody)
_evaluate, _Rforce, and _zforce calculate a hash for the array of points that is passed in by the user. The hash and corresponding potential/force arrays are stored - if a subsequent request matches a previously computed hash, the previous results are returned and not recalculated.
__init__ ( \(s\), num_threads=None, nazimuths=4)
NAME:
\(\qquad\)
PURPOSE:
Initialize a SnapshotRZ potential object

\section*{INPUT:}
s - a simulation snapshot loaded with pynbody num_threads= \((4)\) number of threads to use for calculation nazimuths \(=(4)\) number of azimuths to average over
OUTPUT:
instance

\section*{HISTORY:}
\[
2013 \text { - Written - Rok Roskar (ETH) }
\]

2014-11-24 - Edited for merging into main galpy - Bovy (IAS)
In addition to these classes, a simple Milky-Way-like potential fit to data on the Milky Way is included as galpy. potential.MWPotential2014 (see the galpy paper for details). This potential is defined as
```

>>> bp= PowerSphericalPotentialwCutoff(alpha=1.8,rc=1.9/8., normalize=0.05)
>>> mp= MiyamotoNagaiPotential(a=3./8.,b=0.28/8.,normalize=.6)
>>> np= NFWPotential(a=16/8.,normalize=.35)
>>> MWPotential2014= [bp,mp,np]

```
and can thus be used like any list of Potentials. If one wants to add the supermassive black hole at the Galactic center, this can be done by
```

>>> from galpy.potential import KeplerPotential
>>> from galpy.util import bovy_conversion
>>> MWPotential2014.append(KeplerPotential(amp=4*10**6./bovy_conversion.mass_in_
\hookrightarrowmsol(220.,8.)))

```
for a black hole with a mass of \(4 \times 10^{6} M_{\odot}\).
As explained in this section, without this black hole MWP otential2014 can be used with Dehnen's gyrfalcON code using accname=PowSphwCut+MiyamotoNagai+NFW and accpars=0,1001.79126907,1.8,1.9\#0, \(306770.418682,3.0,0.28 \# 0,16.0,162.958241887\).

An older version galpy.potential. MWPotential of a similar potential that was not fit to data on milky Way is defined as
```

>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375, normalize=.6)
>>> np= NFWPotential(a=4.5,normalize=.35)
>>> hp= HernquistPotential(a=0.6/8, normalize=0.05)
>>> MWPotential= [mp,np,hp]

```
galpy.potential.MWPotential2014 supersedes galpy.potential.MWPotential.

\subsection*{3.2.2 2 D potentials}

\section*{General instance routines}

Use as Potential-instance.method(...)
galpy.potential.planarPotential.__call__
planarPotential.__call__ \((R, p h i=0.0, t=0.0, d R=0, d p h i=0)\)
NAME:
\(\qquad\)
PURPOSE:
evaluate the potential
INPUT:
R - Cylindrica radius
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
\(\mathrm{dR}=\), \(\mathrm{dphi}=\) if set to non-zero integers, return the dR , dphi't derivative
OUTPUT:
Phi(R(,phi,t)))
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarPotential.phiforce
planarPotential.phiforce ( \(R, p h i=0.0, t=0.0\) )
NAME:
phiforce
PURPOSE:
evaluate the phi force
INPUT:

R-Cylindrical radius
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
OUTPUT:
F_phi(R,(phi,t)))
HISTORY:
2010-07-13 - Written - Bovy (NYU)

\section*{galpy.potential.planarPotential.Rforce}
planarPotential. Rforce ( \(R\), phi=0.0, \(t=0.0\) )
NAME:
Rforce
PURPOSE:
evaluate the radial force
INPUT:
R-Cylindrical radius phi= azimuth (optional) \(\mathrm{t}=\) time (optional)
OUTPUT: F_R(R,(phi,t)))

HISTORY:
2010-07-13 - Written - Bovy (NYU)

\section*{General axisymmetric potential instance routines}

Use as Potential-instance.method(...)
galpy.potential.planarAxiPotential.epifreq

Potential.epifreq \((R)\)
NAME:
epifreq
PURPOSE:
calculate the epicycle frequency at R in this potential
INPUT:
R-Galactocentric radius
OUTPUT:
epicycle frequency

HISTORY: 2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.lindbladR

Potential.lindbladR(OmegaP, \(m=2, * * k w a r g s)\)
NAME:
lindbladR
PURPOSE:
calculate the radius of a Lindblad resonance
INPUT:
OmegaP - pattern speed
\(\mathbf{m}=\) order of the resonance (as in \(\mathbf{m}(\mathbf{O}-\mathbf{O p})=\) kappa (negative \(\mathbf{m}\) for outer) use \(m=\) 'corotation' for corotation +scipy.optimize.brentq xtol,rtol,maxiter kwargs

OUTPUT:
radius of Linblad resonance, None if there is no resonance
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.omegac

Potential.omegac ( \(R\) )
NAME:
omegac
PURPOSE:
calculate the circular angular speed at R in this potential
INPUT:
R-Galactocentric radius
OUTPUT:
circular angular speed
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.plot
planarAxiPotential.plot (*args, **kwargs)
NAME: plot
PURPOSE: plot the potential

INPUT: Rrange - range grid - number of points to plot savefilename - save to or restore from this savefile (pickle) +bovy_plot(*args,**kwargs)

OUTPUT: plot to output device
HISTORY: 2010-07-13 - Written - Bovy (NYU)

\section*{galpy.potential.planarAxiPotential.plotEscapecurve}
planarAxiPotential.plotEscapecurve (*args, **kwargs)
NAME:
plotEscapecurve
PURPOSE:
plot the escape velocity curve for this potential
INPUT:
Rrange - range
grid - number of points to plot savefilename - save to or restore from this savefile (pickle) +bovy_plot(*args,**kwargs)

OUTPUT: plot to output device
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarAxiPotential.plotRotcurve
```

planarAxiPotential.plotRotcurve(*args, **kwargs)
NAME:
plotRotcurve
PURPOSE:

```
        plot the rotation curve for this potential
    INPUT:
        Rrange - range
        grid - number of points to plot
        savefilename - save to or restore from this savefile (pickle)
        +bovy_plot(*args,**kwargs)
    OUTPUT:
        plot to output device
    HISTORY:
        2010-07-13 - Written - Bovy (NYU)

\section*{galpy.potential.planarAxiPotential.vcirc}
```

Potential.vcirc(R)

```

NAME:

\section*{vcirc}

PURPOSE:
calculate the circular velocity at R in this potential
INPUT:
R-Galactocentric radius
OUTPUT:
circular rotation velocity
HISTORY:
2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.vesc

Potential.vesc ( \(R\) )
NAME:
vesc
PURPOSE:
calculate the escape velocity at R for this potential
INPUT:
R - Galactocentric radius
OUTPUT: escape velocity

HISTORY:
2011-10-09 - Written - Bovy (IAS)

\section*{General 2D potential routines}

Use as method (. . . )
galpy.potential.evaluateplanarphiforces
galpy.potential.evaluateplanarphiforces ( \(R\), Pot, phi=None, \(t=0.0\) )
NAME:
evaluateplanarphiforces
PURPOSE:
evaluate the phiforce of a (list of) planarPotential instance(s)
INPUT:

R-Cylindrical radius
Pot - (list of) planarPotential instance(s)
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
OUTPUT:
F_phi(R(,phi,t))
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluateplanarPotentials
galpy.potential.evaluateplanarPotentials \((R\), Pot, phi=None, \(t=0.0, d R=0, d p h i=0)\)
NAME:
evaluateplanarPotentials
PURPOSE:
evaluate a (list of) planarPotential instance(s)
INPUT:
R - Cylindrical radius
Pot - (list of) planarPotential instance(s)
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
\(d R=\), dphi= if set to non-zero integers, return the \(d R\), dphi't derivative instead
OUTPUT:
Phi(R(,phi,t))
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluateplanarRforces
galpy.potential.evaluateplanarRforces \((R\), Pot, \(p h i=N o n e, t=0.0)\)
NAME:
evaluateplanarRforces
PURPOSE:
evaluate the Rforce of a (list of) planarPotential instance(s)
INPUT:
R-Cylindrical radius
Pot - (list of) planarPotential instance(s)
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
OUTPUT:
F_R(R(,phi,t))
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluateplanarR2derivs
galpy.potential.evaluateplanarR2derivs ( \(R\), Pot, phi=None, \(t=0.0\) )
NAME: evaluateplanarR2derivs
PURPOSE:
evaluate the second radial derivative of a (list of) planarPotential instance(s)
INPUT:
R-Cylindrical radius
Pot - (list of) planarPotential instance(s)
phi= azimuth (optional)
\(\mathrm{t}=\) time (optional)
OUTPUT:
F_R(R(,phi,t))
HISTORY:
2010-10-09 - Written - Bovy (IAS)
galpy.potential.LinShuReductionFactor
galpy.potential.LinShuReductionFactor(axiPot, \(R\), sigmar, nonaxiPot=None, \(k=\) None, \(m=\) None, Omega \(P=\) None \()\)
NAME:
LinShuReductionFactor
PURPOSE:
Calculate the Lin \& Shu (1966) reduction factor: the reduced linear response of a kinematically-warm stellar disk to a perturbation

\section*{INPUT:}
axiPot - The background, axisymmetric potential
R - Cylindrical radius
sigmar - radial velocity dispersion of the population
Then either provide:
1) \(m=m\) in the perturbation's \(m x\) phi (number of arms for a spiral)
\(\mathrm{k}=\) wavenumber (see Binney \& Tremaine 2008)
OmegaP= pattern speed
2) nonaxiPot= a non-axisymmetric Potential instance (such as SteadyLogSpiralPotential) that has functions that return OmegaP, m, and wavenumber

OUTPUT:
reduction factor
HISTORY:
2014-08-23 - Written - Bovy (IAS)

\section*{galpy.potential.plotplanarPotentials}
galpy.potential.plotplanarPotentials (Pot, *args, **kwargs)
NAME:
plotplanarPotentials
PURPOSE:
plot a planar potential
INPUT:
Rrange - range
xrange, yrange - if relevant
grid, gridx, gridy - number of points to plot
savefilename - save to or restore from this savefile (pickle)
ncontours - number of contours to plot (if applicable)
+bovy_plot(*args,**kwargs) or bovy_dens2d(**kwargs)
OUTPUT:
plot to output device
HISTORY:
2010-07-13 - Written - Bovy (NYU)

\section*{Specific potentials}

All of the 3D potentials above can be used as two-dimensional potentials in the mid-plane.
galpy.potential.RZToplanarPotential
```

galpy.potential.RZToplanarPotential(RZPot)
NAME:
RZToplanarPotential
PURPOSE:

```
convert an RZPotential to a planarPotential in the mid-plane ( \(\mathrm{z}=0\) )
INPUT:
RZPot - RZPotential instance or list of such instances (existing planarPotential instances are just copied to the output)

OUTPUT:
planarPotential instance(s)
HISTORY:
2010-07-13 - Written - Bovy (NYU)
In addition, a two-dimensional bar potential and a two spiral potentials are included

\section*{Dehnen bar potential}
class galpy.potential.DehnenBarPotential(amp=1.0, omegab=None, rb=None, chi=0.8, rolr \(=0.9\), barphi \(=0.4363323129985824\), tform \(=-\) 4.0, \(\quad\) tsteady \(=\) None, \(\quad\) beta \(=0.0, \quad\) alpha \(=0.01\), Af=None)
Class that implements the Dehnen bar potential (Dehnen 2000)
\[
\left.\Phi(R, \phi)=A_{b}(t) \cos \left(2\left(\phi-\Omega_{b} t\right)\right)\right) \times \begin{cases}-\left(R_{b} / R\right)^{3}, & \text { for } R \geq R_{b} \\ \left(R / R_{b}\right)^{3}-2, & \text { for } R \leq R_{b}\end{cases}
\]
where
\[
A_{b}(t)=\frac{\alpha}{3 R_{b}^{3}}\left(\frac{3}{16} \xi^{5}-\frac{5}{8} \xi^{3}+\frac{15}{16} \xi+\frac{1}{2}\right), \xi=2 \frac{t / T_{b}-t_{\text {form }}}{T_{\text {steady }}}-1, \text { if } t_{\text {form }} \leq \frac{t}{T_{b}} \leq t_{\text {form }}+T_{\text {steady }}
\]
and
\[
A_{b}(t)= \begin{cases}0, & \frac{t}{T_{b}}<t_{\text {form }} \\ \frac{\alpha}{3 R_{b}^{3}}, & \frac{t}{T_{b}}>t_{\text {form }}+T_{\text {steady }}\end{cases}
\]
where
\[
T_{b}=\frac{2 \pi}{\Omega_{b}}
\]
is the bar period.
__init__(amp=1.0, omegab=None, \(r b=N o n e, ~ c h i=0.8\), rolr=0.9, barphi=0.4363323129985824, tform \(=-4.0\), tsteady \(=\) None, beta \(=0.0\), alpha \(=0.01\), Af \(=\) None )
NAME:
\(\qquad\)
PURPOSE:
initialize a Dehnen bar potential
INPUT:
amp - amplitude to be applied to the potential (default: 1., see alpha or Ab below)
barphi - angle between sun-GC line and the bar's major axis (in rad; default=25 degree)
tform - start of bar growth / bar period (default: -4)
tsteady - time from tform at which the bar is fully grown / bar period (default: -tform/2, st the perturbation is fully grown at tform/2)
tsteady - time at which the bar is fully grown / bar period (default: tform/2)
Either provide:
a) rolr - radius of the Outer Lindblad Resonance for a circular orbit
chi - fraction R_bar / R_CR (corotation radius of bar)
alpha - relative bar strength (default: 0.01)
beta - power law index of rotation curve (to calculate OLR, etc.)
b) omegab - rotation speed of the bar
rb - bar radius
Af - bar strength

\section*{OUTPUT:}
(none)

\section*{HISTORY:}

2010-11-24 - Started - Bovy (NYU)

\section*{Cos(m phi) disk potential}

Generalization of the lopsided and elliptical disk potentials to any m.
class galpy.potential.CosmphiDiskPotential (amp=1.0, phib=0.4363323129985824, \(p=1.0, \quad\) phio \(=0.01, \quad m=1.0, \quad\) tform=None, tsteady \(=\) None, \(c p=\) None, \(s p=\) None )
Class that implements the disk potential
\[
\Phi(R, \phi)=\phi_{0} R^{p} \cos \left(m\left(\phi-\phi_{b}\right)\right)
\]

This potential can be grown between \(t_{\text {form }}\) and \(t_{\text {form }}+T_{\text {steady }}\) in a similar way as DehnenBarPotential, but times are given directly in galpy time units
__init__( \(a m p=1.0, \quad\) phib \(=0.4363323129985824, \quad p=1.0, \quad\) phio \(=0.01, \quad m=1.0, \quad t\) form=None, tsteady=None, \(c p=\) None, \(s p=\) None )
NAME:


PURPOSE:
initialize an cosmphi disk potential
\(\operatorname{phi}(\mathrm{R}, \mathrm{phi})=\) phio \((\mathrm{R} / \mathrm{Ro})^{\wedge} \mathrm{p} \cos [\mathrm{m}(\) phi- phib\()]\)

\section*{INPUT:}
amp= amplitude to be applied to the potential (default: 1.), see twophio below
tform \(=\) start of growth (to smoothly grow this potential
tsteady \(=\) time delay at which the perturbation is fully grown (default: 2.)
\(\mathrm{m}=\cos (\mathrm{m} *(\mathrm{phi}-\mathrm{phib}))\)
\(p=\) power-law index of the phi \((R)=(R / R o)^{\wedge} p\) part

Either:
a) \(\mathrm{phib}=\) angle (in rad; default \(=25\) degree )
phio \(=\) potential perturbation (in terms of phio \(/ \mathrm{vo}^{\wedge} 2\) if \(\mathrm{vo}=1\) at \(\mathrm{Ro}=1\) )
b) \(\mathrm{cp}, \mathrm{sp}=\mathrm{m} *\) phio \(* \cos (\mathrm{~m} *\) phib \(), \mathrm{m} *\) phio \(* \sin (\mathrm{~m} * \mathrm{phib})\)

OUTPUT:
(none)
HISTORY:
2011-10-27 - Started - Bovy (IAS)

\section*{Elliptical disk potential}

Like in Kuijken \& Tremaine
class galpy.potential.EllipticalDiskPotential(amp=1.0, phib=0.4363323129985824, p=1.0, twophio=0.01, tform=None, tsteady=None, \(c p=\) None, \(s p=\) None )
Class that implements the Elliptical disk potential of Kuijken \& Tremaine (1994)
\[
\Phi(R, \phi)=\phi_{0} R^{p} \cos \left(2\left(\phi-\phi_{b}\right)\right)
\]

This potential can be grown between \(t_{\text {form }}\) and \(t_{\text {form }}+T_{\text {steady }}\) in a similar way as DehnenBarPotential, but times are given directly in galpy time units
__init__(amp \(=1.0\), phib=0.4363323129985824, \(p=1.0\), twophio \(=0.01\), tform \(=\) None, tsteady \(=\) None, \(c p=\) None,\(s p=\) None)
NAME:
\(\qquad\)
PURPOSE:
initialize an Elliptical disk potential
phi \((\mathrm{R}, \mathrm{phi})=\) phio \((\mathrm{R} / \mathrm{Ro})^{\wedge} \mathrm{p} \cos [2(\) phi-phib \()]\)
INPUT:
amp= amplitude to be applied to the potential (default: 1.), see twophio below
tform \(=\) start of growth (to smoothly grow this potential
tsteady \(=\) time delay at which the perturbation is fully grown (default: 2 .)
\(p=\) power-law index of the \(p h i(R)=(R / R o)^{\wedge} p\) part
Either:
a) \(\mathrm{phib}=\) angle (in rad; default \(=25\) degree )
twophio \(=\) potential perturbation (in terms of \(2 \mathrm{phio} / \mathrm{vo}^{\wedge} 2\) if vo \(=1\) at \(\mathrm{Ro}=1\) )
b) \(\mathrm{cp}, \mathrm{sp}=\) twophio \(* \cos (2 \mathrm{phib})\), twophio \(* \sin (2 \mathrm{phib})\)

OUTPUT:
(none)
HISTORY:
2011-10-19 - Started - Bovy (IAS)

\section*{Lopsided disk potential}

Like in Kuijken \& Tremaine, but for \(\mathrm{m}=1\)
class galpy.potential.LopsidedDiskPotential (amp=1.0, phib=0.4363323129985824, \(p=1.0, \quad\) phio \(=0.01, \quad\) tform=None, tsteady=None, \(c p=\) None, \(s p=\) None \()\)
Class that implements the disk potential
\[
\Phi(R, \phi)=\phi_{0} R^{p} \cos \left(\phi-\phi_{b}\right)
\]

See documentation for CosmphiDiskPotential
__init__( \(a m p=1.0, p h i b=0.4363323129985824, p=1.0\), phio=0.01, tform=None, tsteady=None, \(c p=\) None, \(s p=\) None )
NAME:
\(\qquad\)
PURPOSE:
initialize an cosmphi disk potential
\(\operatorname{phi}(\mathrm{R}, \mathrm{phi})=\) phio \((\mathrm{R} / \mathrm{Ro})^{\wedge} \mathrm{p} \cos [\mathrm{m}(\) phi-phib \()]\)
INPUT:
amp= amplitude to be applied to the potential (default: 1.), see twophio below
tform \(=\) start of growth (to smoothly grow this potential
tsteady= time delay at which the perturbation is fully grown (default: 2.)
\(\mathrm{m}=\cos (\mathrm{m} *(\mathrm{phi}-\mathrm{phib}))\)
\(p=\) power-law index of the \(\operatorname{phi}(R)=(R / R o)^{\wedge} p\) part
Either:
a) phib \(=\) angle (in rad; default \(=25\) degree )
phio \(=\) potential perturbation (in terms of phio/vo^2 if vo \(=1\) at \(\mathrm{Ro}=1\) )
b) \(\mathrm{cp}, \mathrm{sp}=\mathrm{m} *\) phio \(* \cos (\mathrm{~m} *\) phib \(), \mathrm{m} *\) phio \(* \sin (\mathrm{~m} * \mathrm{phib})\)

OUTPUT:
(none)

\section*{HISTORY:}

2011-10-27 - Started - Bovy (IAS)

Steady-state logarithmic spiral potential
class galpy.potential.SteadyLogSpiralPotential(amp=1.0, omegas=0.65, \(\quad A=-\) 0.035, alpha=-7.0, \(\quad m=2\), gamma=0.7853981633974483, \(p=\) None, tform=None, tsteady=None)
Class that implements a steady-state spiral potential
\[
\Phi(R, \phi)=\frac{\operatorname{amp} \times A}{\alpha} \cos \left(\alpha \ln R-m\left(\phi-\Omega_{s} t-\gamma\right)\right)
\]

Can be grown in a similar way as the DehnenBarPotential, but using \(T_{s}=2 \pi / \Omega_{s}\) to normalize \(t_{\text {form }}\) and \(T_{\text {steady }}\).
__init__(amp=1.0, omegas \(=0.65, A=-0.035\), alpha \(=-7.0, m=2\), gamma \(=0.7853981633974483\), \(p=\) None, tform \(=\) None, tstead \(y=\) None )
NAME:
\(\qquad\)
PURPOSE:
initialize a logarithmic spiral potential

\section*{INPUT:}
amp - amplitude to be applied to the potential (default: 1., A below)
gamma - angle between sun-GC line and the line connecting the peak of the spiral pattern at the Solar radius (in rad; default=45 degree)

A - force amplitude (alpha*potential-amplitude; default=0.035)
omegas \(=-\) pattern speed \((\) default \(=0.65)\)
\(\mathrm{m}=\) number of arms
Either provide:
a) alpha=
b) \(\mathrm{p}=\) pitch angle ( rad )
tform - start of spiral growth / spiral period (default: -Infinity)
tsteady - time from tform at which the spiral is fully grown / spiral period (default: 2 periods)
OUTPUT:
(none)
HISTORY:
2011-03-27 - Started - Bovy (NYU)

\section*{Transient logarithmic spiral potential}
class galpy.potential.TransientLogSpiralPotential(amp=1.0, omegas=0.65, \(\quad A=-\) 0.035, alpha=-7.0, \(\quad m=2\), gamma \(=0.7853981633974483\), \(p=\) None, sigma \(=1.0\), to \(=0.0\) )
Class that implements a steady-state spiral potential
\[
\Phi(R, \phi)=\frac{\operatorname{amp}(t)}{\alpha} \cos \left(\alpha \ln R-m\left(\phi-\Omega_{s} t-\gamma\right)\right)
\]
where
\[
\operatorname{amp}(t)=\operatorname{amp} \times A \exp \left(-\frac{\left[t-t_{0}\right]^{2}}{2 \sigma^{2}}\right)
\]
__init__(amp \(=1.0\), omegas \(=0.65, A=-0.035\), alpha=-7.0, \(m=2\), gamma=0.7853981633974483, \(p=\) None, sigma \(=1.0\), to \(=0.0\) )
NAME:
\(\qquad\)
PURPOSE:
initialize a transient logarithmic spiral potential localized around to
INPUT:
amp - amplitude to be applied to the potential (default: 1., A below)
gamma - angle between sun-GC line and the line connecting the peak of the spiral pattern at the Solar radius (in rad; default=45 degree)

A - force amplitude (alpha*potential-amplitude; default=0.035)
omegas \(=-\) pattern speed \((\) default \(=0.65)\)
\(\mathrm{m}=\) number of arms
to \(=\) time at which the spiral peaks
sigma \(=\) "spiral duration" (sigma in Gaussian amplitude)
Either provide:
a) alpha=
b) \(\mathrm{p}=\) pitch angle (rad)

OUTPUT:
(none)
HISTORY:
2011-03-27 - Started - Bovy (NYU)

\subsection*{3.2.3 1D potentials}

\section*{General instance routines}

Use as Potential-instance.method(...)
galpy.potential.linearPotential.__call__
```

linearPotential.__call__( }x,t=0.0

```

NAME: \(\qquad\) call_

PURPOSE:
evaluate the potential
INPUT:
x - position
\(\mathrm{t}=\) time (optional)
OUTPUT:
Phi( \(\mathrm{x}, \mathrm{t}\) )
HISTORY:

2010-07-12 - Written - Bovy (NYU)
galpy.potential.linearPotential.force
linearPotential. force ( \(x, t=0.0\) )
NAME:
force
PURPOSE:
evaluate the force
INPUT:
x - position
\(\mathrm{t}=\) time (optional)
OUTPUT:
\(\mathrm{F}(\mathrm{x}, \mathrm{t})\)
HISTORY:
2010-07-12 - Written - Bovy (NYU)
galpy.potential.linearPotential.plot
linearPotential.plot \((t=0.0\), min \(=-15.0\), max \(=15, n s=21\), savefilename \(=\) None \()\)
NAME:
plot
PURPOSE:
plot the potential
INPUT:
t - time to evaluate the potential at
min - minimum \(x\)
max - maximum x
ns - grid in X
savefilename - save to or restore from this savefile (pickle)
OUTPUT:
plot to output device
HISTORY:
2010-07-13 - Written - Bovy (NYU)

\section*{General 1D potential routines}

Use as method (. . . )

\section*{galpy.potential.evaluatelinearForces}
galpy.potential.evaluatelinearForces ( \(x\), Pot, \(t=0.0\) )
NAME:
evaluatelinearForces
PURPOSE:
evaluate the forces due to a list of potentials
INPUT:
x - evaluate forces at this position
Pot - (list of) linearPotential instance(s)
t - time to evaluate at
OUTPUT: force \((x, t)\)

HISTORY: 2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluatelinearPotentials
galpy.potential.evaluatelinearPotentials ( \(x\), Pot, \(t=0.0\) )
NAME:
evaluatelinearPotentials
PURPOSE:
evaluate the sum of a list of potentials
INPUT:
x - evaluate potentials at this position
Pot - (list of) linearPotential instance(s)
t - time to evaluate at
OUTPUT:
\(\operatorname{pot}(x, t)\)
HISTORY:
2010-07-13 - Written - Bovy (NYU)
galpy.potential.plotlinearPotentials
galpy.potential.plotlinearPotentials (Pot, \(t=0.0\), \(\min =-15.0, \max =15\), \(n s=21\), savefilename \(=\) None )
NAME: plotlinearPotentials

PURPOSE:
plot a combination of potentials
INPUT:
t - time to evaluate potential at min - minimum \(x\)
max - maximum x
ns - grid in x
savefilename - save to or restore from this savefile (pickle)
OUTPUT:
plot to output device
HISTORY:
2010-07-13 - Written - Bovy (NYU)

\section*{Specific potentials}

Vertical Kuijken \& Gilmore potential
class galpy.potential.KGPotential ( \(K=1.15, F=0.03, D=1.8, a m p=1.0\) )
Class representing the Kuijken \& Gilmore (1989) potential
\[
\Phi(x)=\operatorname{amp}\left(K\left(\sqrt{x^{2}+D^{2}}-D\right)+F x^{2}\right)
\]
__init__( \(K=1.15, F=0.03, D=1.8, a m p=1.0)\)
NAME:
__init_
PURPOSE:
Initialize a KGPotential
INPUT:
\(\mathrm{K}=\mathrm{K}\) parameter
\(\mathrm{F}=\mathrm{F}\) parameter
\(\mathrm{D}=\mathrm{D}\) parameter
amp - an overall amplitude
OUTPUT:
instance
HISTORY:
2010-07-12 - Written - Bovy (NYU)
One-dimensional potentials can also be derived from 3D axisymmetric potentials as the vertical potential at a certain Galactocentric radius

\section*{galpy.potential.RZToverticalPotential}
```

galpy.potential.RZToverticalPotential(RZPot,R)

```

NAME:
RZToverticalPotential
PURPOSE:
convert a RZPotential to a vertical potential at a given R
INPUT:
RZPot - RZPotential instance or list of such instances
R - Galactocentric radius at which to evaluate the vertical potential
OUTPUT:
(list of) linearPotential instance(s)
HISTORY:
2010-07-21 - Written - Bovy (NYU)

\subsection*{3.3 DF}

\subsection*{3.3.1 Two-dimensional, axisymmetric disk distribution functions}

Distribution function for orbits in the plane of a galactic disk.

\section*{General instance routines}
galpy.df.diskdf.__call__
galpy.df.diskdf.asymmetricdrift
galpy.df.diskdf.kurtosisvR
galpy.df.diskdf.kurtosisvT
galpy.df.diskdf.meanvR
galpy.df.diskdf.meanvT
galpy.df.diskdf.oortA
galpy.df.diskdf.oortB
galpy.df.diskdf.oortC
galpy.df.diskdf.oortK
```

galpy.df.diskdf.sigma2surfacemass
galpy.df.diskdf.sigma2
galpy.df.diskdf.sigmaR2
galpy.df.diskdf.sigmaT2
galpy.df.diskdf.skewvR
galpy.df.diskdf.skewvT
galpy.df.diskdf.surfacemass
galpy.df.diskdf.surfacemassLOS
galpy.df.diskdf.targetSigma2
galpy.df.diskdf.targetSurfacemass
galpy.df.diskdf.targetSurfacemassLOS
galpy.df.diskdf.vmomentsurfacemass
Sampling routines
galpy.df.diskdf.sample
galpy.df.diskdf.sampledSurfacemassLOS
hhgalpy.df.diskdf.sampleLOS
galpy.df.diskdf.sampleVRVT
Specific distribution functions
Dehnen DF

```
Shu DF

\subsection*{3.3.2 Two-dimensional, non-axisymmetric disk distribution functions}

Distribution function for orbits in the plane of a galactic disk in non-axisymmetric potentials. These are calculated using the technique of Dehnen 2000, where the DF at the current time is obtained as the evolution of an initiallyaxisymmetric DF at time to in the non-axisymmetric potential until the current time.
```

General instance routines
galpy.df.evolveddiskdf.__call__
The DF of a two-dimensional, non-axisymmetric disk
galpy.df.evolveddiskdf.meanvR
galpy.df.evolveddiskdf.meanvT
galpy.df.evolveddiskdf.oortA
galpy.df.evolveddiskdf.oortB
galpy.df.evolveddiskdf.oortC
galpy.df.evolveddiskdf.oortK
galpy.df.evolveddiskdf.sigmaR2
galpy.df.evolveddiskdf.sigmaRT
galpy.df.evolveddiskdf.sigmaT2
galpy.df.evolveddiskdf.vertexdev
galpy.df.evolveddiskdf.vmomentsurfacemass

```

\subsection*{2.3.3 Three-dimensional disk distribution functions}

Distribution functions for orbits in galactic disks, including the vertical motion for stars reaching large heights above the plane. Currently only the quasi-isothermal \(D F\).

General instance routines
galpy.df.quasiisothermaldf.__call__
galpy.df.quasiisothermaldf.density
galpy.df.quasiisothermaldf.estimate_hr
galpy.df.quasiisothermaldf.estimate_hsr
galpy.df.quasiisothermaldf.estimate_hsz
galpy.df.quasiisothermaldf.estimate_hz
```

galpy.df.quasiisothermaldf.jmomentdensity
galpy.df.quasiisothermaldf.meanjr
galpy.df.quasiisothermaldf.meanjz
galpy.df.quasiisothermaldf.meanlz
galpy.df.quasiisothermaldf.meanvR
galpy.df.quasiisothermaldf.meanvT
galpy.df.quasiisothermaldf.meanvz
galpy.df.quasiisothermaldf.pvR
galpy.df.quasiisothermaldf.pvRvT
galpy.df.quasiisothermaldf.pvRvz
galpy.df.quasiisothermaldf.pvT
galpy.df.quasiisothermaldf.pvTvz
galpy.df.quasiisothermaldf.pvz
galpy.df.quasiisothermaldf.sampleV
galpy.df.quasiisothermaldf.sigmaR2
galpy.df.quasiisothermaldf.sigmaRz
galpy.df.quasiisothermaldf.sigmaT2
galpy.df.quasiisothermaldf.sigmaz2
galpy.df.quasiisothermaldf.surfacemass_z
galpy.df.quasiisothermaldf.tilt
galpy.df.quasiisothermaldf.vmomentdensity
Specific distribution functions

```

\section*{Quasi-isothermal DF}

\subsection*{3.3.4 The distribution function of a tidal stream}

From Bovy 2014; see Dynamical modeling of tidal streams.

General instance routines
galpy.df.streamdf. \(\qquad\)
\(\qquad\)

The stream DF
galpy.df.streamdf.calc_stream_lb
galpy.df.streamdf.callMarg
galpy.df.streamdf.estimateTdisrupt
galpy.df.streamdf.find_closest_trackpoint
galpy.df.streamdf.find_closest_trackpointLB
galpy.df.streamdf.freqEigvalRatio
galpy.df.streamdf.gaussApprox
galpy.df.streamdf.meanangledAngle
galpy.df.streamdf.meanOmega
galpy.df.streamdf.meantdAngle
galpy.df.streamdf.misalignment
galpy.df.streamdf.pangledAngle
galpy.df.streamdf.plotCompareTrackAAModel
galpy.df.streamdf.plotProgenitor
galpy.df.streamdf.plotTrack
galpy.df.streamdf.ptdAngle
galpy.df.streamdf.sample
galpy.df.streamdf.sigangledAngle
galpy.df.streamdf.sigOmega
galpy.df.streamdf.sigtdAngle

\section*{3.4 actionAngle}

\subsection*{3.4.1 General instance routines}

Not necessarily supported for all different types of actionAngle calculations. These have extra arguments for different actionAngle modules, so check the documentation of the module-specific functions for more info (e.g., ?actionAngleIsochrone.__call__)
galpy.actionAngle.actionAngle._call__
actionAngle. \(\qquad\) (*args, **kwargs)

NAME: \(\qquad\) call_

PURPOSE: evaluate the actions (jr,lz,jz)
INPUT:
Either:
a) \(\mathrm{R}, \mathrm{vR}, \mathrm{vT}, \mathrm{z}, \mathrm{vz}[, \mathrm{phi}]:\)
1) floats: phase-space value for single object (phi is optional)
2) numpy.ndarray: [ N\(]\) phase-space values for N objects
b) Orbit instance: initial condition used if that's it, orbit \((\mathrm{t})\) if there is a time given as well as the second argument

OUTPUT: (jr,lz,jz)
HISTORY: 2014-01-03 - Written for top level - Bovy (IAS)
galpy.actionAngle.actionAngle.actionsFreqs
galpy.actionAngle.actionAngle.actionsFreqsAngles
actionAngle.actionsFreqsAngles (*args, **kwargs)
NAME: actionsFreqsAngles
PURPOSE: evaluate the actions, frequencies, and angles (jr,lz,jz,Omegar,Omegaphi,Omegaz,angler,anglephi,anglez)
INPUT:
Either:
a) \(\mathrm{R}, \mathrm{vR}, \mathrm{vT}, \mathrm{z}, \mathrm{vz}, \mathrm{phi}\) :
1) floats: phase-space value for single object (phi needs to be specified)
2) numpy.ndarray: \([\mathrm{N}]\) phase-space values for \(N\) objects
b) Orbit instance: initial condition used if that's it, orbit \((\mathrm{t})\) if there is a time given as well as the second argument

OUTPUT: (jr,lz,jz,Omegar,Omegaphi,Omegaz,angler,anglephi,anglez)
HISTORY: 2014-01-03 - Written for top level - Bovy (IAS)

\subsection*{3.4.2 Specific actionAngle modules}

\section*{actionAnglelsochrone}
class galpy.actionAngle.actionAngleIsochrone(*args, **kwargs)
Action-angle formalism for the isochrone potential, on the Jphi, Jtheta system of Binney \& Tremaine (2008)
__init__ (*args, **kwargs)
NAME: \(\qquad\)
\(\qquad\)
PURPOSE: initialize an actionAngleIsochrone object
INPUT: Either:
\(b=\) scale parameter of the isochrone parameter
ip= instance of a IsochronePotential
OUTPUT: HISTORY: 2013-09-08 - Written - Bovy (IAS)
actionAngleSpherical
class galpy.actionAngle.actionAngleSpherical(*args, **kwargs)
Action-angle formalism for spherical potentials
__init_ (*args, **kwargs)
NAME: \(\qquad\) nit \(\qquad\)
PURPOSE: initialize an actionAngleSpherical object
INPUT: pot= a Spherical potential
OUTPUT: HISTORY:
2013-12-28 - Written - Bovy (IAS)
actionAngleAdiabatic
class galpy.actionAngle.actionAngleAdiabatic(*args, **kwargs)
Action-angle formalism for axisymmetric potentials using the adiabatic approximation
__init__ (*args, **kwargs)
NAME: __init \(\qquad\)
PURPOSE: initialize an actionAngleAdiabatic object
INPUT:
pot \(=\) potential or list of potentials (planarPotentials)
gamma \(=(\) default \(=1\).\() replace L z\) by \(L z+\) gamma \(J z\) in effective potential
OUTPUT: HISTORY:
2012-07-26 - Written - Bovy (IAS @MPIA)

\section*{actionAngleAdiabaticGrid}
class galpy.actionAngle.actionAngleAdiabaticGrid(pot=None, zmax=1.0, gamma=1.0, Rmax \(=5.0, n R=16, n E z=16, n E r=31\), \(n L z=31\), numcores \(=1, * *\) wargs)
Action-angle formalism for axisymmetric potentials using the adiabatic approximation, grid-based interpolation
__init__(pot=None, zmax=1.0, gamma=1.0, Rmax=5.0, \(n R=16, n E z=16, n E r=31\), \(n L z=31\), numcores \(=1, * *\) kwargs)

NAME: __init__
PURPOSE: initialize an actionAngleAdiabaticGrid object
INPUT:
pot \(=\) potential or list of potentials
zmax \(=\) zmax for building Ez grid
\(R \max =R \max\) for building grids
gamma \(=(\) default \(=1\).\() replace L z\) by \(L z+\) gamma \(J z\) in effective potential
\(\mathrm{nEz}=, \mathrm{nEr}=, \mathrm{nLz}, \mathrm{nR}=\) grid size
numcores \(=\) number of cpus to use to parallellize
\(\mathrm{c}=\) if True, use C to calculate actions
+scipy.integrate.quad keywords
OUTPUT: HISTORY:
2012-07-27 - Written - Bovy (IAS @MPIA)

\section*{actionAngleStaeckel}
class galpy.actionAngle.actionAngleStaeckel(*args, **kwargs)
Action-angle formalism for axisymmetric potentials using Binney (2012)'s Staeckel approximation
\(\qquad\)
\(\qquad\) (*args, **kwargs)

NAME: __init__
PURPOSE: initialize an actionAngleStaeckel object
INPUT: pot= potential or list of potentials (3D)
delta \(=\) focus
useu0 - use \(u 0\) to calculate dV (NOT recommended)
\(\mathrm{c}=\) if True, always use C for calculations
OUTPUT: HISTORY:

2012-11-27 - Written - Bovy (IAS)

\section*{actionAngleStaeckelGrid}
class galpy.actionAngle.actionAngleStaeckelGrid(pot=None, delta=None, Rmax=5.0, \(n E=25, \quad n p s i=25, \quad n L z=30\), numcores \(=1\), **kwargs)
Action-angle formalism for axisymmetric potentials using Binney (2012)'s Staeckel approximation, grid-based interpolation
__init__(pot=None, delta=None, Rmax=5.0, \(n E=25\), npsi=25, \(n L z=30\), numcores \(=1\), ***wargs)
NAME: __init__
PURPOSE: initialize an actionAngleStaeckelGrid object
INPUT: pot= potential or list of potentials
delta \(=\) focus of prolate confocal coordinate system
\(R \max =R \max\) for building grids
\(\mathrm{nE}=, \mathrm{npsi}=, \mathrm{nLz}=\) grid size
numcores \(=\) number of cpus to use to parallellize
+scipy.integrate.quad keywords

\section*{OUTPUT: HISTORY:}

2012-11-29 - Written - Bovy (IAS)

\section*{actionAnglelsochroneApprox}
```

class galpy.actionAngle.actionAngleIsochroneApprox(*args, **kwargs)

```

Action-angle formalism using an isochrone potential as an approximate potential and using a Fox \& Binney (2014?) like algorithm to calculate the actions using orbit integrations and a torus-machinery-like angle-fit to get the angles and frequencies (Bovy 2014)
\(\qquad\) _ (*args, **kwargs)

NAME: \(\qquad\)
\(\qquad\)
PURPOSE: initialize an actionAngleIsochroneApprox object

\section*{INPUT:}

Either:
\(b=\) scale parameter of the isochrone parameter
ip= instance of a IsochronePotential
\(\mathrm{aAI}=\) instance of an actionAngleIsochrone
pot= potential to calculate action-angle variables for
tintJ= (default: 100) time to integrate orbits for to estimate actions ntintJ= (default: 10000) number of time-integration points
integrate_method= (default: 'dopr54_c') integration method to use
OUTPUT: HISTORY:

2013-09-10 - Written - Bovy (IAS)

\subsection*{3.5 Utilities}

\subsection*{3.5.1 galpy.util.bovy_plot}

Various plotting routines:
galpy.util.bovy_plot.bovy_dens2d
galpy.util.bovy_plot.bovy_dens2d(X, **kwargs)
NAME:
bovy_dens2d
PURPOSE:
plot a 2 d density with optional contours
INPUT:
first argument is the density
matplotlib.pyplot.imshow keywords (see http://matplotlib.sourceforge.net/api/axes_api.html\# matplotlib.axes.Axes.imshow)
xlabel - (raw string!) x-axis label, LaTeX math mode, no \$s needed
ylabel - (raw string!) y-axis label, LaTeX math mode, no \$s needed
xrange
yrange
noaxes - don't plot any axes
overplot - if True, overplot
colorbar - if True, add colorbar
shrink= colorbar argument: shrink the colorbar by the factor (optional)
conditional - normalize each column separately (for probability densities, i.e., cntrmass=True)
Contours:
justcontours - if True, only draw contours
contours - if True, draw contours (10 by default)
levels - contour-levels
cntrmass - if True, the density is a probability and the levels are probability masses contained within the contour
entrcolors - colors for contours (single color or array)
cntrlabel - label the contours
cntrlw, cntrls - linewidths and linestyles for contour
cntrlabelsize, cntrlabelcolors, entrinline - contour arguments
cntrSmooth - use ndimage.gaussian_filter to smooth before contouring
onedhists - if True, make one-d histograms on the sides onedhistcolor - histogram color retAxes= return all Axes instances
retCont= return the contour instance

\section*{OUTPUT:}
plot to output device, Axes instances depending on input
HISTORY:
2010-03-09 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_end_print
galpy.util.bovy_plot.bovy_end_print (filename, **kwargs)
NAME:
bovy_end_print
PURPOSE:
saves the current figure(s) to filename
INPUT:
filename - filename for plot (with extension)
OPTIONAL INPUTS:
format - file-format
OUTPUT:
(none)
HISTORY:
2009-12-23 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_hist
galpy.util.bovy_plot.bovy_hist ( \(x\), xlabel=None, ylabel=None, overplot=False, **kwargs)
NAME:
bovy_hist
PURPOSE:
wrapper around matplotlib's hist function
INPUT:
x - array to histogram
xlabel - (raw string!) x-axis label, LaTeX math mode, no \$s needed
ylabel - (raw string!) y-axis label, LaTeX math mode, no \$s needed
yrange - set the \(y\)-axis range
+all pyplot.hist keywords

OUTPUT: (from the matplotlib docs: http://matplotlib.sourceforge.net/api/pyplot_api.html\#matplotlib.pyplot. hist)

The return value is a tuple ( \(n\), bins, patches) or ([n0, n1, ...], bins, [patches0, patches1,...]) if the input contains multiple data

HISTORY:
2009-12-23 - Written - Bovy (NYU)

\section*{galpy.util.bovy_plot.bovy_plot}
```

galpy.util.bovy_plot.bovy_plot(*args, **kwargs)

```

NAME:
bovy_plot
PURPOSE:
wrapper around matplotlib's plot function
INPUT:
see http://matplotlib.sourceforge.net/api/pyplot_api.html\#matplotlib.pyplot.plot
xlabel - (raw string!) x-axis label, LaTeX math mode, no \$s needed
ylabel - (raw string!) y-axis label, LaTeX math mode, no \$s needed
xrange
yrange
scatter= if True, use pyplot.scatter and its options etc.
colorbar \(=\) if True, and scatter==True, add colorbar
crange - range for colorbar of scatter==True
clabel= label for colorbar
overplot=True does not start a new figure and does not change the ranges and labels
\(\mathrm{gcf}=\) True does not start a new figure (does change the ranges and labels)
onedhists - if True, make one-d histograms on the sides
onedhistcolor, onedhistfc, onedhistec
onedhistxnormed, onedhistynormed - normed keyword for one-d histograms
onedhistxweights, onedhistyweights - weights keyword for one-d histograms
bins= number of bins for onedhists
semilog \(x=\), semilogy \(=, \log \log =\) if True, plot \(\log s\)

\section*{OUTPUT:}
plot to output device, returns what pyplot.plot returns, or 3 Axes instances if onedhists=True HISTORY:

2009-12-28 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_print
galpy.util.bovy_plot.bovy_print (fig_width=5, fig_height=5, axes_labelsize=16, text_fontsize=11, legend_fontsize \(=12, \quad x t i c k \_l a b e l s i z e=10\), ytick_labelsize \(=10\), xtick_minor_size \(=2\), ytick_minor_size \(=2\), \(x t i c k \_m a j o r \_s i z e=4, y t i c k \_m a j o r \_s i z e=4\) )
NAME:
bovy_print
PURPOSE:
setup a figure for plotting
INPUT:
fig_width - width in inches
fig_height - height in inches
axes_labelsize - size of the axis-labels
text_fontsize - font-size of the text (if any)
legend_fontsize - font-size of the legend (if any)
xtick_labelsize - size of the x-axis labels
ytick_labelsize - size of the y-axis labels
xtick_minor_size - size of the minor x-ticks
ytick_minor_size - size of the minor y-ticks
OUTPUT:
(none)
HISTORY:
2009-12-23 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_text
galpy.util.bovy_plot.bovy_text (*args, **kwargs)
NAME:
bovy_text
PURPOSE:
thin wrapper around matplotlib's text and annotate
use keywords:
```

'bottom_left=True'
'bottom_right=True'
'top_left=True'
'top_right=True'
'title=True'

```
to place the text in one of the corners or use it as the title

\section*{INPUT:}
see matplotlib's text (http://matplotlib.sourceforge.net/api/pyplot_api.html\#matplotlib.pyplot.text)
OUTPUT:
prints text on the current figure
HISTORY:
2010-01-26 - Written - Bovy (NYU)

\section*{galpy.util.bovy_plot.scatterplot}
galpy.util.bovy_plot.scatterplot ( \(x, y, * \operatorname{args}, * * k w a r g s)\)
NAME:
scatterplot
PURPOSE:
make a 'smart' scatterplot that is a density plot in high-density regions and a regular scatterplot for outliers

INPUT:
\(\mathrm{x}, \mathrm{y}\)
xlabel - (raw string!) x-axis label, LaTeX math mode, no \$s needed
ylabel - (raw string!) y-axis label, LaTeX math mode, no \$s needed
xrange
yrange
bins - number of bins to use in each dimension
weights - data-weights
aspect - aspect ratio
conditional - normalize each column separately (for probability densities, i.e., cntrmass=True)
contours - if False, don't plot contours
justcontours - if True, only draw contours, no density
cntrcolors - color of contours (can be array as for bovy_dens2d)
cntrlw, cntrls - linewidths and linestyles for contour
cntrSmooth - use ndimage.gaussian_filter to smooth before contouring
levels - contour-levels; data points outside of the last level will be individually shown (so, e.g., if this list is descending, contours and data points will be overplotted)
onedhists - if True, make one-d histograms on the sides
onedhistx - if True, make one-d histograms on the side of the \(x\) distribution
onedhisty - if True, make one-d histograms on the side of the \(y\) distribution
onedhistcolor, onedhistfc, onedhistec
onedhistxnormed, onedhistynormed - normed keyword for one-d histograms
onedhistxweights, onedhistyweights - weights keyword for one-d histograms
cmap= cmap for density plot
hist \(=\) and edges \(=-\) you can supply the histogram of the data yourself, this can be useful if you want to censor the data, both need to be set and calculated using scipy.histogramdd with the given range retAxes= return all Axes instances

\section*{OUTPUT:}
plot to output device, Axes instance(s) or not, depending on input
HISTORY:
2010-04-15 - Written - Bovy (NYU)
galpy also contains a new matplotlib projection 'galpolar' that can be used when working with older versions of matplotlib like 'polar' to create a polar plot in which the azimuth increases clockwise (like when looking at the Milky Way from the north Galactic pole). In newer versions of matplotlib, this does not work, but the 'polar' projection now supports clockwise azimuths by doing, e.g.,
```

>>> ax= pyplot.subplot(111,projection='polar')
>>> ax.set_theta_direction(-1)

```

\subsection*{3.5.2 galpy.util.bovy_conversion}

Utility functions that provide conversions between galpy's natural units and physical units. These can be used to translate galpy outputs in natural coordinates to physical units by multiplying with the appropriate function.

These could also be used to figure out the conversion between different units. For example, if you want to know how many \(\mathrm{GeV} \mathrm{cm}{ }^{-3}\) correspond to \(1 M_{\odot} \mathrm{pc}^{-3}\), you can calculate
```

>>> from galpy.util import bovy_conversion
>>> bovy_conversion.dens_in_gevcc(1.,1.)/bovy_conversion.dens_in_msolpc3(1.,1.)
37.978342941703616

```
or \(1 M_{\odot} \mathrm{pc}^{-3} \approx 40 \mathrm{GeV} \mathrm{cm}{ }^{-3}\).

\section*{Functions:}
galpy.util.bovy_conversion.dens_in_criticaldens
```

galpy.util.bovy_conversion.dens_in_criticaldens(vo,ro, H=70.0)

```

NAME:
dens_in_criticaldens

\section*{PURPOSE:}
convert density to units of the critical density
INPUT:
vo - velocity unit in km/s
ro - length unit in kpc
\(\mathrm{H}=\) (default: 70) Hubble constant in \(\mathrm{km} / \mathrm{s} / \mathrm{Mpc}\)
OUTPUT:
conversion from units where \(v o=1\). at ro \(=1\). to units of the critical density

HISTORY:
2014-01-28 - Written - Bovy (IAS)
galpy.util.bovy_conversion.dens_in_gevcc
galpy.util.bovy_conversion.dens_in_gevcc (vo, ro)
NAME:
dens_in_gevcc
PURPOSE:
convert density to \(\mathrm{GeV} / \mathrm{cm}^{\wedge} 3\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where vo \(=1\). at ro \(=1\). to \(\mathrm{GeV} / \mathrm{cm}^{\wedge} 3\)
HISTORY:
2014-06-16 - Written - Bovy (IAS)
galpy.util.bovy_conversion.dens_in_meanmatterdens
galpy.util.bovy_conversion.dens_in_meanmatterdens (vo, ro, \(H=70.0, O m=0.3\) )
NAME:
dens_in_meanmatterdens
PURPOSE:
convert density to units of the mean matter density
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
\(\mathrm{H}=\) (default: 70) Hubble constant in \(\mathrm{km} / \mathrm{s} / \mathrm{Mpc}\)
Om= (default: 0.3) Omega matter
OUTPUT:
conversion from units where vo=1. at ro=1. to units of the mean matter density
HISTORY: 2014-01-28 - Written - Bovy (IAS)

\section*{galpy.util.bovy_conversion.dens_in_msolpc3}
```

galpy.util.bovy_conversion.dens_in_msolpc3(vo,ro)

```

NAME: dens_in_msolpc3

PURPOSE:
convert density to Msolar / pc^3
INPUT:
vo - velocity unit in km/s
ro - length unit in kpc
OUTPUT:
conversion from units where \(\mathrm{vo}=1\). at \(\mathrm{ro}=1\). to Msolar/pc^3
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_2piGmsolpc2
galpy.util.bovy_conversion.force_in_2piGmsolpc2 (vo, ro)
NAME:
force_in_2piGmsolpc2
PURPOSE:
convert a force or acceleration to \(2 \mathrm{piG} \times\) Msolar / \(\mathrm{pc}^{\wedge} 2\)
INPUT:
vo - velocity unit in km/s
ro - length unit in kpc
OUTPUT:
conversion from units where \(\mathrm{vo}=1\). at \(\mathrm{ro}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_pcMyr2
galpy.util.bovy_conversion.force_in_pcMyr2 (vo, ro)
NAME:
force_in_pcMyr2
PURPOSE:
convert a force or acceleration to \(\mathrm{pc} / \mathrm{Myr}^{\wedge} 2\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where \(\mathrm{vo}=1\). at \(\mathrm{ro}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_10m13kms2
galpy.util.bovy_conversion.force_in_10m13kms2(vo, ro)
NAME:
force_in_10m13kms2
PURPOSE:
convert a force or acceleration to \(10^{\wedge}(-13) \mathrm{km} / \mathrm{s}^{\wedge} 2\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT: conversion from units where vo=1. at ro=1.

HISTORY:
2014-01-22 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_kmsMyr
galpy.util.bovy_conversion.force_in_kmsMyr (vo, ro)
NAME:
force_in_kmsMyr
PURPOSE:
convert a force or acceleration to \(\mathrm{km} / \mathrm{s} / \mathrm{Myr}\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where vo \(=1\). at \(\mathrm{ro}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)

\section*{galpy.util.bovy_conversion.freq_in_Gyr}
galpy.util.bovy_conversion.freq_in_Gyr (vo, ro)
NAME:
freq_in_Gyr
PURPOSE:
convert a frequency to \(1 / \mathrm{Gyr}\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where vo \(=1\). at ro \(=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.freq_in_kmskpc
galpy.util.bovy_conversion.freq_in_kmskpc (vo, ro)
NAME:
freq_in_kmskpc
PURPOSE:
convert a frequency to \(\mathrm{km} / \mathrm{s} / \mathrm{kpc}\)
INPUT:
vo - velocity unit in km/s
ro - length unit in kpc
OUTPUT:
conversion from units where vo=1. at ro=1.
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.surfdens_in_msolpc2
galpy.util.bovy_conversion.surfdens_in_msolpc2 (vo, ro)
NAME:
surfdens_in_msolpc2
PURPOSE:
convert a surface density to Msolar / pc^2
INPUT:

\section*{vo - velocity unit in km/s \\ ro - length unit in kpc \\ OUTPUT:}
conversion from units where \(\mathrm{vo}=1\). at \(\mathrm{ro}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.mass_in_msol
galpy.util.bovy_conversion.mass_in_msol (vo, ro)
NAME:
mass_in_msol
PURPOSE:
convert a mass to Msolar
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT: conversion from units where vo=1. at ro=1.

HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.mass_in_1010msol
galpy.util.bovy_conversion.mass_in_1010msol(vo, ro)
NAME:
mass_in_1010msol
PURPOSE:
convert a mass to \(10^{\wedge} 10 \times\) Msolar
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where vo \(=1\). at \(\mathrm{ro}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.time_in_Gyr
galpy.util.bovy_conversion.time_in_Gyr (vo, ro)
NAME:
time_in_Gyr
PURPOSE:
convert a time to Gyr
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where \(\mathrm{vo}=1\). at \(\mathrm{r} \mathrm{o}=1\).
HISTORY:
2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.velocity_in_kpcGyr
galpy.util.bovy_conversion.velocity_in_kpcGyr(vo, ro)
NAME:
velocity_in_kpcGyr
PURPOSE:
convert a velocity to \(\mathrm{kpc} / \mathrm{Gyr}\)
INPUT:
vo - velocity unit in \(\mathrm{km} / \mathrm{s}\)
ro - length unit in kpc
OUTPUT:
conversion from units where vo=1. at ro=1.
HISTORY:
2014-12-19 - Written - Bovy (IAS)

\subsection*{3.5.3 galpy.util.bovy_coords}

Various coordinate transformation routines with fairly self-explanatory names:
galpy.util.bovy_coords.cov_dvrpmllbb_to_vxyz
galpy.util.bovy_coords.cov_dvrpmllbb_to_vxyz( \(d, e_{-} d, e_{-} v r, p m l l, p m b b, c o v \_p m l l b b, l, b\), plx=False, degree \(=\) False )
NAME:
cov_dvrpmllbb_to_vxyz

\section*{PURPOSE:}
propagate distance, radial velocity, and proper motion uncertainties to Galactic coordinates
INPUT:
d - distance [kpc, as/mas for plx]
e_d - distance uncertainty [kpc, [as/mas] for plx]
e_vr - low velocity uncertainty [km/s]
pmll - proper motion in 1 ( \({ }^{\cos (b))}\) [ [as/mas]/yr ]
pmbb - proper motion in b [ [as/mas]/yr ]
cov_pmllbb - uncertainty covariance for proper motion
1-Galactic longitude
b- Galactic lattitude
KEYWORDS:
plx - if True, \(d\) is a parallax, and e_d is a parallax uncertainty
degree - if True, 1 and \(b\) are given in degree
OUTPUT:
\(\operatorname{cov}(\mathrm{vx}, \mathrm{vy}, \mathrm{vz})[3,3]\) or \([:, 3,3]\)
HISTORY:
2010-04-12 - Written - Bovy (NYU)
galpy.util.bovy_coords.cov_pmrapmdec_to_pmllpmbb
galpy.util.bovy_coords.cov_pmrapmdec_to_pmllpmbb (cov_pmradec, ra, dec, degree=False, epoch=2000.0)
NAME:
cov_pmrapmdec_to_pmllpmbb
PURPOSE:
propagate the proper motions errors through the rotation from (ra,dec) to (1,b)
INPUT:
covar_pmradec - uncertainty covariance matrix of the proper motion in ra (multplied with \(\cos (\mathrm{dec})\) ) and dec [2,2] or [:,2,2]
ra - right ascension
dec - declination
degree - if True, ra and dec are given in degrees (default=False)
epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported)
OUTPUT:
covar_pmllbb [2,2] or [:,2,2]
HISTORY:
2010-04-12 - Written - Bovy (NYU)
galpy.util.bovy_coords.cyl_to_rect
galpy.util.bovy_coords.cyl_to_rect ( \(R\), phi, \(Z\) )
NAME:
cyl_to_rect
PURPOSE:
convert from cylindrical to rectangular coordinates
INPUT:
R, phi, Z - cylindrical coordinates
OUTPUT:
X,Y,Z
HISTORY: 2011-02-23 - Written - Bovy (NYU)
galpy.util.bovy_coords.cyl_to_rect_vec
galpy.util.bovy_coords.cyl_to_rect_vec \((v r, v t, v z, p h i)\)
NAME:
cyl_to_rect_vec
PURPOSE:
transform vectors from cylindrical to rectangular coordinate vectors
INPUT:
vr - radial velocity
vt - tangential velocity
vz - vertical velocity phi - azimuth

OUTPUT:
vx,vy,vz
HISTORY:
2011-02-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.dl_to_rphi_2d
galpy.util.bovy_coords.dl_to_rphi_2d(d,l, degree=False, ro=1.0, phio=0.0)
NAME:
dl_to_rphi_2d
PURPOSE:
convert Galactic longitude and distance to Galactocentric radius and azimuth
INPUT:
d-distance
1- Galactic longitude [rad/deg if degree]
KEYWORDS:
degree \(=(\) False \(): 1\) is in degrees rather than rad
ro= (1) Galactocentric radius of the observer
phio \(=(0)\) Galactocentric azimuth of the observer [rad/deg if degree]
OUTPUT:
( \(\mathrm{R}, \mathrm{phi}\) ); phi in degree if degree
HISTORY:
2012-01-04 - Written - Bovy (IAS)
galpy.util.bovy_coords.galcencyl_to_XYZ
galpy.util.bovy_coords.galcencyl_to_XYZ (R, phi, Z, Xsun=1.0, Ysun=0.0, Zsun=0.0)
NAME:
galcencyl_to_XYZ
PURPOSE:
transform cylindrical Galactocentric coordinates to XYZ coordinates (wrt Sun)
INPUT:
R, phi, Z - Galactocentric cylindrical coordinates
OUTPUT:
X,Y,Z
HISTORY:
2011-02-23 - Written - Bovy (NYU)
galpy.util.bovy_coords.galcencyl_to_vxvyvz
galpy.util.bovy_coords.galcencyl_to_vxvyvz ( \(v R, v T, v Z, p h i, v s u n=[0.0,1.0,0.0])\) NAME:
galcencyl_to_vxvyvz
PURPOSE:
transform cylindrical Galactocentric coordinates to XYZ (wrt Sun) coordinates for velocities
INPUT:
vR - Galactocentric radial velocity
vT - Galactocentric tangential velocity
vZ - Galactocentric vertical velocity
phi - Galactocentric azimuth
vsun - velocity of the sun in the GC frame ndarray[3]

\section*{OUTPUT:}
vx,vy,vz
HISTORY:
2011-02-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.galcenrect_to_XYZ
galpy.util.bovy_coords.galcenrect_to_XYZ (X,Y,Z,Xsun=1.0, Ysun=0.0, Zsun=0.0)
NAME:
galcenrect_to_XYZ
PURPOSE:
transform rectangular Galactocentric to XYZ coordinates (wrt Sun) coordinates
INPUT:
X, Y, Z - Galactocentric rectangular coordinates
OUTPUT:
(X, Y, Z)
HISTORY:
2011-02-23 - Written - Bovy (NYU)
galpy.util.bovy_coords.galcenrect_to_vxvyvz
galpy.util.bovy_coords.galcenrect_to_vxvyvz (vXg, vYg, vZg, vsun=[0.0, 1.0, 0.0])
NAME:
galcenrect_to_vxvyvz
PURPOSE:
transform rectangular Galactocentric coordinates to XYZ coordinates (wrt Sun) for velocities
INPUT:
vXg - Galactocentric x-velocity
vYg-Galactocentric y-velocity
vZg - Galactocentric z-velocity
vsun - velocity of the sun in the GC frame ndarray[3]
OUTPUT:
\([: 3]=v x, v y, v z\)
HISTORY:
2011-02-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.lb_to_radec
galpy.util.bovy_coords.lb_to_radec (l, b, degree=False, epoch=2000.0)
NAME:
lb_to_radec

\section*{PURPOSE:}
transform from Galactic coordinates to equatorial coordinates
INPUT:
1- Galactic longitude
b- Galactic lattitude
degree - (Bool) if True, 1 and \(b\) are given in degree and ra and dec will be as well
epoch - epoch of target ra,dec (right now only 2000.0 and 1950.0 are supported)
OUTPUT:
ra,dec
For vector inputs [:2]
HISTORY:
2010-04-07 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.lb_to_radec
galpy.util.bovy_coords.lbd_to_XYZ (l,, , d, degree=False)
NAME:
lbd_to_XYZ
PURPOSE:
transform from spherical Galactic coordinates to rectangular Galactic coordinates (works with vector inputs)

INPUT:
1 - Galactic longitude (rad)
b- Galactic lattitude (rad)
d - distance (arbitrary units) degree - (bool) if True, 1 and \(b\) are in degrees
OUTPUT:
\([\mathrm{X}, \mathrm{Y}, \mathrm{Z}]\) in whatever units d was in
For vector inputs [:3]
HISTORY:
2009-10-24- Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)

\section*{galpy.util.bovy_coords.pmllpmbb_to_pmrapmdec}
galpy.util.bovy_coords.pmllpmbb_to_pmrapmdec (pmll, pmbb, \(\quad l, \quad b, \quad\) degree \(=\) False, epoch=2000.0)
NAME:
pmllpmbb_to_pmrapmdec

\section*{PURPOSE:}
rotate proper motions in (l,b) into proper motions in (ra,dec)
INPUT:
pmll - proper motion in 1 (multplied with \(\cos (\mathrm{b}))\) [mas/yr]
pmbb - proper motion in b [mas/yr]
1-Galactic longitude
b-Galactic lattitude
degree - if True, 1 and \(b\) are given in degrees (default=False)
epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported)
OUTPUT: (pmra,pmdec), for vector inputs [:,2]
HISTORY:
2010-04-07 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)

\section*{galpy.util.bovy_coords.pmrapmdec_to_pmllpmbb}
```

galpy.util.bovy_coords.pmrapmdec_to_pmllpmbb (pmra, pmdec, ra, dec, degree=False, epoch=2000.0)

```

NAME:
pmrapmdec_to_pmllpmbb
PURPOSE:
rotate proper motions in (ra,dec) into proper motions in (l,b)
INPUT:
pmra - proper motion in ra (multplied with \(\cos (\mathrm{dec})\) ) [mas/yr]
pmdec - proper motion in dec [mas/yr]
ra - right ascension
dec - declination
degree - if True, ra and dec are given in degrees (default=False)
epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported)
OUTPUT:
(pmll,pmbb) for vector inputs [:,2]
HISTORY:

2010-04-07 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.radec_to_lb
galpy.util.bovy_coords.radec_to_lb (ra, dec, degree=False, epoch=2000.0)
NAME:
radec_to_lb
PURPOSE:
transform from equatorial coordinates to Galactic coordinates
INPUT:
ra - right ascension dec-declination degree - (Bool) if True, ra and dec are given in degree and 1 and \(b\) will be as well epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported)
OUTPUT:
1,b
For vector inputs [:,2]
HISTORY:
2009-11-12 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.rectgal_to_sphergal
galpy.util.bovy_coords.rectgal_to_sphergal ( \(X, Y, Z, v x, v y, v z\), degree \(=\) False \()\)
NAME:
rectgal_to_sphergal
PURPOSE:
transform phase-space coordinates in rectangular Galactic coordinates to spherical Galactic coordinates (can take vector inputs)

INPUT:
X - component towards the Galactic Center (kpc)
Y - component in the direction of Galactic rotation (kpc)
Z - component towards the North Galactic Pole (kpc)
vx - velocity towards the Galactic Center (km/s)
vy - velocity in the direction of Galactic rotation ( \(\mathrm{km} / \mathrm{s}\) )
vz - velocity towards the North Galactic Pole (km/s)
degree - (Bool) if True, return 1 and \(b\) in degrees
OUTPUT:
(l,b,d,vr,pmll,pmbb) in (rad,rad,kpc,km/s,mas/yr,mas/yr)
HISTORY:
2009-10-25 - Written - Bovy (NYU)
galpy.util.bovy_coords.rect_to_cyl
galpy.util.bovy_coords.rect_to_cyl \((X, Y, Z)\)
NAME:
rect_to_cyl
PURPOSE:
convert from rectangular to cylindrical coordinates
INPUT:
X, Y, Z - rectangular coordinates
OUTPUT:
R,phi,z
HISTORY:
2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.rect_to_cyl_vec
galpy.util.bovy_coords.rect_to_cyl_vec ( \(v x, v y, v z, X, Y, Z, c y l=F a l s e)\)
NAME:
rect_to_cyl_vec
PURPOSE:
transform vectors from rectangular to cylindrical coordinates vectors
INPUT:
vx -
vy -
vz -
X - X
Y - Y
Z - Z
cyl - if True, \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}\) are already cylindrical
OUTPUT:
vR,vT,vz
HISTORY:
2010-09-24 - Written - Bovy (NYU)

\section*{galpy.util.bovy_coords.rphi_to_dl_2d}
galpy.util.bovy_coords.rphi_to_dl_2d (R, phi, degree=False, ro=1.0, phio=0.0)
NAME:
rphi_to_dl_2d
PURPOSE:
convert Galactocentric radius and azimuth to distance and Galactic longitude
INPUT:
R - Galactocentric radius phi - Galactocentric azimuth [rad/deg if degree]

\section*{KEYWORDS:}
degree \(=(\) False \():\) phi is in degrees rather than rad ro= (1) Galactocentric radius of the observer phio \(=(0)\) Galactocentric azimuth of the observer [rad/deg if degree]

OUTPUT:
(d,l); phi in degree if degree
HISTORY:
2012-01-04 - Written - Bovy (IAS)
galpy.util.bovy_coords.Rz_to_coshucosv
galpy.util.bovy_coords.Rz_to_coshucosv( \(R, z\), delta=1.0)
NAME:
Rz_to_coshucosv
PURPOSE:
calculate prolate confocal \(\cosh (\mathrm{u})\) and \(\cos (\mathrm{v})\) coordinates from \(\mathrm{R}, \mathrm{z}\), and delta
INPUT:
R - radius
z - height
delta \(=\) focus
OUTPUT:
( \(\cosh (\mathrm{u}), \cos (\mathrm{v}))\)
HISTORY:
2012-11-27 - Written - Bovy (IAS)
galpy.util.bovy_coords.Rz_to_uv
galpy.util.bovy_coords.Rz_to_uv ( \(R, z\), delta=1.0)
NAME:
Rz_to_uv
PURPOSE:
calculate prolate confocal \(u\) and \(v\) coordinates from \(\mathrm{R}, \mathrm{z}\), and delta
INPUT:
R - radius
z - height
delta= focus
OUTPUT:
(u,v)
HISTORY:
2012-11-27 - Written - Bovy (IAS)
galpy.util.bovy_coords.sphergal_to_rectgal
galpy.util.bovy_coords.sphergal_to_rectgal (l, b, d, vr, pmll, pmbb, degree=False)
NAME:
sphergal_to_rectgal
PURPOSE:
transform phase-space coordinates in spherical Galactic coordinates to rectangular Galactic coordinates (can take vector inputs)
INPUT:
1- Galactic longitude (rad)
b- Galactic lattitude (rad)
d - distance (kpc)
vr - line-of-sight velocity (km/s)
pmll - proper motion in the Galactic longitude direction (mu_l* \(\cos (b)\) ) (mas/yr)
pmbb - proper motion in the Galactic lattitude (mas \(/ \mathrm{yr}\) ) degree - (bool) if True, 1 and \(b\) are in degrees
OUTPUT:
( \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}, \mathrm{vx}, \mathrm{vy}, \mathrm{vz}\) ) in ( \(\mathrm{kpc}, \mathrm{kpc}, \mathrm{kpc}, \mathrm{km} / \mathrm{s}, \mathrm{km} / \mathrm{s}, \mathrm{km} / \mathrm{s}\) )
HISTORY:
2009-10-25 - Written - Bovy (NYU)
galpy.util.bovy_coords.uv_to_Rz
galpy.util.bovy_coords.uv_to_Rz(u, v, delta=1.0)
NAME:
uv_to_Rz
PURPOSE:
calculate R and z from prolate confocal u and v coordinates
INPUT:
u - confocal u
v - confocal v
delta \(=\) focus
OUTPUT:
(R,z)
HISTORY:
2012-11-27 - Written - Bovy (IAS)
galpy.util.bovy_coords.vrpmllpmbb_to_vxvyvz
galpy.util.bovy_coords.vrpmllpmbb_to_vxvyvz(vr, pmil, pmbb, \(l, b, d, X Y Z=F a l s e, d e-\) gree=False )
NAME:
vrpmllpmbb_to_vxvyvz
PURPOSE:
Transform velocities in the spherical Galactic coordinate frame to the rectangular Galactic coordinate frame (can take vector inputs)

INPUT:
vr - line-of-sight velocity (km/s)
pmll - proper motion in the Galactic longitude (mu_1 * \(\cos (\mathrm{b})\) )(mas/yr)
pmbb - proper motion in the Galactic lattitude (mas/yr)
1-Galactic longitude
b- Galactic lattitude
d - distance (kpc)
XYZ - (bool) If True, then \(1, \mathrm{~b}, \mathrm{~d}\) is actually \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}\) (rectangular Galactic coordinates)
degree - (bool) if True, 1 and \(b\) are in degrees
OUTPUT:
( \(\mathrm{vx}, \mathrm{vy}, \mathrm{vz}\) ) in (km/s,km/s,km/s)
For vector inputs \([; 3]\)
HISTORY:

2009-10-24 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.vxvyvz_to_galcencyl
galpy.util.bovy_coords.vxvyvz_to_galcencyl ( \(v x, v y, v z, X, Y, Z, v s u n=[0.0,1.0,0.0]\), galcen=False)
NAME:
vxvyvz_to_galcencyl
PURPOSE:
transform velocities in XYZ coordinates (wrt Sun) to cylindrical Galactocentric coordinates for velocities

INPUT:
\(v x-U\)
vy - V
\(v z-W\)
X - X in Galactocentric rectangular coordinates
Y - Y in Galactocentric rectangular coordinates
\(\mathrm{Z}-\mathrm{Z}\) in Galactocentric rectangular coordinates
vsun - velocity of the sun in the GC frame ndarray[3]
galcen - if True, then \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}\) are in cylindrical Galactocentric coordinates rather than rectangular coordinates

OUTPUT:
vRg, vTg, vZg
HISTORY:
2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.vxvyvz_to_galcenrect
galpy.util.bovy_coords.vxvyvz_to_galcenrect ( \(v x, v y, v z, v s u n=[0.0,1.0,0.0]\) )
NAME:
vxvyvz_to_galcenrect
PURPOSE:
transform velocities in XYZ coordinates (wrt Sun) to rectangular Galactocentric coordinates for velocities

INPUT:
vx-U
vy - V
vz - W
vsun - velocity of the sun in the GC frame ndarray[3]

OUTPUT:
\([: 3]=v X g, v Y g, v Z g\)
HISTORY:
2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.vxvyvz_to_vrpmllpmbb
galpy.util.bovy_coords.vxvyvz_to_vrpmllpmbb ( \(v x, v y, v z, l, b, d, X Y Z=F a l s e\), degree \(=\) False \()\)
NAME:
vxvyvz_to_vrpmllpmbb
PURPOSE:
Transform velocities in the rectangular Galactic coordinate frame to the spherical Galactic coordinate frame (can take vector inputs)
INPUT:
vx - velocity towards the Galactic Center (km/s)
vy - velocity in the direction of Galactic rotation ( \(\mathrm{km} / \mathrm{s}\) )
vz - velocity towards the North Galactic Pole ( \(\mathrm{km} / \mathrm{s}\) )
1-Galactic longitude
b - Galactic lattitude
d - distance (kpc)
XYZ - (bool) If True, then \(1, \mathrm{~b}, \mathrm{~d}\) is actually \(\mathrm{X}, \mathrm{Y}, \mathrm{Z}\) (rectangular Galactic coordinates)
degree - (bool) if True, 1 and \(b\) are in degrees
OUTPUT:
(vr,pmll,pmbb) in (km/s,mas/yr,mas/yr); pmll = mu_l \({ }^{1} \cos (\mathrm{~b})\)
For vector inputs [:3]
HISTORY:
2009-10-24 - Written - Bovy (NYU)
2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.XYZ_to_galcencyl
galpy.util.bovy_coords.xYZ_to_galcencyl ( \(X, Y, Z, X\) sun \(=1.0, Y \operatorname{sun}=0.0, Z \operatorname{sun}=0.0\) )
NAME:
XYZ_to_galcencyl
PURPOSE:
transform XYZ coordinates (wrt Sun) to cylindrical Galactocentric coordinates
INPUT:

X - X
Y-Y
Z - Z
OUTPUT:
R,phi,z
HISTORY:
2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.XYZ_to_galcenrect
galpy.util.bovy_coords.XYZ_to_galcenrect ( \(X, Y, Z, X \operatorname{sun}=1.0, Y \operatorname{sun}=0.0, Z \operatorname{sun}=0.0\) )
NAME:
XYZ_to_galcenrect
PURPOSE:
transform XYZ coordinates (wrt Sun) to rectangular Galactocentric coordinates
INPUT:
X - X
Y - Y
Z - Z
OUTPUT:
( \(\mathrm{Xg}, \mathrm{Yg}, \mathrm{Zg}\) )
HISTORY:
2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.XYZ_to_lbd
galpy.util.bovy_coords.XYZ_to_lbd ( \(X, Y, Z\), degree=False \()\)
NAME:
XYZ_to_lbd
PURPOSE:
transform from rectangular Galactic coordinates to spherical Galactic coordinates (works with vector inputs)

INPUT:
X - component towards the Galactic Center (in kpc; though this obviously does not matter))
Y - component in the direction of Galactic rotation (in kpc )
Z - component towards the North Galactic Pole (kpc)
degree - (Bool) if True, return 1 and \(b\) in degrees
OUTPUT:
\([1, \mathrm{~b}, \mathrm{~d}]\) in (rad or degree, rad or degree, kpc )
For vector inputs [:,3]
HISTORY:
2009-10-24 - Written - Bovy (NYU)
2014-06-14-Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)

\subsection*{3.5.4 galpy.util.bovy_ars.bovy_ars}
galpy.util.bovy_ars.bovy_ars (domain, isDomainFinite, abcissae, hx, hpx, nsamples=1, hxparams=(), maxn=100)
bovy_ars: Implementation of the Adaptive-Rejection Sampling algorithm by Gilks \& Wild (1992): Adaptive Rejection Sampling for Gibbs Sampling, Applied Statistics, 41, 337 Based on Wild \& Gilks (1993), Algorithm AS 287: Adaptive Rejection Sampling from Log-concave Density Functions, Applied Statistics, 42, 701

Input:
domain - [.,.] upper and lower limit to the domain
isDomainFinite - [.,.] is there a lower/upper limit to the domain?
abcissae - initial list of abcissae (must lie on either side of the peak in hx if the domain is unbounded
\(h x\) - function that evaluates \(h(x)=\ln g(x)\)
\(h p x\) - function that evaluates \(h p(x)=d h(x) / d x\)
nsamples - (optional) number of desired samples (default=1)
hxparams - (optional) a tuple of parameters for \(\mathrm{h}(\mathrm{x})\) and h '( x )
maxn - (optional) maximum number of updates to the hull (default=100)
Output:
list with nsamples of samples from \(\exp (\mathrm{h}(\mathrm{x}))\)
External dependencies:
math scipy scipy.stats
History: 2009-05-21 - Written - Bovy (NYU)

\section*{CHAPTER 4}

\section*{Papers using galpy}
galpy is described in detail in this publication:
- galpy: A Python Library for Galactic Dynamics, Jo Bovy (2015), Astrophys. J. Supp., 216, 29 (arXiv/1412.3451).

The following is a list of publications using galpy; please let me (bovy -at- ias.edu) know if you make use of galpy in a publication.
1. Tracing the Hercules stream around the Galaxy, Jo Bovy (2010), Astrophys. J. 725, 1676 (2010ApJ. . 725.1676B):

Uses what later became the orbit integration routines and Dehnen and Shu disk distribution functions.
2. The spatial structure of mono-abundance sub-populations of the Milky Way disk, Jo Bovy, Hans-Walter Rix, Chao Liu, et al. Employs galpy orbit integration in galpy.potential. MWPotential to characterize the orbits in the SEGUE G dwarf sample.
3. On the local dark matter density, Jo Bovy \& Scott Tremaine (2012), Astrophys. J. 756, 89 (2012ApJ. . 756...89B): Uses galpy.potential force and density routines to characterize the difference between the vertical force and the surface density at large heights above the MW midplane.
4. The Milky Way's circular velocity curve between 4 and 14 kpc from APOGEE data, Jo Bovy, Carlos Allende Prieto, Timothy Utilizes the Dehnen distribution function to inform a simple model of the velocity distribution of APOGEE stars in the Milky Way disk and to create mock data.
5. A direct dynamical measurement of the Milky Way's disk surface density profile, disk scale length, and dark matter profile at 4 Makes use of potential models, the adiabatic and Staeckel actionAngle modules, and the quasiisothermal DF to model the dynamics of the SEGUE G dwarf sample in mono-abundance bins.
6. The peculiar pulsar population of the central parsec, Jason Dexter \& Ryan M. O'Leary (2013), Astrophys. J. Lett., 783, L7 ( Uses galpy for orbit integration of pulsars kicked out of the Galactic center.
7. Chemodynamics of the Milky Way. I. The first year of APOGEE data, Friedrich Anders, Christina Chiappini, Basilio X. San Employs galpy to perform orbit integrations in galpy.potential. MWPotential to characterize the orbits of stars in the APOGEE sample.
8. Dynamical modeling of tidal streams, Jo Bovy (2014), Astrophys. J., 795, 95 (2014ApJ...795. . 95B):

Introduces galpy.df.streamdf and galpy.actionAngle.
actionAngleIsochroneApprox for modeling tidal streams using simple models formulated in action-angle space (see the tutorial above).
9. The Milky Way Tomography with SDSS. V. Mapping the Dark Matter Halo, Sarah R. Loebman, Zeljko Ivezic Thomas R. Qu Uses galpy. potential functions to calculate the acceleration field of the best-fit potential in Bovy \& Rix (2013) above.
10. The power spectrum of the Milky Way: Velocity fluctuations in the Galactic disk, Jo Bovy, Jonathan C. Bird, Ana E. Garcia I Uses galpy.df.evolveddiskdf to calculate the mean non-axisymmetric velocity field due to different non-axisymmetric perturbations and compares it to APOGEE data.
11. Generation of mock tidal streams, Mark A. Fardal, Shuiyao Huang, \& Martin D. Weinberg (2014), Mon. Not. Roy. Astron. Uses galpy.potential and galpy.orbit for orbit integration in creating a particle-spray model for tidal streams.
12. The nature and orbit of the Ophiuchus stream, Branimir Sesar, Jo Bovy, Edouard J. Bernard, et al. (2015), Astrophys. J., su Uses the Orbit.fit routine in galpy. orbit to fit the orbit of the Ophiuchus stream to newly obtained observational data and the routines in galpy.df. streamdf to model the creation of the stream.
13. The LMC geometry and outer stellar populations from early DES data, Eduardo Balbinot, B. X. Santiago, L. Girardi, et al. Employs galpy.potential. MWPotential as a mass model for the Milky Way to constrain the mass of the LMC.
14. Young Pulsars and the Galactic Center GeV Gamma-ray Excess, Ryan M. O’Leary, Matthew D. Kistler, Matthew Kerr, \& J Uses galpy orbit integration and galpy.potential. MWPotential2014 as part of a Monte Carlo simulation of the Galactic young-pulsar population.

\section*{chapter 5}

\section*{Acknowledging galpy}

If you use galpy in a publication, please cite the following paper
- galpy: A Python Library for Galactic Dynamics, Jo Bovy (2015), Astrophys. J. Supp., 216, 29 (arXiv/1412.3451).
and link to http://github.com/jobovy/galpy. Please also send me a reference to the paper or send a pull request including your paper in the list of galpy papers on this page (this page is at doc/source/index.rst). Thanks!

When using the galpy.actionAngle.actionAngleAdiabatic and galpy.actionAngle. actionAngleStaeckel modules, please cite 2013ApJ...779..115B in addition to the papers describing the algorithm used. When using galpy.actionAngle.actionAngleIsochroneApprox, please cite 2014ApJ... 795...95B, which introduced this technique.

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[^0]:    >>> plot(jfa[6],jfa[8],'b.')

