galpy Documentation

Release v1.6.0

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galpy is a Python 2 and 3 package for galactic dynamics. It supports orbit integration in a variety of potentials, evaluating and sampling various distribution functions, and the calculation of action-angle coordinates for all static potentials. galpy is an astropy affiliated package and provides full support for astropy's Quantity framework for variables with units.

galpy is developed on GitHub. If you are looking to report an issue or for information on how to contribute to the code, please head over to galpy's GitHub page for more information.

As a preview of the kinds of things you can do with galpy, here's a video introducing some of the new features in galpy v1.5:

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CHAPTER 1

Quick-start guide

1.1 Installation

1.1.1 Dependencies

galpy requires the numpy, scipy, and matplotlib packages; these must be installed or the code will not be able to be imported.

Optional dependencies are: astropy for Quantity support (used throughout galpy when installed), astroquery for the Orbit.from_name initialization method (to initialize using a celestial object's name), numexpr for plotting arbitrary expressions of Orbit quantities, and pynbody for use of SnapshotRZPotential and InterpSnapshotRZPotential.

To be able to use the fast C extensions for orbit integration and action-angle calculations, the GNU Scientific Library (GSL) needs to be installed (*see below*).

1.1.2 With conda

The easiest way to install the latest released version of galpy is using conda or pip (see below):

```
conda install galpy -c conda-forge
```

or:

```
conda config --add channels conda-forge conda install galpy
```

Installing with conda will automatically install the required dependencies (numpy, scipy, and matplotlib) and the GSL, but not the optional dependencies.

1.1.3 With pip

galpy can also be installed using pip. Since v1.6.0, the pip installation will install binary *wheels* for most major operating systems (Mac, Windows, and Linux) and commonly-used Python 3 versions. When this is the case, you do not need to separately install the GSL.

When you are on a platform or Python version for which no binary wheel is available, pip will compile the source code on your machine. Some advanced features require the GNU Scientific Library (GSL; *see below*). If you want to use these with a pip-from-source install, install the GSL first (or install it later and re-install using the upgrade command. Then do:

```
pip install galpy
```

or to upgrade without upgrading the dependencies:

```
pip install -U --no-deps galpy
```

Installing with pip will automatically install the required dependencies (numpy, scipy, and matplotlib), but not the optional dependencies.

1.1.4 Latest version

The latest updates in galpy can be installed using:

```
pip install -U --no-deps git+git://github.com/jobovy/galpy.git#egg=galpy
```

or:

for a local installation. The latest updates can also be installed from the source code downloaded from github using the standard python setup.py installation:

```
python setup.py install
```

or:

```
python setup.py install --prefix=~/local
```

for a local installation.

Note that these latest-version commands all install directly fromm the source code and thus require you to have the GSL and a C compiler installed to build the C extension(s). If you are having issues with this, you can also download a binary wheel for the latest master version, which are available here for Mac/Windows wheels and here for Linux wheels (note that you need to be logged into GitHub to access the artifacts, which otherwise just show up as a gray non-link). To install these wheels, click on the latest run, download the "artifact" for the platform/Python version that you are using, unzip the file, and install the wheel with:

```
pip install WHEEL_FILE.whl
```

1.1.5 Installing from a branch

If you want to use a feature that is currently only available in a branch, do:

```
pip install -U --no-deps git+git://github.com/jobovy/galpy.git@dev#egg=galpy
```

to, for example, install the dev branch.

Note that we currently do not build binary wheels for branches other than master. If you *really* wanted this, you could fork galpy, edit the GitHub Actions workflow file that generates the wheel to include the branch that you want to build (in the on: section), and push to GitHub; then the binary wheel will be built as part of your fork.

1.1.6 Installing from source on Windows

Tip: You can install a pre-compiled Windows "wheel" of the latest master version that is automatically built on AppVeyor for all recent Python versions. Navigate to the latest master build, click on the first job and then on "Artifacts", download the wheel for your version of Python, and install with pip install WHEEL_FILE.whl. Similar wheels are also available here (see above), but require you to be logged into GitHub.

Versions >1.3 should be able to be compiled on Windows systems using the Microsoft Visual Studio C compiler (>= 2015). For this you need to first install the GNU Scientific Library (GSL), for example using Anaconda (*see below*). Similar to on a UNIX system, you need to set paths to the header and library files where the GSL is located. On Windows, using the CDM commandline, this is done as:

```
set INCLUDE=%CONDA_PREFIX%\Library\include;%INCLUDE%
set LIB=%CONDA_PREFIX%\Library\lib;%LIB%
set LIBPATH=%CONDA_PREFIX%\Library\lib;%LIBPATH%
```

If you are using the Windows PowerShell (which newer versions of the Anaconda prompt might set as the default), do:

```
$env:INCLUDE="$env:CONDA_PREFIX\Library\include"
$env:LIB="$env:CONDA_PREFIX\Library\lib"
$env:LIBPATH="$env:CONDA_PREFIX\Library\lib"
```

where in this example CONDA_PREFIX is the path of your current conda environment (the path that ends in \ENV_NAME). If you have installed the GSL somewhere else, adjust these paths (but do not use YOUR_PATH\include\gsl or YOUR_PATH\lib\gsl as the paths, simply use YOUR_PATH\include and YOUR_PATH\lib).

To compile with OpenMP on Windows, you have to install Intel OpenMP via:

```
conda install -c anaconda intel-openmp
```

and then to compile the code:

```
python setup.py install
```

If you encounter any issue related to OpenMP during compilation, you can do:

```
python setup.py install --no-openmp
```

1.1.7 Installing from source with Intel Compiler

Compiling galpy with an Intel Compiler can give significant performance improvements on 64-bit Intel CPUs. Moreover students can obtain a free copy of an Intel Compiler at this link.

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To compile the galpy C extensions with the Intel Compiler on 64bit MacOS/Linux do:

```
python setup.py build_ext --inplace --compiler=intelem
```

and to compile the galpy C extensions with the Intel Compiler on 64bit Windows do:

```
python setup.py build_ext --inplace --compiler=intel64w
```

Then you can simply install with:

```
python setup.py install
```

or other similar installation commands, or you can build your own wheels with:

```
python setup.py sdist bdist_wheel
```

1.1.8 Installing the TorusMapper code

Warning: The TorusMapper code is *not* part of any of galpy's binary distributions (installed using conda or pip); if you want to gain access to the TorusMapper, you need to install from source as explained in this section and above.

Since v1.2, galpy contains a basic interface to the TorusMapper code of Binney & McMillan (2016). This interface uses a stripped-down version of the TorusMapper code, that is not bundled with the galpy code, but kept in a fork of the original TorusMapper code. Installation of the TorusMapper interface is therefore only possible when installing from source after downloading or cloning the galpy code and using the python setup.py install method above.

To install the TorusMapper code, *before* running the installation of galpy, navigate to the top-level galpy directory (which contains the setup.py file) and do:

```
git clone https://github.com/jobovy/Torus.git galpy/actionAngle/actionAngleTorus_c_

--ext/torus
cd galpy/actionAngle/actionAngleTorus_c_ext/torus
git checkout galpy
cd --
```

Then proceed to install galpy using the python setup.py install technique or its variants as usual.

1.1.9 Installation FAQ

What is the required numpy version?

galpy should mostly work for any relatively recent version of numpy, but some advanced features, including calculating the normalization of certain distribution functions using Gauss-Legendre integration require numpy version 1.7.0 or higher.

I get warnings like "galpyWarning: libgalpy C extension module not loaded, because libgalpy.so image was not found"

This typically means that the GNU Scientific Library (GSL) was unavailable during galpy's installation, causing the C extensions not to be compiled. Most of the galpy code will still run, but slower because it will run in pure Python.

The code requires GSL versions >= 1.14. If you believe that the correct GSL version is installed for galpy, check that the library can be found during installation (see *below*).

I get the warning "galpyWarning: libgalpy_actionAngleTorus C extension module not loaded, because libgalpy_actionAngleTorus.so image was not found"

This is typically because the TorusMapper code was not compiled, because it was unavailable during installation. This code is only necessary if you want to use <code>galpy.actionAngle.actionAngleTorus</code>. See *above* for instructions on how to install the TorusMapper code. Note that in recent versions of galpy, you should *not* be getting this warning, unless you set <code>verbose=True</code> in the *configuration file*.

How do I install the GSL?

Certain advanced features require the GNU Scientific Library (GSL), with action calculations requiring version 1.14 or higher. The easiest way to install this is using its Anaconda build:

```
conda install -c conda-forge gsl
```

If you do not want to go that route, on a Mac, the next easiest way to install the GSL is using Homebrew as:

```
brew install gsl --universal
```

You should be able to check your version using (on Mac/Linux):

```
gsl-config --version
```

On Linux distributions with apt-get, the GSL can be installed using:

```
apt-get install libgs10-dev
```

or on distros with yum, do:

```
yum install gsl-devel
```

The galpy installation fails because of C compilation errors

galpy's installation can fail due to compilation errors, which look like:

```
error: command 'gcc' failed with exit status 1
```

or:

```
error: command 'clang' failed with exit status 1
```

or:

```
error: command 'cc' failed with exit status 1
```

This is typically because the compiler cannot locate the GSL header files or the GSL library. You can tell the installation about where you've installed the GSL library by defining (for example, when the GSL was installed under /usr; the LD_LIBRARY_PATH part of this may or may not be necessary depending on your system):

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```
export CFLAGS=-I/usr/include
export LDFLAGS=-L/usr/lib
export LD_LIBRARY_PATH=-L/usr/lib
```

or:

```
setenv CFLAGS -I/usr/include
setenv LDFLAGS -L/usr/lib
setenv LD_LIBRARY_PATH -L/usr/lib
```

depending on your shell type (change the actual path to the include and lib directories that have the gsl directory). If you already have CFLAGS, LDFLAGS, and LD_LIBRARY_PATH defined you just have to add the '-I/usr/include' and '-L/usr/lib' to them.

If you are on a Mac or UNIX system (e.g., Linux), you can find the correct CFLAGS and LDFLAGS/LD_LIBRARY_path entries by doing:

```
gsl-config --cflags
gsl-config --libs
```

(don't include the -lgsl lgslcblas portion of the latter output.)

I have defined CFLAGS, LDFLAGS, and LD_LIBRARY_PATH, but the compiler does not seem to include these and still returns with errors

This typically happens if you install using sudo, but have defined the CFLAGS etc. environment variables without using sudo. Try using sudo —E instead, which propagates your own environment variables to the sudo user.

I'm having issues with OpenMP

galpy uses OpenMP to parallelize various of the computations done in C. galpy can be installed without OpenMP by specifying the option --no-openmp when running the python setup.py commands above:

```
python setup.py install --no-openmp
```

or when using pip as follows:

```
pip install -U --no-deps --install-option="--no-openmp" git+git://github.com/jobovy/

--galpy.git#egg=galpy
```

or:

```
pip install -U --no-deps --install-option="--prefix=~/local" --install-option="--no-

→openmp" git+git://github.com/jobovy/galpy.git#egg=galpy
```

for a local installation. This might be useful if one is using the clang compiler, which is the new default on macs with OS X > 10.8, but does not support OpenMP. clang might lead to errors in the installation of galpy such as:

```
ld: library not found for -lgomp
clang: error: linker command failed with exit code 1 (use -v to see invocation)
```

If you get these errors, you can use the commands given above to install without OpenMP, or specify to use gcc by specifying the CC and LDSHARED environment variables to use gcc. Note that clang does not seem to have this issue anymore in more recent versions, but it still does not support OpenMP.

1.1.10 Configuration file

Since v1.2, galpy uses a configuration file to set a small number of configuration variables. This configuration file is parsed using ConfigParser/configparser. It is currently used:

- to set a default set of distance and velocity scales (ro and vo throughout galpy) for conversion between physical and internal galpy unit
- to decide whether to use seaborn plotting with galpy's defaults (which affects *all* plotting after importing galpy.util.bovy_plot),
- to specify whether output from functions or methods should be given as an astropy Quantity with units as much as possible or not, and whether or not to use astropy's coordinate transformations (these are typically somewhat slower than galpy's own coordinate transformations, but they are more accurate and more general)
- to set the level of verbosity of galpy's warning system (the default verbose=False turns off non-crucial warnings).

The current configuration file therefore looks like this:

```
[normalization]
ro = 8.
vo = 220.

[plot]
seaborn-bovy-defaults = False

[astropy]
astropy-units = False
astropy-coords = True

[warnings]
verbose = False
```

where ro is the distance scale specified in kpc, vo the velocity scale in km/s, and the setting is to *not* return output as a Quantity. These are the current default settings.

A user-wide configuration file should be located at \$HOME/.galpyrc. This user-wide file can be overridden by a \$PWD/.galpyrc file in the current directory. If no configuration file is found, the code will automatically write the default configuration to \$HOME/.galpyrc. Thus, after installing galpy, you can simply use some of its simplest functionality (e.g., integrate an orbit), and after this the default configuration file will be present at \$HOME/.galpyrc. If you want to change any of the settings (for example, you want Quantity output), you can edit this file. The default configuration file can also be found here.

1.2 What's new?

This page gives some of the key improvements in each galpy version. See the HISTORY.txt file in the galpy source for full details on what is new and different in each version.

1.2.1 v1.6

This version mainly consists of changes to the internal functioning of galpy; some of the new outward-facing features are:

• ChandrasekharDynamicalFrictionForce is now implemented in C, leading to 100x to 1000x speed-ups for orbit integrations using dynamical friction compared to the prior pure-Python version.

1.2. What's new?

- New potentials:
 - HomogeneousSpherePotential: the potential of a constant density sphere out to some radius R.
 - DehnenSphericalPotential: the Dehnen Spherical Potential from Dehnen (1993).
 - DehnenCoreSphericalPotential: the Dehnen Spherical Potential from (Dehnen 1993) with alpha=0 (corresponding to an inner core).
- Some notable internal changes:
 - Fixed a bug in how DiskSCFPotential instances are passed to C for orbit integration that in particular
 affected the McMillan17 Milky-Way potential (any hole in the surface density was effectively ignored
 in the C code in v1.5).
 - The performance of orbit animations is significantly improved.
 - All main galpy C extensions are now compiled into a single shared-object library libgalpy.
 - Binary wheels are now automatically built for Windows, Mac, and most major Linux distributions upon
 every push to the master branch and these are automatically uploaded to PyPI upon release. See the
 Installation Instructions for more info. Binary wheels on Windows are also built for every push on AppVeyor, see the Windows installation instructions.

1.2.2 v1.5

This version will be the last to support Python 2.7 as this version of Python is reaching end-of-life on January 1 2020.

- This version's highlight is a fully re-written implementation of galpy.orbit.Orbit such that it can now contain and manipulate multiple objects at once. galpy.orbit.Orbit can be initialized with an arbitrary shape of input objects in a *variety of ways*, manipulated in a manner similar to Numpy arrays, and all Orbit methods work efficiently on Orbit instances containing multiple objects. Some methods, such as *orbit integration* and those for *fast orbital characterization* are parallelized on multi-core machines. Orbit instances can contain and manipulate millions of objects simultaneously now.
- Added the galpy.potentials.mwpotentials module with various Milky-Way-like potentials. Currently included are MWPotential2014, McMillan17 for the potential from McMillan (2017), models 1 through 4 from Dehnen & Binney (1998), and the three models from Irrgang et al. (2013). See this section of the API documentation for details.
- Added a (JSON) list with the phase-space coordinates of known objects (mainly Milky Way globular clusters
 and dwarf galaxies) for easy *Orbit.from_name initialization*. For ease of use, Orbit.from_name also supports
 tab completion for known objects in this list in IPython/Jupyter.
- Added galpy.potential.to_amuse to create an AMUSE representation of any galpy potential, allowing galpy potentials to be used as external gravitational fields in AMUSE N-body simulations.
- New or improved potentials and potential wrappers:
 - MovingObjectPotential: Re-wrote potential.MovingObjectPotential to allow general mass distributions for the moving object, implemented now as standard galpy potentials. Also added a C implementation of this potential for fast orbit integration.
 - IsothermalDiskPotential: The one-dimensional potential of an isothermal self-gravitating disk (sech^2 profile).
 - NumericalPotentialDerivativesMixin: a Mixin class to add numerically-computed forces and second
 derivatives to any Potential class, allowing new potentials to be implemented quickly by only implementing
 the potential itself and obtaining all forces and second derivatives numerically.
 - DehnenSmoothWrapperPotential: Can now decay rather than grow a potential by setting decay=True.

- Added support to combine Potential instances or lists thereof through the addition operator. E.g., pot=pot1+pot2+pot3 to create the combined potential of the three component potentials (pot1,pot2,pot3).
 Each of these components can be a combined potential itself. As before, combined potentials are simply lists of potentials, so this is simply an alternative (and perhaps more intuitive) way to create these lists.
- Added support to adjust the amplitude of a Potential instance through multiplication of the instance by a number or through division by a numer. E.g., pot= 2.*pot1 returns a Potential instance that is the same as pot1, except that the amplitude is twice larger. Similarly, pot= pot1/2. decreases the amplitude by a factor of two. This is useful, for example, to quickly change the mass of a potential. Only works for Potential instances, not for lists of Potential instances.
- New or improved galpy.orbit.Orbit functionality and methods:
 - Added support for 1D orbit integration in C.
 - Added support to plot arbitrary combinations of the basic Orbit attributes by giving them as an expression (e.g., orb.plot (d2="vR*R/r+vz*z/r")); requires the numexpr package.
 - Switched default Sun's vertical height zo parameter for Orbit initialization to be the value of 20.8 pc from Bennett & Bovy (2019).
 - Add Python and C implementation of Dormand-Prince 8(5,3) integrator.

1.2.3 v1.4

- Added dynamical friction as the ChandrasekharDynamicalFrictionForce class, an implementation of dynamical friction based on the classical Chandrasekhar formula (with recent tweaks from the literature to better represent the results from N-body simulations).
- A general EllipsoidalPotential superclass for implementing potentials with densities that are constant on ellipsoids (functions of $m^2 = x^2 + y^2/b^2 + z^2/c^2$). Also implemented in C. Implementing new types of ellipsoidal potentials now only requires three simple functions to be defined: the density as a function of m, its derivative with respect to m, and its integral with respect to m^2. Makes implementing any ellipsoidal potential a breeze. See examples in the new-potentials section below.
- New or improved potentials and *potential wrappers*:
 - CorotatingRotationWrapperPotential: wrapper to make a pattern (e.g., a SpiralArmsPotential) wind up over time such that it is always corotating (see Hunt et al. (2018) for an example of this).
 - GaussianAmplitudeWrapperPotential: wrapper to modulate the amplitude of a (list of) Potential (s) with a Gaussian.
 - PerfectEllipsoidPotential: Potential of a perfect triaxial ellipsoid (de Zeeuw 1985).
 - SphericalShellPotential: Potential of a thin, spherical shell.
 - RingPotential: Potential of a circular ring.
 - Re-implemented TwoPowerTriaxialPotential, TriaxialHernquistPotential, TriaxialJaffePotential, and TriaxialNFWPotential using the general EllipsoidalPotential class.
- New Potential methods and functions:
 - Use nested lists of Potential instances wherever lists of Potential instances can be used. Allows easy adding of components (e.g., a bar) to previously defined potentials (which may be lists themselves): new_pot=[pot,bar_pot].
 - rtide and ttensor: compute the tidal radius of an object and the full tidal tensor.
 - surfdens method and evaluateSurfaceDensities function to evaluate the surface density up to a given z.

1.2. What's new?

- r2deriv and evaluater2derivs: 2nd derivative wrt spherical radius.
- evaluatephi2derivs: second derivative wrt phi.
- evaluateRphiderivs: mixed (R,phi) derivative.
- New or improved galpy.orbit.Orbit functionality and methods:
 - Orbit.from_name to initialize an Orbit instance from an object's name. E.g., orb= Orbit. from name('LMC').
 - Orbit initialization without arguments is now the orbit of the Sun.
 - Orbits can be initialized with a SkyCoord.
 - Default solarmotion = parameter is now 'schoenrich' for the Solar motion of Schoenrich et al. (2010).
 - rguiding: Guiding-center radius.
 - Lz: vertical component of the angular momentum.
 - If astropy version > 3, Orbit.SkyCoord method returns a SkyCoord object that includes the velocity information and the Galactocentric frame used by the Orbit instance.
- galpy.df.jeans module with tools for Jeans modeling. Currently only contains the functions sigmar and sigmalos to calculate the velocity dispersion in the radial or line-of-sight direction using the spherical Jeans equation in a given potential, density profile, and anisotropy profile (anisotropy can be radially varying).
- Support for compilation on Windows with MSVC.

1.2.4 v1.3

- A fast and precise method for approximating an orbit's eccentricity, peri- and apocenter radii, and maximum height above the midplane using the Staeckel approximation (see Mackereth & Bovy 2018). Can determine these parameters to better than a few percent accuracy in as little as $10 \mu s$ per object, more than 1,000 times faster than through direct orbit integration. See *this section* of the documentation for more info.
- A general method for modifying Potential classes through potential wrappers—simple classes that wrap existing potentials to modify their behavior. See *this section* of the documentation for examples and *this section* for information on how to easily define new wrappers. Example wrappers include SolidBodyRotationWrapper-Potential to allow *any* potential to rotate as a solid body and DehnenSmoothWrapperPotential to smoothly grow *any* potential. See *this section of the galpy.potential API page* for an up-to-date list of wrappers.
- New or improved potentials:
 - DiskSCFPotential: a general Poisson solver well suited for galactic disks
 - Bar potentials SoftenedNeedleBarPotential and FerrersPotential (latter only in Python for now)
 - 3D spiral arms model SpiralArmsPotential
 - Henon & Heiles (1964) potential HenonHeilesPotential
 - Triaxial version of LogarithmicHaloPotential
 - 3D version of DehnenBarPotential
 - Generalized version of CosmphiDiskPotential
- New or improved galpy.orbit.Orbit methods:
 - Method to display an animation of an integrated orbit in jupyter notebooks: Orbit animate. See this section
 of the documentation.

- Improved default method for fast calculation of eccentricity, zmax, rperi, rap, actions, frequencies, and angles by switching to the Staeckel approximation with automatically-estimated approximation parameters.
- Improved plotting functions: plotting of spherical radius and of arbitrary user-supplied functions of time in Orbit.plot, Orbit.plot3d, and Orbit.animate.
- actionAngleStaeckel upgrades:
 - actionAngleStaeckel methods now allow for different focal lengths delta for different phase-space
 points and for the order of the Gauss-Legendre integration to be specified (default: 10, which is good
 enough when using actionAngleStaeckel to compute approximate actions etc. for an axisymmetric potential).
 - Added an option to the estimateDeltaStaeckel function to facilitate the return of an estimated delta parameter at every phase space point passed, rather than returning a median of the estimate at each point.
- galpy.df.schwarzschilddf:the simple Schwarzschild distribution function for a razor-thin disk (useful for teaching).

1.2.5 v1.2

- Full support for providing inputs to all initializations, methods, and functions as astropy Quantity with units and for providing outputs as astropy Quantities.
- galpy.potential.TwoPowerTriaxialPotential, a set of triaxial potentials with iso-density contours that are arbitrary, similar, coaxial ellipsoids whose 'radial' density is a (different) power-law at small and large radii: 1/m^alpha/(1+m)^beta-alpha (the triaxial generalization of TwoPowerSphericalPotential, with flattening in the density rather than in the potential; includes triaxial Hernquist and NFW potentials.
- galpy.potential.SCFPotential, a class that implements general density/potential pairs through the basis expansion approach to solving the Poisson equation of Hernquist & Ostriker (1992). Also implemented functions to compute the coefficients for a given density function. See more explanation *here*.
- galpy.actionAngle.actionAngleTorus: an experimental interface to Binney & McMillan's TorusMapper code for computing positions and velocities for given actions and angles. See the installation instructions for how to properly install this. See *this section* and the galpy.actionAngle API page for documentation.
- galpy.actionAngle.actionAngleIsochroneApprox (Bovy 2014) now implemented for the general case of a time-independent potential.
- galpy.df.streamgapdf, a module for modeling the effect of a dark-matter subhalo on a tidal stream. See Sanders et al. (2016). Also includes the fast methods for computing the density along the stream and the stream track for a perturbed stream from Bovy et al. (2016).
- Orbit.flip can now flip the velocities of an orbit in-place by specifying inplace=True. This allows correct velocities to be easily obtained for backwards-integrated orbits.
- galpy.potential.PseudoIsothermalPotential, a standard pseudo-isothermal-sphere potential. galpy.potential.KuzminDiskPotential, a razor-thin disk potential.
- Internal transformations between equatorial and Galactic coordinates are now performed by default using astropy's coordinates module. Transformation of (ra,dec) to Galactic coordinates for general epochs.

1.2.6 v1.1

• Full support for Python 3.

1.2. What's new?

- galpy.potential.SnapshotRZPotential, a potential class that can be used to get a frozen snapshot of the potential of an N-body simulation.
- Various other potentials: PlummerPotential, a standard Plummer potential; MN3ExponentialDiskPotential, an approximation to an exponential disk using three Miyamoto-Nagai potentials (Smith et al. 2015); KuzminKutuzovStaeckelPotential, a Staeckel potential that can be used to approximate the potential of a disk galaxy (Batsleer & Dejonghe 1994).
- Support for converting potential parameters to NEMO format and units.
- Orbit fitting in custom sky coordinates.

1.3 Introduction

The most basic features of galpy are its ability to display rotation curves and perform orbit integration for arbitrary combinations of potentials. This section introduce the most basic features of galpy.potential and galpy.orbit.

1.3.1 Rotation curves

The following code example shows how to initialize a Miyamoto-Nagai disk potential and plot its rotation curve

```
>>> from galpy.potential import MiyamotoNagaiPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=1.)
>>> mp.plotRotcurve(Rrange=[0.01,10.],grid=1001)
```

The normalize=1. option normalizes the potential such that the radial force is a fraction normalize=1. of the radial force necessary to make the circular velocity 1 at R=1. Starting in v1.2 you can also initialize potentials with amplitudes and other parameters in physical units; see below and other parts of this documentation.

Tip: You can copy all of the code examples in this documentation to your clipboard by clicking the button in the top, right corner of each example. This can be directly pasted into a Python interpreter (including the >>>).

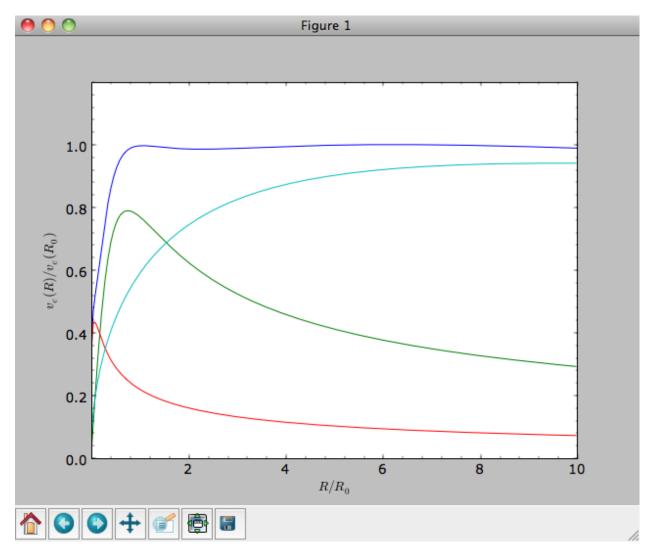
Similarly we can initialize other potentials and plot the combined rotation curve

```
>>> from galpy.potential import NFWPotential, HernquistPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=.6)
>>> np= NFWPotential(a=4.5,normalize=.35)
>>> hp= HernquistPotential(a=0.6/8,normalize=0.05)
>>> from galpy.potential import plotRotcurve
>>> plotRotcurve(hp+mp+np,Rrange=[0.01,10.],grid=1001,yrange=[0.,1.2])
```

Note that the normalize values add up to 1. such that the circular velocity will be 1 at R=1. Potentials can be combined into a composite potential either by combining them in a list as [hp,mp,np] or by adding them up hp+mp+np (the latter simply returns the list [hp,mp,np]). The resulting rotation curve is approximately flat. To show the rotation curves of the three components do

```
>>> mp.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
>>> hp.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
>>> np.plotRotcurve(Rrange=[0.01,10.],grid=1001,overplot=True)
```

You'll see the following



As a shortcut the [hp, mp, np] Milky-Way-like potential is defined as

```
>>> from galpy.potential import MWPotential
```

This is *not* the recommended Milky-Way-like potential in galpy. The (currently) recommended Milky-Way-like potential is MWPotential2014:

```
>>> from galpy.potential import MWPotential2014
```

MWPotential2014 has a more realistic bulge model and is actually fit to various dynamical constraints on the Milky Way (see *here* and the galpy paper).

1.3.2 Units in galpy

Internal (natural) units

Above we normalized the potentials such that they give a circular velocity of 1 at R=1. These are the standard galpy units (sometimes referred to as *natural units* in the documentation). galpy will work most robustly when using these natural units. When using galpy to model a real galaxy with, say, a circular velocity of 220 km/s at R=8 kpc, all of the

velocities should be scaled as v = V/[220 km/s] and all of the positions should be scaled as x = X/[8 kpc] when using galpy's natural units.

For convenience, a utility module bovy_conversion is included in galpy that helps in converting between physical units and natural units for various quantities. Alternatively, you can use the astropy units module to specify inputs in physical units and get outputs with units (see the *next subsection* below). For example, in natural units the orbital time of a circular orbit at R=1 is 2π ; in physical units this corresponds to

```
>>> from galpy.util import bovy_conversion
>>> print(2.*numpy.pi*bovy_conversion.time_in_Gyr(220.,8.))
# 0.223405444283
```

or about 223 Myr. We can also express forces in various physical units. For example, for the Milky-Way-like potential defined in galpy, we have that the vertical force at 1.1 kpc is

```
>>> from galpy.potential import MWPotential2014, evaluatezforces
>>> -evaluatezforces(MWPotential2014, 1.,1.1/8.)*bovy_conversion.force_in_pcMyr2(220., 48.)
# 2.0259181908629933
```

which we can also express as an equivalent surface-density by dividing by $2\pi G$

```
>>> -evaluatezforces(MWPotential2014, 1.,1.1/8.)*bovy_conversion.force_in_

-2piGmsolpc2(220.,8.)
# 71.658016957792356
```

Because the vertical force at the solar circle in the Milky Way at 1.1 kpc above the plane is approximately $70 (2\pi G M_{\odot} \,\mathrm{pc^{-2}})$ (e.g., $2013 \,\mathrm{arXiv} 1309.0809 \,\mathrm{B}$), this shows that our Milky-Way-like potential has a realistic disk (at least in this respect).

bovy_conversion further has functions to convert densities, masses, surface densities, and frequencies to physical units (actions are considered to be too obvious to be included); see *here* for a full list. As a final example, the local dark matter density in the Milky-Way-like potential is given by

```
>>> MwPotential2014[2].dens(1.,0.)*bovy_conversion.dens_in_msolpc3(220.,8.) # 0.0075419566970079373
```

or

```
>>> MwPotential2014[2].dens(1.,0.)*bovy_conversion.dens_in_gevcc(220.,8.) # 0.28643101789044584
```

or about $0.0075 M_{\odot} \,\mathrm{pc^{-3}} \approx 0.3 \,\mathrm{GeV \, cm^{-3}}$, in line with current measurements (e.g., 2012ApJ...756...89B).

When galpy Potentials, Orbits, actionAngles, or DFs are initialized using a distance scale ro= and a velocity scale vo= output quantities returned and plotted in physical coordinates. Specifically, positions are returned in the units in the table below. If astropy-units = True in the *configuration file*, then an astropy Quantity which includes the units is returned instead (see below).

Quantity	Default unit
position	kpc
velocity	km/s
energy	(km/s)^2
Jacobi integral	(km/s)^2
angular momentum	km/s x kpc
actions	km/s x kpc
frequencies	1/Gyr
time	Gyr
period	Gyr
potential	(km/s)^2
force	km/s/Myr
force derivative	1/Gyr^2
density	Msun/pc^3
number density	1/pc^3
surface density	Msun/pc^2
mass	Msun
angle	rad
proper motion	mas/yr
phase-space density	1/(kpc x km/s)^3

Physical units

Tip: With apy-units = True in the configuration file and specifying all inputs using astropy Quantity with units, galpy will return outputs in convenient, unambiguous units.

Full support for unitful quantities using astropy Quantity was added in v1.2. Thus, any input to a galpy Potential, Orbit, actionAngle, or DF instantiation, method, or function can now be specified in physical units as a Quantity. For example, we can set up a Miyamoto-Nagai disk potential with a mass of $5 \times 10^{10} M_{\odot}$, a scale length of 3 kpc, and a scale height of 300 pc as follows

```
>>> from galpy.potential import MiyamotoNagaiPotential
>>> from astropy import units
>>> mp= MiyamotoNagaiPotential(amp=5*10**10*units.Msun,a=3.*units.kpc,b=300.*units.pc)
```

Internally, galpy uses a set of normalized units, where positions are divided by a scale ro and velocities are divided by a scale vo. If these are not specified, the default set from the *configuration file* is used. However, they can also be specified on an instance-by-instance manner for all Potential, Orbit, actionAngle, and DF instances. For example

```
>>> mp= MiyamotoNagaiPotential(amp=5*10**10*units.Msun,a=3.*units.kpc,b=300.*units.pc, 

->ro=9*units.kpc,vo=230.*units.km/units.s)
```

uses differently normalized internal units. When you specify the parameters of a Potential, Orbit, etc. in physical units (e.g., the Miyamoto-Nagai setup above), the internal set of units is unimportant as long as you receive output in physical units (see below) and it is unnecessary to change the values of ro and vo, unless you are modeling a system with very different distance and velocity scales from the default set (for example, if you are looking at internal globular cluster dynamics rather than galaxy dynamics). If you find an input to any galpy function that does not take a Quantity as an input (or that does it wrong), please report an Issue.

Warning: If you combine potentials by adding them (comb_pot= pot1+pot2), galpy uses the ro and vo scales from the first potential in the list for physical <-> internal unit conversion. If you add potentials using the '+' operator, galpy will check that the units are compatible. galpy does **not** always check whether the unit systems of various objects are consistent when they are combined (but does check this for many common cases, e.g., integrating an Orbit in a Potential, setting up an actionAngle object for a given potential, setting up a DF object for a given potential, etc.).

galpy can also return values with units as an astropy Quantity. Whether or not this is done is specified by the apy-units option in the *configuration file*. If you want to get return values as a Quantity, set apy-units = True in the configuration file. Then you can do for the Miyamoto-Nagai potential above

```
>>> mp.vcirc(10.*units.kpc)
# <Quantity 135.72399857308042 km / s>
```

Note that if you do not specify the argument as a Quantity with units, galpy will assume that it is given in natural units, viz.

```
>>> mp.vcirc(10.)
# <Quantity 51.78776595740726 km / s>
```

because this input is considered equal to 10 times the distance scale (this is for the case using the default ro and vo, the first Miyamoto-Nagai instantiation of this subsection)

```
>>> mp.vcirc(10.*8.*units.kpc)
# <Quantity 51.78776595740726 km / s>
```

Warning: If you do not specify arguments of methods and functions using a Quantity with units, galpy assumes that the argument has internal (natural) units.

If you do not use astropy Quantities (apy-units = False in the configuration file), you can still get output in physical units when you have specified ro= and vo= during instantiation of the Potential, Orbit, etc. For example, for the Miyamoto-Nagai potential above in a session with apy-units = False

```
>>> mp= MiyamotoNagaiPotential(amp=5*10**10*units.Msun,a=3.*units.kpc,b=300.*units.pc)
>>> mp.vcirc(10.*units.kpc)
# 135.72399857308042
```

This return value is in km/s (see the *table* at the end of the previous section for default units for different quantities). Note that as long as astropy is installed, we can still provide arguments as a Quantity, but the return value will not be a Quantity when apy-units = False. If you setup a Potential, Orbit, actionAngle, or DF object with parameters specified as a Quantity, the default is to return any output in physical units. This is why mp.vcirc returns the velocity in km/s above. Potential and Orbit instances (or lists of Potentials) also support the functions turn_physical_off and turn_physical_on to turn physical output off or on. For example, if we do

```
>>> mp.turn_physical_off()
```

outputs will be in internal units

```
>>> mp.vcirc(10.*units.kpc)
# 0.61692726624127459
```

If you setup a Potential, Orbit, etc. object without specifying the parameters as a Quantity, the default is to return output in natural units, except when ro= and vo= scales are specified (exception: when you wrap a potential that has

physical outputs on, the wrapped potential will also have them on). ro= and vo= can always be given as a Quantity themselves. ro= and vo= can always also be specified on a method-by-method basis, overwriting an object's default. For example

```
>>> mp.vcirc(10.*units.kpc,ro=12.*units.kpc)
# 0.69273212489609337
```

Physical output can also be turned off on a method-by-method or function-by-function basis, for example

```
>>> mp.turn_physical_on() # turn overall physical output on
>>> mp.vcirc(10.*units.kpc)
135.72399857308042 # km/s
>>> mp.vcirc(10.*units.kpc,use_physical=False)
# 0.61692726624127459 # in natural units
```

Further examples of specifying inputs with units will be given throughout the documentation.

1.3.3 Orbit integration

Warning: galpy uses a left-handed Galactocentric coordinate frame, as is common in studies of the kinematics of the Milky Way. This means that in particular cross-products, like the angular momentum $\vec{L} = \vec{r} \times \vec{p}$, behave differently than in a right-handed coordinate frame.

We can also integrate orbits in all galpy potentials. Going back to a simple Miyamoto-Nagai potential, we initialize an orbit as follows

```
>>> from galpy.orbit import Orbit
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,amp=1.,normalize=1.)
>>> o= Orbit([1.,0.1,1.1,0.,0.1])
```

Since we gave Orbit () a five-dimensional initial condition [R, VR, VT, z, VZ], we assume we are dealing with a three-dimensional axisymmetric potential in which we do not wish to track the azimuth. We then integrate the orbit for a set of times ts

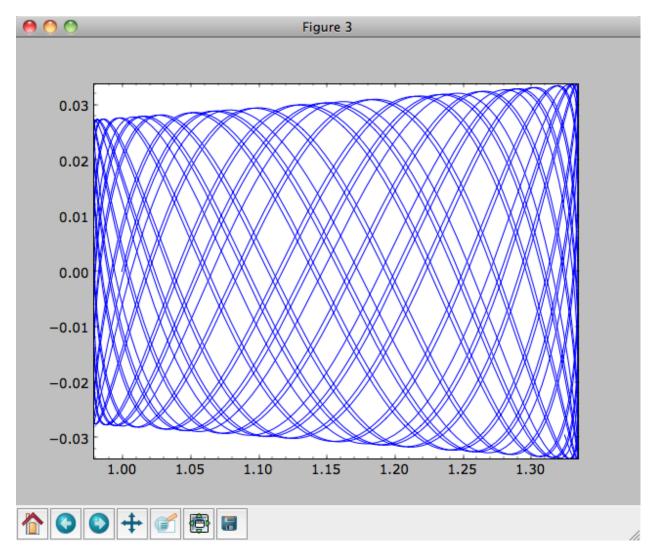
```
>>> import numpy
>>> ts= numpy.linspace(0,100,10000)
>>> o.integrate(ts,mp,method='odeint')
```

Tip: Like for the Miyamoto-Nagai example in the section above, the Orbit and integration times can also be specified in physical units, e.g., o= Orbit([8.*units.kpc,22.*units.km/units.s,242.*units.km/units.s.0.*units.pc,20.*units.km/s]) and ts= numpy.linspace(0.,10.,10000)*units. Gyr

Now we plot the resulting orbit as

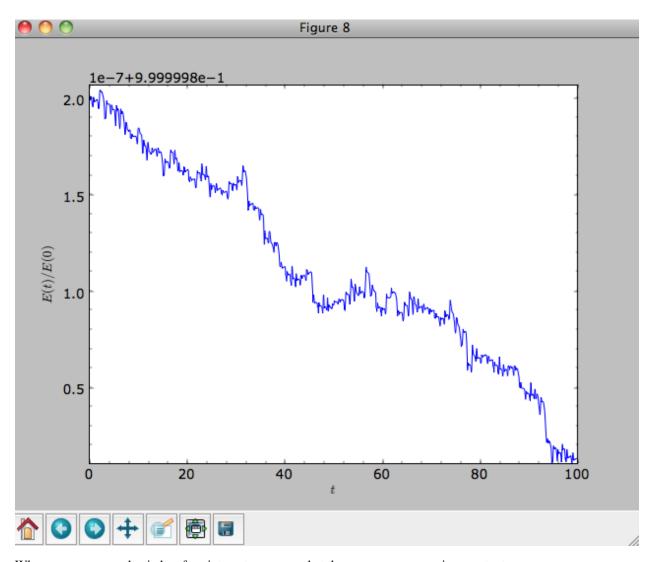
```
>>> o.plot()
```

Which gives



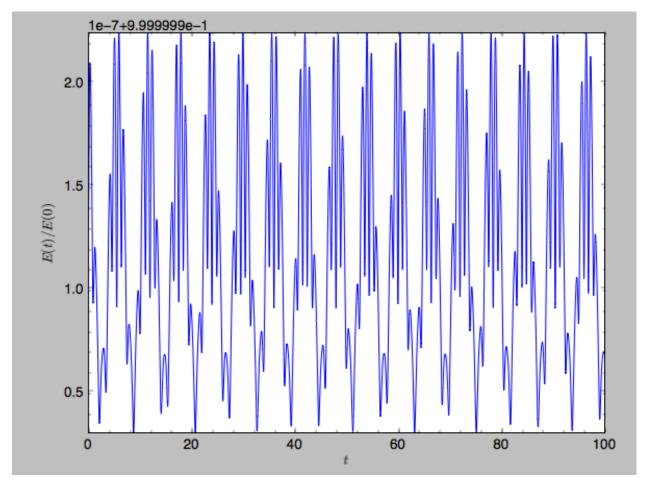
The integrator used is not symplectic, so the energy error grows with time, but is small nonetheless

>>> o.plotE(normed=True)



When we use a symplectic leapfrog integrator, we see that the energy error remains constant

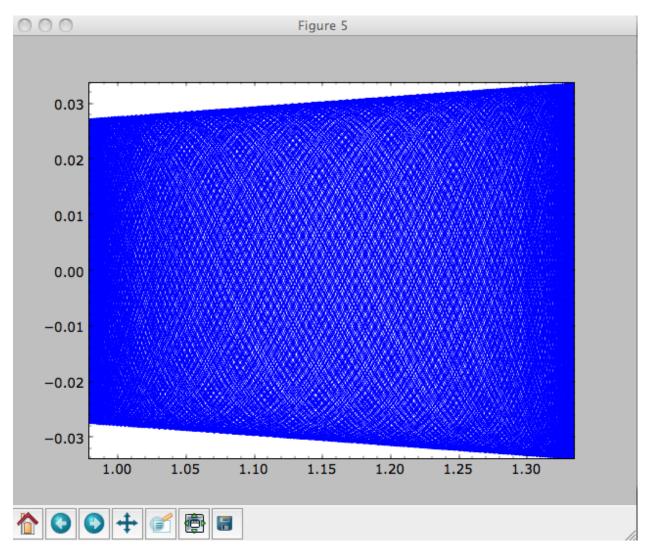
```
>>> o.integrate(ts,mp,method='leapfrog')
>>> o.plotE(xlabel=r'$t$',ylabel=r'$E(t)/E(0)$')
```



Because stars have typically only orbited the center of their galaxy tens of times, using symplectic integrators is mostly unnecessary (compared to planetary systems which orbits millions or billions of times). galpy contains fast integrators written in C, which can be accessed through the method= keyword (e.g., integrate(..., method='dopr54_c') is a fast high-order Dormand-Prince method).

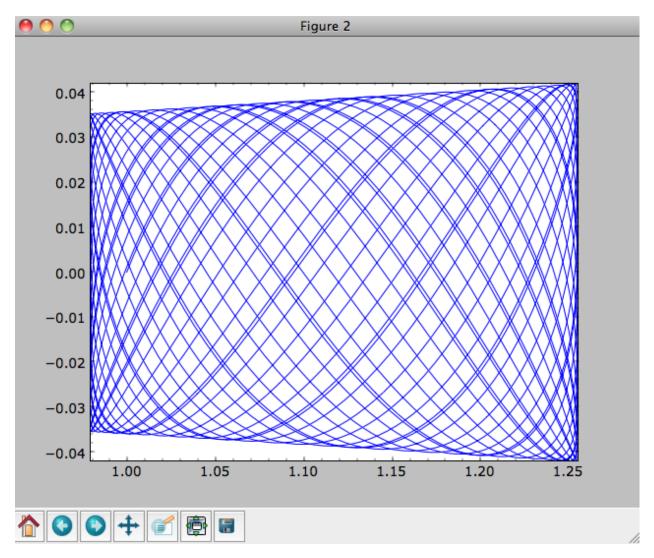
When we integrate for much longer we see how the orbit fills up a torus (this could take a minute)

```
>>> ts= numpy.linspace(0,1000,10000)
>>> o.integrate(ts,mp,method='odeint')
>>> o.plot()
```



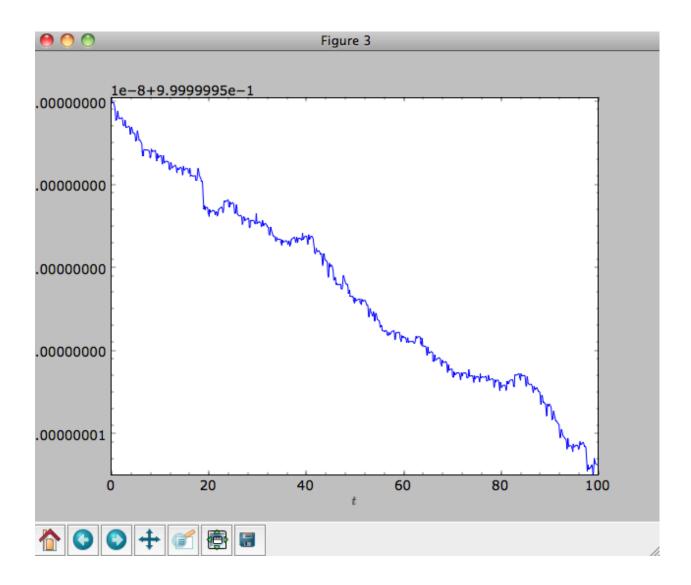
As before, we can also integrate orbits in combinations of potentials. Assuming mp, np, and hp were defined as above, we can

```
>>> ts= numpy.linspace(0,100,10000)
>>> o.integrate(ts,mp+hp+np)
>>> o.plot()
```



Energy is again approximately conserved

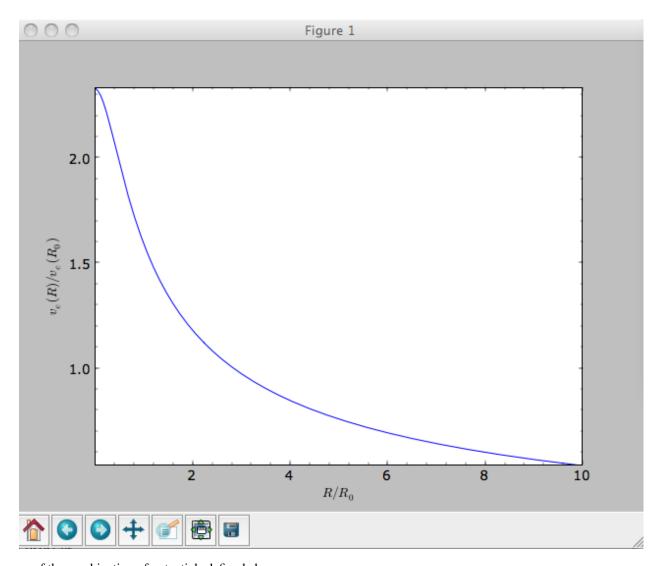
>>> o.plotE(xlabel=r'\$t\$',ylabel=r'\$E(t)/E(0)\$')



1.3.4 Escape velocity curves

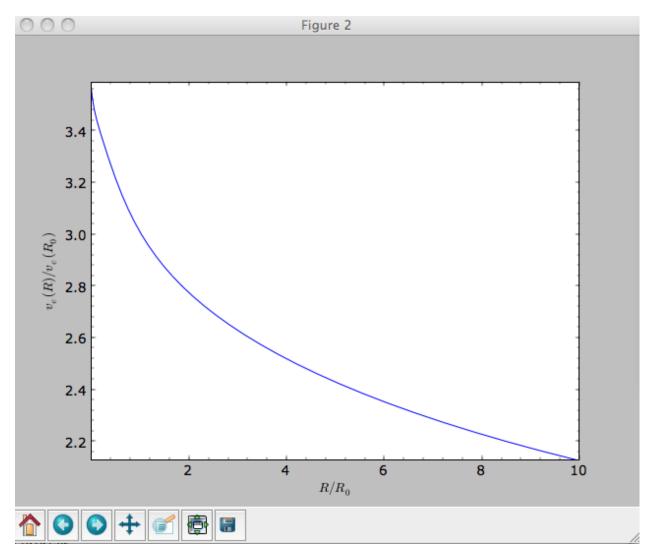
Just like we can plot the rotation curve for a potential or a combination of potentials, we can plot the escape velocity curve. For example, the escape velocity curve for the Miyamoto-Nagai disk defined above

>>> mp.plotEscapecurve(Rrange=[0.01,10.],grid=1001)



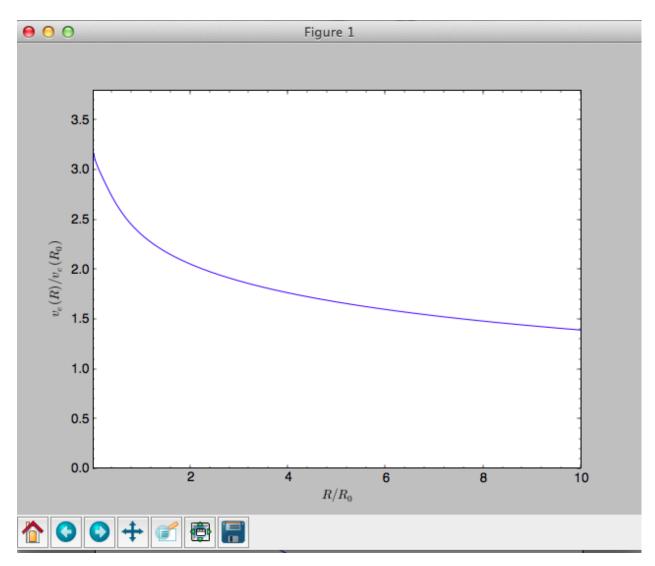
or of the combination of potentials defined above

```
>>> from galpy.potential import plotEscapecurve
>>> plotEscapecurve(mp+hp+np,Rrange=[0.01,10.],grid=1001)
```



For the Milky-Way-like potential MWPotential 2014, the escape-velocity curve is

>>> plotEscapecurve(MWPotential2014,Rrange=[0.01,10.],grid=1001)



At the solar radius, the escape velocity is

```
>>> from galpy.potential import vesc
>>> vesc(MWPotential2014,1.)
2.3316389848832784
```

Or, for a local circular velocity of 220 km/s

```
>>> vesc(MWPotential2014,1.) *220.
# 512.96057667432126
```

similar to direct measurements of this (e.g., 2007MNRAS.379..755S and 2014A%26A...562A..91P).

1.4 Potentials in galpy

galpy contains a large variety of potentials in <code>galpy.potential</code> that can be used for orbit integration, the calculation of action-angle coordinates, as part of steady-state distribution functions, and to study the properties of gravitational potentials. This section introduces some of these features.

1.4.1 Potentials and forces

Various 3D and 2D potentials are contained in galpy, list in the API page. Another way to list the latest overview of potentials included with galpy is to run

```
>>> import galpy.potential
>>> print([p for p in dir(galpy.potential) if 'Potential' in p])
# ['CosmphiDiskPotential',
# 'DehnenBarPotential',
# 'DoubleExponentialDiskPotential',
# 'EllipticalDiskPotential',
# 'FlattenedPowerPotential',
# 'HernquistPotential',
# ....]
```

(list cut here for brevity). Section *Rotation curves* explains how to initialize potentials and how to display the rotation curve of single Potential instances or of combinations of such instances. Similarly, we can evaluate a Potential instance

```
>>> from galpy.potential import MiyamotoNagaiPotential
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=1.)
>>> mp(1.,0.)
# -1.2889062500000001
```

Most member functions of Potential instances have corresponding functions in the galpy.potential module that allow them to be evaluated for lists of multiple Potential instances (and in versions >=1.4 even for nested lists of Potential instances). galpy.potential.MWPotential2014 is such a list of three Potential instances

```
>>> from galpy.potential import MWPotential2014
>>> print (MWPotential2014)
# [<galpy.potential.PowerSphericalPotentialwCutoff.PowerSphericalPotentialwCutoff_
instance at 0x1089b23b0>, <galpy.potential.MiyamotoNagaiPotential.

instance at 0x1089b23b0>, <galpy.potential.

TwoPowerSphericalPotential.NFWPotential instance at 0x1089b2248>]
```

and we can evaluate the potential by using the evaluatePotentials function

```
>>> from galpy.potential import evaluatePotentials
>>> evaluatePotentials(MWPotential2014,1.,0.)
# -1.3733506513947895
```

Tip: Lists of Potential instances can be nested, allowing you to easily add components to existing gravitational-potential models. For example, to add a DehnenBarPotential to MWPotential2014, you can do: pot= [MWPotential2014, DehnenBarPotential()] and then use this pot everywhere where you can use a list of Potential instances. You can also add potential simply as pot= MWPotential2014+DehnenBarPotential().

Warning: galpy potentials do *not* necessarily approach zero at infinity. To compute, for example, the escape velocity or whether or not an orbit is unbound, you need to take into account the value of the potential at infinity. E.g., $v_{\rm esc}(r) = \sqrt{2[\Phi(\infty) - \Phi(r)]}$.

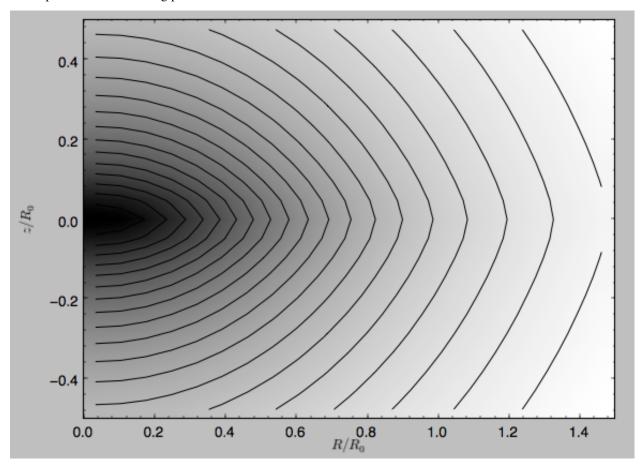
Tip: As discussed in the section on *physical units*, potentials can be initialized and evaluated with arguments specified as a astropy Quantity with units. Use the configuration parameter apy-units = True to get output values as a

Quantity. See also the subsection on Initializing potentials with parameters with units below.

We can plot the potential of axisymmetric potentials (or of non-axisymmetric potentials at phi=0) using the plot member function

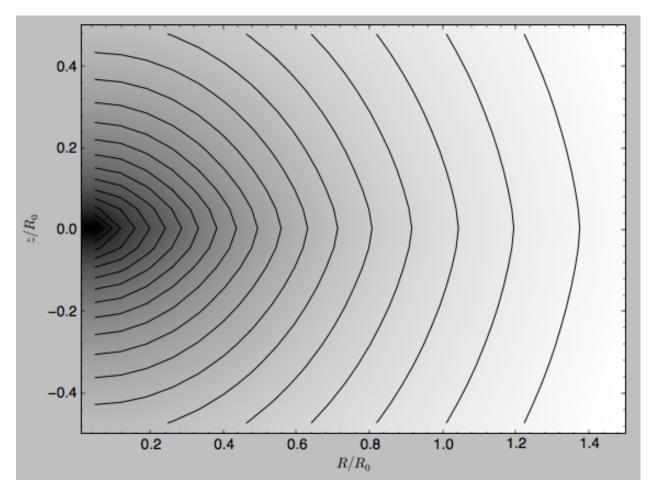
```
>>> mp.plot()
```

which produces the following plot



Similarly, we can plot combinations of Potentials using plotPotentials, e.g.,

```
>>> from galpy.potential import plotPotentials
>>> plotPotentials (MWPotential2014,rmin=0.01)
```



These functions have arguments that can provide custom \mathbb{R} and z ranges for the plot, the number of grid points, the number of contours, and many other parameters determining the appearance of these figures.

galpy also allows the forces corresponding to a gravitational potential to be calculated. Again for the Miyamoto-Nagai Potential instance from above

```
>>> mp.Rforce(1.,0.)
# -1.0
```

This value of -1.0 is due to the normalization of the potential such that the circular velocity is 1. at R=1. Similarly, the vertical force is zero in the mid-plane

```
>>> mp.zforce(1.,0.)
# -0.0
```

but not further from the mid-plane

```
>>> mp.zforce(1.,0.125)
# -0.53488743705310848
```

As explained in *Units in galpy*, these forces are in standard galpy units, and we can convert them to physical units using methods in the galpy.util.bovy_conversion module. For example, assuming a physical circular velocity of 220 km/s at R=8 kpc

```
>>> from galpy.util import bovy_conversion

>>> mp.zforce(1.,0.125)*bovy_conversion.force_in_kmsMyr(220.,8.)

# -3.3095671288657584 #km/s/Myr

>>> mp.zforce(1.,0.125)*bovy_conversion.force_in_2piGmsolpc2(220.,8.)

# -119.72021771473301 #2 \pi G Msol / pc^2
```

Again, there are functions in galpy.potential that allow for the evaluation of the forces for lists of Potential instances, such that

We can evaluate the flattening of the potential as $\sqrt{|z F_R/R F_Z|}$ for a Potential instance as well as for a list of such instances

```
>>> mp.flattening(1.,0.125)
# 0.4549542914935209
>>> from galpy.potential import flattening
>>> flattening(MWPotential2014,1.,0.125)
# 0.61231675305658628
```

1.4.2 Densities

galpy can also calculate the densities corresponding to gravitational potentials. For many potentials, the densities are explicitly implemented, but if they are not, the density is calculated using the Poisson equation (second derivatives of the potential have to be implemented for this). For example, for the Miyamoto-Nagai potential, the density is explicitly implemented

```
>>> mp.dens(1.,0.)
# 1.1145444383277576
```

and we can also calculate this using the Poisson equation

```
>>> mp.dens(1.,0.,forcepoisson=True)
# 1.1145444383277574
```

which are the same to machine precision

```
>>> mp.dens(1.,0.,forcepoisson=True)-mp.dens(1.,0.)
# -2.2204460492503131e-16
```

Similarly, all of the potentials in galpy.potential.MWPotential2014 have explicitly-implemented densities, so we can do

```
>>> from galpy.potential import evaluateDensities
>>> evaluateDensities (MWPotential2014,1.,0.)
# 0.57508603122264867
```

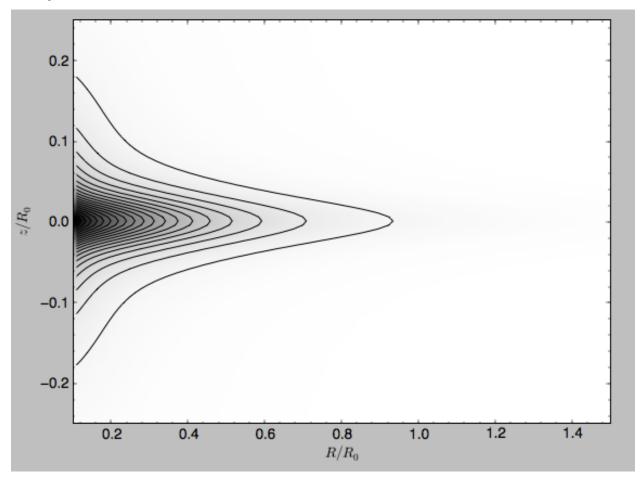
In physical coordinates, this becomes

```
>>> evaluateDensities(MWPotential2014,1.,0.)*bovy_conversion.dens_in_msolpc3(220.,8.) # 0.1010945632524705 #Msol / pc^3
```

We can also plot densities

```
>>> from galpy.potential import plotDensities
>>> plotDensities (MWPotential2014,rmin=0.1,zmax=0.25,zmin=-0.25,nrs=101,nzs=101)
```

which gives



Another example of this is for an exponential disk potential

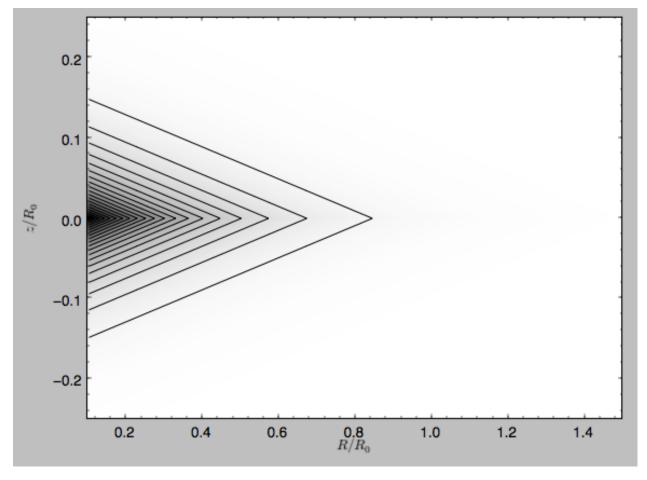
```
>>> from galpy.potential import DoubleExponentialDiskPotential
>>> dp= DoubleExponentialDiskPotential(hr=1./4.,hz=1./20.,normalize=1.)
```

The density computed using the Poisson equation now requires multiple numerical integrations, so the agreement between the analytical density and that computed using the Poisson equation is slightly less good, but still better than a percent

```
>>> (dp.dens(1.,0.,forcepoisson=True)-dp.dens(1.,0.))/dp.dens(1.,0.)
# 0.0032522956769123019
```

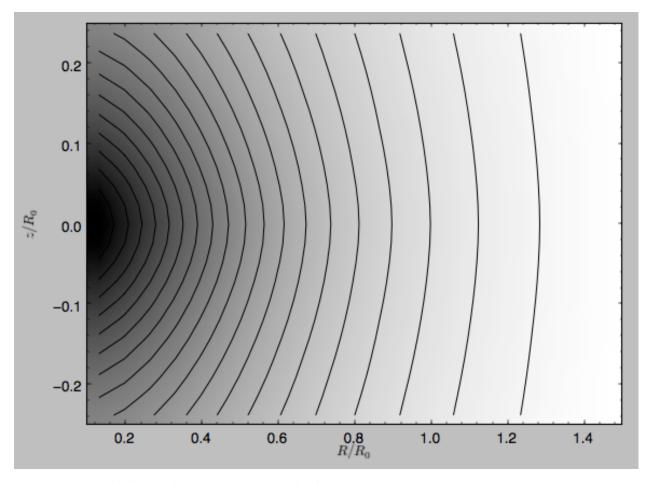
The density is

```
>>> dp.plotDensity(rmin=0.1,zmax=0.25,zmin=-0.25,nrs=101,nzs=101)
```



and the potential is

>>> dp.plot(rmin=0.1, zmin=-0.25, zmax=0.25)



Clearly, the potential is much less flattened than the density.

1.4.3 Modifying potential instances using wrappers

Potentials implemented in galpy can be modified using different kinds of wrappers. These wrappers modify potentials to, for example, change their amplitude as a function of time (e.g., to grow or decay the bar contribution to a potential) or to make a potential rotate. Specific kinds of wrappers are listed on the *Potential wrapper API page*. These wrappers can be applied to instances of *any* potential implemented in galpy (including other wrappers). An example is to grow a bar using the polynomial smoothing of Dehnen (2000). We first setup an instance of a DehnenBarPotential that is essentially fully grown already

```
>>> from galpy.potential import DehnenBarPotential
>>> dpn= DehnenBarPotential(tform=-100.,tsteady=0.) # DehnenBarPotential has a custom_
implementation of growth that we ignore by setting tform to -100
```

and then wrap it

```
>>> from galpy.potential import DehnenSmoothWrapperPotential
>>> dswp= DehnenSmoothWrapperPotential(pot=dpn,tform=-4.*2.*numpy.pi/dpn.OmegaP(),

--tsteady=2.*2.*numpy.pi/dpn.OmegaP())
```

This grows the DehnenBarPotential starting at 4 bar periods before t=0 over a period of 2 bar periods. DehnenBarPotential has an older, custom implementation of the same smoothing and the (tform, tsteady) pair used here corresponds to the default setting for DehnenBarPotential. Thus we can compare the two

```
>>> dp= DehnenBarPotential()
>>> print(dp(0.9,0.3,phi=3.,t=-2.)-dswp(0.9,0.3,phi=3.,t=-2.))
# 0.0
>>> print(dp.Rforce(0.9,0.3,phi=3.,t=-2.)-dswp.Rforce(0.9,0.3,phi=3.,t=-2.))
# 0.0
```

The wrapper SolidBodyRotationWrapperPotential allows one to make any potential rotate around the z axis. This can be used, for example, to make general bar-shaped potentials, which one could construct from a basisfunction expansion with SCFPotential, rotate without having to implement the rotation directly. As an example consider this SoftenedNeedleBarPotential (which has a potential-specific implementation of rotation)

```
>>> sp= SoftenedNeedleBarPotential(normalize=1.,omegab=1.8,pa=0.)
```

The same potential can be obtained from a non-rotating SoftenedNeedleBarPotential run through the SolidBodyRotationWrapperPotential to add rotation

```
>>> sp_still= SoftenedNeedleBarPotential(omegab=0.,pa=0.,normalize=1.)
>>> swp= SolidBodyRotationWrapperPotential(pot=sp_still,omega=1.8,pa=0.)
```

Compare for example

```
>>> print(sp(0.8,0.2,phi=0.2,t=3.)-swp(0.8,0.2,phi=0.2,t=3.))
# 0.0
>>> print(sp.Rforce(0.8,0.2,phi=0.2,t=3.)-swp.Rforce(0.8,0.2,phi=0.2,t=3.))
# 8.881784197e-16
```

Wrapper potentials can be used anywhere in galpy where general potentials can be used. They can be part of lists of Potential instances. They can also be used in C for orbit integration provided that both the wrapper and the potentials that it wraps are implemented in C. For example, a static LogarithmicHaloPotential with a bar potential grown as above would be

```
>>> from galpy.potential import LogarithmicHaloPotential, evaluateRforces
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> pot= lp+dswp
>>> print(evaluateRforces(pot,0.9,0.3,phi=3.,t=-2.))
# -1.00965326579
```

Tip: To simply adjust the amplitude of a Potential instance, you can multiply the instance with a number or divide it by a number. For example, pot= 2.*LogarithmicHaloPotential(amp=1.) is equivalent to pot= LogarithmicHaloPotential(amp=2.). This is useful if you want to, for instance, quickly adjust the mass of a potential.

Warning: When wrapping a potential that has *physical outputs turned on*, the wrapper object inherits the units of the wrapped potential and automatically turns them on, even when you do not explictly set ro= and vo=.

1.4.4 Close-to-circular orbits and orbital frequencies

We can also compute the properties of close-to-circular orbits. First of all, we can calculate the circular velocity and its derivative

```
>>> mp.vcirc(1.)
# 1.0
>>> mp.dvcircdR(1.)
# -0.163777427566978
```

or, for lists of Potential instances

```
>>> from galpy.potential import vcirc
>>> vcirc(MWPotential2014,1.)
# 1.0
>>> from galpy.potential import dvcircdR
>>> dvcircdR(MWPotential2014,1.)
# -0.10091361254334696
```

We can also calculate the various frequencies for close-to-circular orbits. For example, the rotational frequency

```
>>> mp.omegac(0.8)

# 1.2784598203204887

>>> from galpy.potential import omegac

>>> omegac(MWPotential2014,0.8)

# 1.2733514576122869
```

and the epicycle frequency

```
>>> mp.epifreq(0.8)
# 1.7774973530267848
>>> from galpy.potential import epifreq
>>> epifreq(MWPotential2014,0.8)
# 1.7452189766287691
```

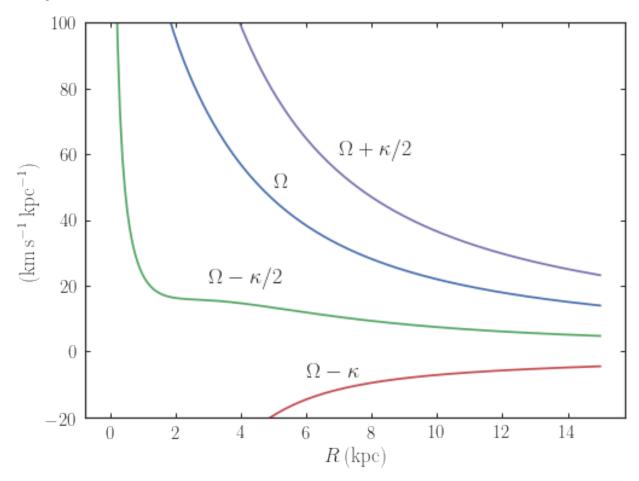
as well as the vertical frequency

```
>>> mp.verticalfreq(1.0)
# 3.7859388972001828
>>> from galpy.potential import verticalfreq
>>> verticalfreq(MWPotential2014,1.)
# 2.7255405754769875
```

We can also for example easily make the diagram of $\Omega - n\kappa/m$ that is important for understanding kinematic spiral density waves. For example, for MWPotential2014

```
>>> from galpy.potential import MWPotential2014, omegac, epifreq
>>> def OmegaMinusKappa(pot, Rs, n, m, ro=8., vo=220.):
        # ro,vo for physical units, Rs in units of ro
        return omegac(pot,Rs/ro,ro=ro,vo=vo)-n/m*epifreq(pot,Rs/ro,ro=ro,vo=vo)
>>> Rs= numpy.linspace(0.,16.,101) # kpc
>>> plot(Rs,OmegaMinusKappa(MWPotential2014,Rs,0,1))
>>> plot(Rs,OmegaMinusKappa(MWPotential2014,Rs,1,2))
>>> plot(Rs,OmegaMinusKappa(MWPotential2014,Rs,1,1))
>>> plot (Rs, OmegaMinusKappa (MWPotential2014, Rs, 1, -2))
>>> ylim(-20.,100.)
>>> xlabel(r'$R\,(\mathrm{kpc})$')
>>> ylabel(r'$(\mathrm{km\,s}^{-1}\,\mathrm{kpc}^{-1})$')
>>> text(3.,21.,r'$\Omega-\kappa/2$',size=18.)
>>> text(5.,50.,r'$\Omega$',size=18.)
>>> text(7.,60.,r'$\Omega+\kappa/2$',size=18.)
>>> text(6.,-7.,r'$\Omega-\kappa$',size=18.)
```





For close-to-circular orbits, we can also compute the radii of the Lindblad resonances. For example, for a frequency similar to that of the Milky Way's bar

```
>>> mp.lindbladR(5./3.,m='corotation') #args are pattern speed and m of pattern # 0.6027911166042229 #~ 5kpc  
>>> print(mp.lindbladR(5./3.,m=2))  
# None  
>>> mp.lindbladR(5./3.,m=-2)  
# 0.9906190683480501
```

The None here means that there is no inner Lindblad resonance, the m=-2 resonance is in the Solar neighborhood (see the section on the *Hercules stream* in this documentation).

1.4.5 Using interpolations of potentials

galpy contains a general Potential class interpRZPotential that can be used to generate interpolations of potentials that can be used in their stead to speed up calculations when the calculation of the original potential is computationally expensive (for example, for the DoubleExponentialDiskPotential). Full details on how to set this up are given *here*. Interpolated potentials can be used anywhere that general three-dimensional galpy potentials can be used. Some care must be taken with outside-the-interpolation-grid evaluations for functions that use C to speed up computations.

1.4.6 Initializing potentials with parameters with units

As already discussed in the section on *physical units*, potentials in galpy can be specified with parameters with units since v1.2. For most inputs to the initialization it is straightforward to know what type of units the input Quantity needs to have. For example, the scale length parameter a= of a Miyamoto-Nagai disk needs to have units of distance.

The amplitude of a potential is specified through the amp= initialization parameter. The units of this parameter vary from potential to potential. For example, for a logarithmic potential the units are velocity squared, while for a Miyamoto-Nagai potential they are units of mass. Check the documentation of each potential on the *API page* for the units of the amp= parameter of the potential that you are trying to initialize and please report an Issue if you find any problems with this.

1.4.7 General density/potential pairs with basis-function expansions

galpy allows for the potential and forces of general, time-independent density functions to be computed by expanding the potential and density in terms of basis functions. This is supported for ellipsoidal-ish as well as for disk-y density distributions, in both cases using the basis-function expansion of the self-consistent-field (SCF) method of Hernquist & Ostriker (1992). On its own, the SCF technique works well for ellipsoidal-ish density distributions, but using a trick due to Kuijken & Dubinski (1995) it can also be made to work well for disky potentials. We first describe the basic SCF implementation and then discuss how to use it for disky potentials.

The basis-function approach in the SCF method is implemented in the SCFPotential class, which is also implemented in C for fast orbit integration. The coefficients of the basis-function expansion can be computed using the scf_compute_coeffs_spherical (for spherically-symmetric density distribution), scf_compute_coeffs_axi (for axisymmetric densities), and scf_compute_coeffs (for the general case). The coefficients obtained from these functions can be directly fed into the SCFPotential initialization. The basis-function expansion has a free scale parameter a, which can be specified for the scf_compute_coeffs_XX functions and for the SCFPotential itself. Make sure that you use the same a! Note that the general functions are quite slow.

The simplest example is that of the Hernquist potential, which is the lowest-order basis function. When we compute the first ten radial coefficients for this density we obtain that only the lowest-order coefficient is non-zero

```
>>> from galpy.potential import HernquistPotential
>>> from galpy.potential import scf_compute_coeffs_spherical
>>> hp= HernquistPotential(amp=1.,a=2.)
>>> Acos, Asin= scf_compute_coeffs_spherical(hp.dens,10,a=2.)
>>> print (Acos)
# array([[[ 1.0000000e+00]],
          [[-2.83370393e-17]]
             3.31150709e-19]],
          [[ -6.66748299e-18]],
          [[ 8.19285777e-18]],
#
          [[ -4.26730651e-19]],
#
          [[ -7.16849567e-19]],
          [[ 1.52355608e-18]],
#
#
          [[-2.24030288e-18]]
          [[ -5.24936820e-19]]])
```

As a more complicated example, consider a prolate NFW potential

```
>>> from galpy.potential import TriaxialNFWPotential
>>> np= TriaxialNFWPotential(normalize=1.,c=1.4,a=1.)
```

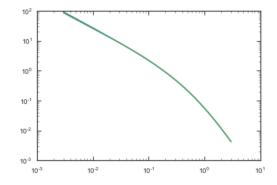
and we compute the coefficients using the axisymmetric scf_compute_coeffs_axi

```
>>> a_SCF= 50. # much larger a than true scale radius works well for NFW
>>> Acos, Asin= scf_compute_coeffs_axi(np.dens,80,40,a=a_SCF)
>>> sp= SCFPotential(Acos=Acos,Asin=Asin,a=a_SCF)
```

If we compare the densities along the R=Z line as

```
>>> xs= numpy.linspace(0.,3.,1001)
>>> loglog(xs,np.dens(xs,xs))
>>> loglog(xs,sp.dens(xs,xs))
```

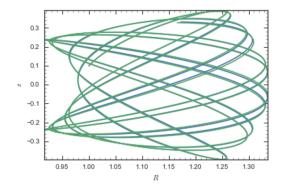
we get



If we then integrate an orbit, we also get good agreement

```
>>> from galpy.orbit import Orbit
>>> o= Orbit([1.,0.1,1.1,0.1,0.3,0.])
>>> ts= numpy.linspace(0.,100.,10001)
>>> o.integrate(ts,hp)
>>> o.plot()
>>> o.integrate(ts,sp)
>>> o.plot(overplot=True)
```

which gives



Near the end of the orbit integration, the slight differences between the original potential and the basis-expansion version cause the two orbits to deviate from each other.

To use the SCF method for disky potentials, we use the trick from Kuijken & Dubinski (1995). This trick works by approximating the disk density as $\rho_{\rm disk}(R,\phi,z) \approx \sum_i \Sigma_i(R) \, h_i(z)$, with $h_i(z) = {\rm d}^2 H(z)/{\rm d}z^2$ and searching for solutions of the form

$$\Phi(R, \phi, z = \Phi_{\rm ME}(R, \phi, z) + 4\pi G \sum_{i} \Sigma_{i}(r) H_{i}(z),$$

where r is the spherical radius $r^2 = R^2 + z^2$. The density which gives rise to $\Phi_{\rm ME}(R,\phi,z)$ is not strongly confined to a plane when $\rho_{\rm disk}(R,\phi,z) \approx \sum_i \Sigma_i(R) \, h_i(z)$ and can be obtained using the SCF basis-function-expansion technique discussed above. See the documentation of the *DiskSCFPotential* class for more details on this procedure.

As an example, consider a double-exponential disk, which we can compare to the DoubleExponentialDiskPotentialimplementation

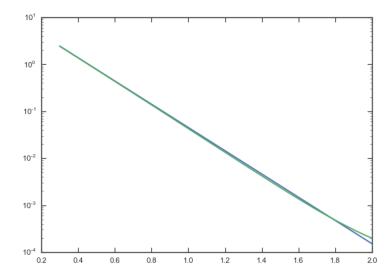
```
>>> from galpy import potential
>>> dp= potential.DoubleExponentialDiskPotential(amp=13.5,hr=1./3.,hz=1./27.)
```

and then setup the DiskSCFPotential approximation to this as

The dens= keyword specifies the target density, while the Sigma= and hz= inputs specify the approximation functions $\Sigma_i(R)$ and $h_i(z)$. These are specified as dictionaries here for a few pre-defined approximation functions, but general functions are supported as well. Care should be taken that the dens= input density and the approximation functions have the same normalization. We can compare the density along the R=10 z line as

```
>>> xs= numpy.linspace(0.3,2.,1001)
>>> semilogy(xs,dp.dens(xs,xs/10.))
>>> semilogy(xs,dscfp.dens(xs,xs/10.))
```

which gives



The agreement is good out to 5 scale lengths and scale heights and then starts to degrade. We can also integrate orbits and compare them

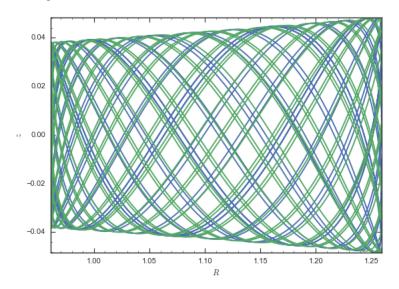
```
>>> from galpy.orbit import Orbit
>>> o= Orbit([1.,0.1,0.9,0.,0.1,0.])
>>> ts= numpy.linspace(0.,100.,10001)
```

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```
>>> o.integrate(ts,dp)
>>> o.plot()
>>> o.integrate(ts,dscfp)
>>> o.plot(overplot=True)
```

which gives



The orbits diverge slightly because the potentials are not quite the same, but have very similar properties otherwise (peri- and apogalacticons, eccentricity, ...). By increasing the order of the SCF approximation, the potential can be gotten closer to the target density. Note that orbit integration in the DiskSCFPotential is much faster than that of the DoubleExponentialDisk potential

```
>>> timeit(o.integrate(ts,dp))
# 1 loops, best of 3: 5.83 s per loop
>>> timeit(o.integrate(ts,dscfp))
# 1 loops, best of 3: 286 ms per loop
```

The SCFPotential and DiskSCFPotential can be used wherever general potentials can be used in galpy.

1.4.8 The potential of N-body simulations

galpy can setup and work with the frozen potential of an N-body simulation. This allows us to study the properties of such potentials in the same way as other potentials in galpy. We can also investigate the properties of orbits in these potentials and calculate action-angle coordinates, using the galpy framework. Currently, this functionality is limited to axisymmetrized versions of the N-body snapshots, although this capability could be somewhat straightforwardly expanded to full triaxial potentials. The use of this functionality requires pynbody to be installed; the potential of any snapshot that can be loaded with pynbody can be used within galpy.

As a first, simple example of this we look at the potential of a single simulation particle, which should correspond to galpy's KeplerPotential. We can create such a single-particle snapshot using pynbody by doing

```
>>> import pynbody
>>> s= pynbody.new(star=1)
>>> s['mass']= 1.
>>> s['eps']= 0.
```

and we get the potential of this snapshot in galpy by doing

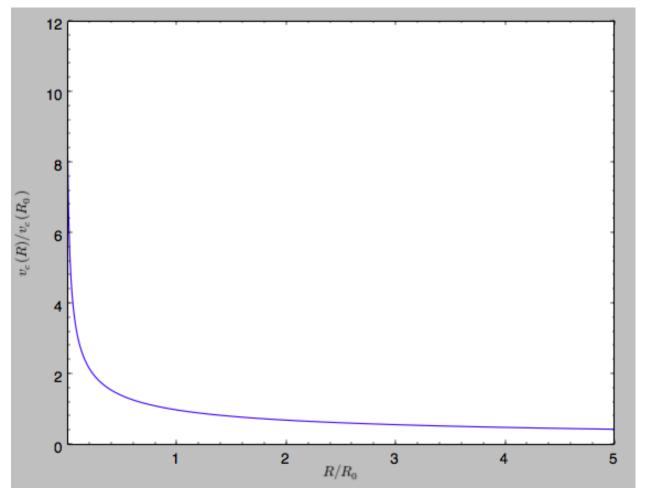
```
>>> from galpy.potential import SnapshotRZPotential
>>> sp= SnapshotRZPotential(s,num_threads=1)
```

With these definitions, this snapshot potential should be the same as KeplerPotential with an amplitude of one, which we can test as follows

```
>>> from galpy.potential import KeplerPotential
>>> kp= KeplerPotential(amp=1.)
>>> print(sp(1.1,0.),kp(1.1,0.),sp(1.1,0.)-kp(1.1,0.))
# (-0.909090909090906, -0.90909090909091, 0.0)
>>> print(sp.Rforce(1.1,0.),kp.Rforce(1.1,0.),sp.Rforce(1.1,0.)-kp.Rforce(1.1,0.))
# (-0.82644628099173545, -0.8264462809917353, -1.1102230246251565e-16)
```

SnapshotRZPotential instances can be used wherever other galpy potentials can be used (note that the second derivatives have not been implemented, such that functions depending on those will not work). For example, we can plot the rotation curve

```
>>> sp.plotRotcurve()
```



Because evaluating the potential and forces of a snapshot is computationally expensive, most useful applications of frozen N-body potentials employ interpolated versions of the snapshot potential. These can be setup in galpy using an InterpSnapshotRZPotential class that is a subclass of the interpRZPotential described above and that can be used in the same manner. To illustrate its use we will make use of one of pynbody's example snapshots, g15784. This snapshot is used here to illustrate pynbody's use. Please follow the instructions there on how to

download this snapshot.

Once you have downloaded the pynbody testdata, we can load this snapshot using

```
>>> s = pynbody.load('testdata/g15784.lr.01024.gz')
```

(please adjust the path according to where you downloaded the pynbody testdata). We get the main galaxy in this snapshot, center the simulation on it, and align the galaxy face-on using

```
>>> h = s.halos()
>>> h1 = h[1]
>>> pynbody.analysis.halo.center(h1,mode='hyb')
>>> pynbody.analysis.angmom.faceon(h1, cen=(0,0,0),mode='ssc')
```

we also convert the simulation to physical units, but set G=1 by doing the following

```
>>> s.physical_units()
>>> from galpy.util.bovy_conversion import _G
>>> g= pynbody.array.SimArray(_G/1000.)
>>> g.units= 'kpc Msol**-1 km**2 s**-2 G**-1'
>>> s._arrays['mass']= s._arrays['mass']*g
```

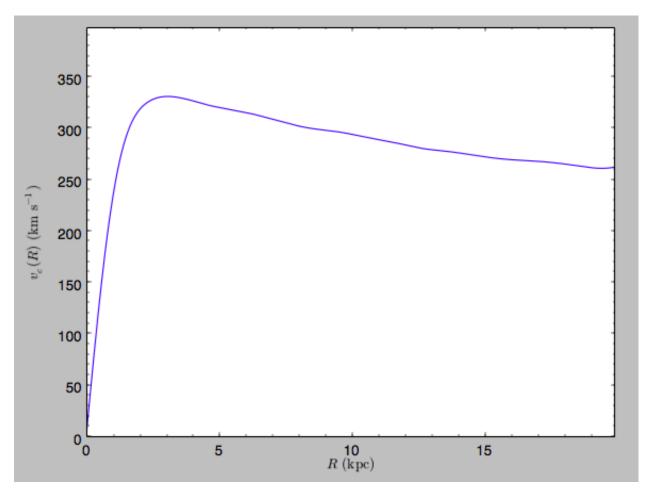
We can now load an interpolated version of this snapshot's potential into galpy using

```
>>> from galpy.potential import InterpSnapshotRZPotential
>>> spi= InterpSnapshotRZPotential(h1,rgrid=(numpy.log(0.01),numpy.log(20.),101),

-logR=True,zgrid=(0.,10.,101),interpPot=True,zsym=True)
```

where we further assume that the potential is symmetric around the mid-plane (z=0). This instantiation will take about ten to fiteen minutes. This potential instance has *physical* units (and thus the rgrid= and zgrid= inputs are given in kpc if the simulation's distance unit is kpc). For example, if we ask for the rotation curve, we get the following:

```
>>> spi.plotRotcurve(Rrange=[0.01,19.9],xlabel=r'$R\,(\mathrm{kpc})$',ylabel=r'$v_ \hookrightarrowc(R)\,(\mathrm{km\,s}^{-1})$')
```



This can be compared to the rotation curve calculated by pynbody, see here.

Because galpy works best in a system of *natural units* as explained in *Units in galpy*, we will convert this instance to natural units using the circular velocity at R=10 kpc, which is

```
>>> spi.vcirc(10.)
# 294.62723076942245
```

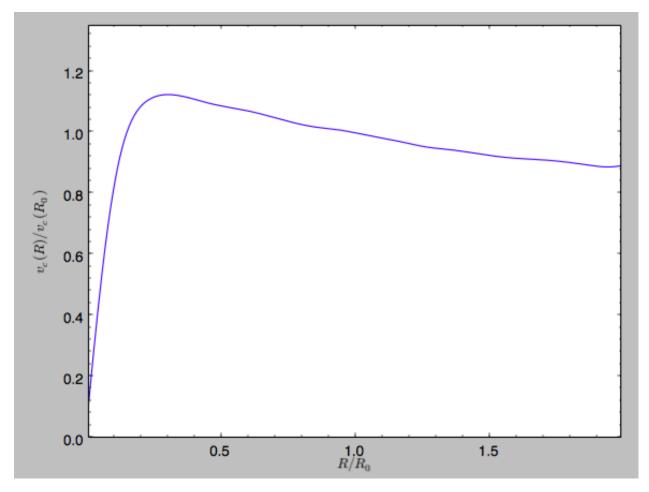
To convert to natural units we do

```
>>> spi.normalize(R0=10.)
```

We can then again plot the rotation curve, keeping in mind that the distance unit is now R_0

```
>>> spi.plotRotcurve(Rrange=[0.01,1.99])
```

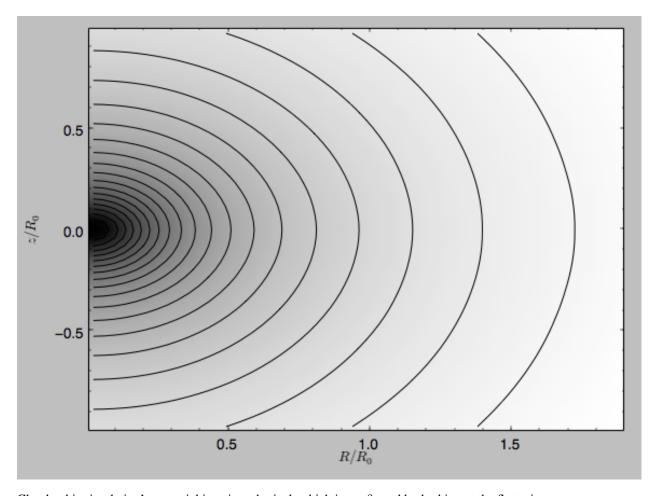
which gives



in particular

We can also plot the potential

```
>>> spi.plot(rmin=0.01,rmax=1.9,nrs=51,zmin=-0.99,zmax=0.99,nzs=51)
```



Clearly, this simulation's potential is quite spherical, which is confirmed by looking at the flattening

```
>>> spi.flattening(1.,0.1)
# 0.86675711023391921
>>> spi.flattening(1.5,0.1)
# 0.94442750306256895
```

The epicyle and vertical frequencies can also be interpolated by setting the interpepifreq=True or interpverticalfreq=True keywords when instantiating the InterpSnapshotRZPotential object.

1.4.9 Conversion to NEMO potentials

NEMO is a set of tools for studying stellar dynamics. Some of its functionality overlaps with that of galpy, but many of its programs are very complementary to galpy. In particular, it has the ability to perform N-body simulations with a variety of poisson solvers, which is currently not supported by galpy (and likely will never be directly supported). To encourage interaction between galpy and NEMO it is quite useful to be able to convert potentials between these two frameworks, which is not completely trivial. In particular, NEMO contains Walter Dehnen's fast collisionless gyrfalcon code (see 2000ApJ...536L..39D and 2002JCoPh.179...27D) and the discussion here focuses on how to run N-body simulations using external potentials defined in galpy.

Some galpy potential instances support the functions nemo_accname and nemo_accpars that return the name of the NEMO potential corresponding to this galpy Potential and its parameters in NEMO units. These functions assume that you use NEMO with WD_units, that is, positions are specified in kpc, velocities in kpc/Gyr, times in Gyr, and G=1. For the Miyamoto-Nagai potential above, you can get its name in the NEMO framework as

```
>>> mp.nemo_accname()
# 'MiyamotoNagai'
```

and its parameters as

```
>>> mp.nemo_accpars(220.,8.)
# '0,592617.11132,4.0,0.3'
```

assuming that we scale velocities by vo=220 km/s and positions by ro=8 kpc in galpy. These two strings can then be given to the gyrfalcoN accname= and accpars= keywords.

We can do the same for lists of potentials. For example, for MWPotential2014 we do

```
>>> from galpy.potential import nemo_accname, nemo_accpars
>>> nemo_accname (MWPotential2014)
# 'PowSphwCut+MiyamotoNagai+NFW'
>>> nemo_accpars (MWPotential2014,220.,8.)
# '0,1001.79126907,1.8,1.9#0,306770.418682,3.0,0.28#0,16.0,162.958241887'
```

Therefore, these are the accname= and accpars= that one needs to provide to gyrfalcON to run a simulation in MWPotential 2014.

Note that the NEMO potential PowSphwCut is *not* a standard NEMO potential. This potential can be found in the nemo/ directory of the galpy source code; this directory also contains a Makefile that can be used to compile the extra NEMO potential and install it in the correct NEMO directory (this requires one to have NEMO running, i.e., having sourced nemo_start).

You can use the PowSphwCut.cc file in the nemo/ directory as a template for adding additional potentials in galpy to the NEMO framework. To figure out how to convert the normalized galpy potential to an amplitude when scaling to physical coordinates (like kpc and kpc/Gyr), one needs to look at the scaling of the radial force with R. For example, from the definition of MiyamotoNagaiPotential, we see that the radial force scales as R^{-2} . For a general scaling $R^{-\alpha}$, the amplitude will scale as $V_0^2 R_0^{\alpha-1}$ with the velocity V_0 and position R_0 of the v=1 at R=1 normalization. Therefore, for the MiyamotoNagaiPotential, the physical amplitude scales as $V_0^2 R_0$. For the LogarithmicHaloPotential, the radial force scales as R^{-1} , so the amplitude scales as V_0^2 .

Currently, only the MiyamotoNagaiPotential, NFWPotential, PowerSphericalPotentialwCutoff, HernquistPotential, PlummerPotential, MN3ExponentialDiskPotential, and the LogarithmicHaloPotential have this NEMO support. Combinations of all but the LogarithmicHaloPotential are allowed in general (e.g., MWPotential2014); they can also be combined with spherical LogarithmicHaloPotentials. Because of the definition of the logarithmic potential in NEMO, it cannot be flattened in z, so to use a flattened logarithmic potential, one has to flip y and z between galpy and NEMO (one can flatten in y).

1.4.10 Conversion to AMUSE potentials

AMUSE is a Python software framework for astrophysical simulations, in which existing codes from different domains, such as stellar dynamics, stellar evolution, hydrodynamics and radiative transfer can be easily coupled. AMUSE allows you to run N-body simulations that include a wide range of physics (gravity, stellar evolution, hydrodynamics, radiative transfer) with a large variety of numerical codes (collisionless, collisional, etc.).

The galpy.potential.to_amuse function allows you to create an AMUSE representation of any galpy potential. This is useful, for instance, if you want to run a simulation of a stellar cluster in an external gravitational field, because galpy has wide support for representing external gravitational fields. Creating the AMUSE representation is as simple as (for MWPotential2014):

```
>>> from galpy.potential import to_amuse, MWPotential2014
>>> mwp_amuse= to_amuse(MWPotential2014)
>>> print(mwp_amuse)
# <galpy.potential.amuse.galpy_profile object at 0x7f6b366d13c8>
```

Schematically, this potential can then be used in AMUSE as

```
>>> gravity = bridge.Bridge(use_threading=False)
>>> gravity.add_system(cluster_code, (mwp_amuse,))
>>> gravity.add_system(mwp_amuse,)
```

where cluster_code is a code to perform the N-body integration of a system (e.g., a BHTree in AMUSE). A fuller example is given below.

AMUSE uses physical units when interacting with the galpy potential and it is therefore necessary to make sure that the correct physical units are used. The to_amuse function takes the galpy unit conversion parameters ro= and vo= as keyword parameters to perform the conversion between internal galpy units and physical units; if these are not explicitly set, to_amuse attempts to set them automatically using the potential that you input using the galpy. util.bovy_conversion.get_physical function.

Another difference between <code>galpy</code> and AMUSE is that in AMUSE integration times can only be positive and they have to increase in time. <code>to_amuse</code> takes as input the <code>t=</code> and <code>tgalpy=</code> keywords that specify (a) the initial time in AMUSE and (b) the initial time in <code>galpy</code> that this time corresponds to. Typically these will be the same (and equal to zero), but if you want to run a simulation where the initial time in <code>galpy</code> is negative it is useful to give them different values. The time inputs can be either given in <code>galpy</code> internal units or using AMUSE's units. Similarly, to integrate backwards in time in AMUSE, <code>to_amuse</code> has a keyword <code>reverse=(default: False)</code> that reverses the time direction given to the <code>galpy</code> potential; <code>reverse=True</code> does this (note that you also have to flip the velocities to actually go backwards).

A full example of setting up a Plummer-sphere cluster and evolving its N-body dynamics using an AMUSE BHTree in the external MWPotential2014 potential is:

```
>>> from amuse.lab import *
>>> from amuse.couple import bridge
>>> from amuse.datamodel import Particles
>>> from galpy.potential import to_amuse, MWPotential2014
>>> from galpy.util import bovy_plot
>>>
>>> # Convert galpy MWPotential 2014 to AMUSE representation
>>> mwp_amuse= to_amuse (MWPotential2014)
>>>
>>> # Set initial cluster parameters
>>> N= 1000
>>> Mcluster= 1000. | units.MSun
>>> Rcluster= 10. | units.parsec
>>> Rinit= [10.,0.,0.] | units.kpc
>>> Vinit= [0.,220.,0.] | units.km/units.s
>>> # Setup star cluster simulation
>>> tend= 100.0 | units.Myr
>>> dtout= 5.0 | units.Myr
>>> dt= 1.0 | units.Myr
>>>
>>> def setup_cluster(N, Mcluster, Rcluster, Rinit, Vinit):
       converter= nbody_system.nbody_to_si(Mcluster,Rcluster)
       stars= new_plummer_sphere(N, converter)
       stars.x+= Rinit[0]
>>>
       stars.y+= Rinit[1]
>>>
```

(continues on next page)

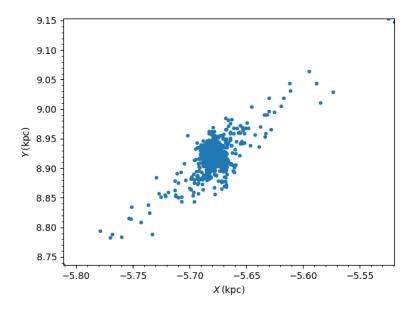
(continued from previous page)

```
>>>
       stars.z+= Rinit[2]
>>>
        stars.vx+= Vinit[0]
        stars.vy+= Vinit[1]
>>>
        stars.vz+= Vinit[2]
>>>
>>>
        return stars, converter
>>> # Setup cluster
>>> stars,converter= setup_cluster(N,Mcluster,Rcluster,Rinit,Vinit)
>>> cluster_code= BHTree(converter,number_of_workers=1) #Change number of workers_
\hookrightarrow depending no. of CPUs
>>> cluster_code.parameters.epsilon_squared= (3. | units.parsec) **2
>>> cluster_code.parameters.opening_angle= 0.6
>>> cluster_code.parameters.timestep= dt
>>> cluster_code.particles.add_particles(stars)
>>> # Setup channels between stars particle dataset and the cluster code
>>> channel_from_stars_to_cluster_code= stars.new_channel_to(cluster_code.particles,
                                            attributes=["mass", "x", "y", "z", "vx",
>>>

'''\\''', "\\'Z"])

>>> channel_from_cluster_code_to_stars= cluster_code.particles.new_channel_to(stars,
                                            attributes=["mass", "x", "y", "z", "vx",
→"VV", "VZ"])
>>>
>>> # Setup gravity bridge
>>> gravity= bridge.Bridge(use_threading=False)
>>> # Stars in cluster_code depend on gravity from external potential mwp_amuse (i.e.,
→ MWPotential2014)
>>> gravity.add_system(cluster_code, (mwp_amuse,))
>>> # External potential mwp_amuse still needs to be added to system so it evolves_
⇔with time
>>> gravity.add_system(mwp_amuse,)
>>> # Set how often to update external potential
>>> gravity.timestep= cluster_code.parameters.timestep/2.
>>> # Evolve
>>> time= 0.0 | tend.unit
>>> while time<tend:
       gravity.evolve_model(time+dt)
>>>
        # If you want to output or analyze the simulation, you need to copy
>>>
       # stars from cluster_code
        #channel_from_cluster_code_to_stars.copy()
>>>
       # If you edited the stars particle set, for example to remove stars from the
>>>
       # array because they have been kicked far from the cluster, you need to
       # copy the array back to cluster_code:
>>>
>>>
        #channel_from_stars_to_cluster_code.copy()
>>>
        # Update time
       time= gravity.model_time
>>>
>>> channel_from_cluster_code_to_stars.copy()
>>> gravity.stop()
>>>
>>> bovy_plot.bovy_plot(stars.x.value_in(units.kpc),stars.y.value_in(units.kpc),'.',
>>>
                        xlabel=r'$X\,(\mathbf{kpc})$',ylabel=r'$Y\,(\mathbf{kpc})$')
```

After about 30 seconds, you should get a plot like the following, which shows a cluster in the first stages of disruption:



1.4.11 Dissipative forces

While almost all of the forces that you can use in galpy derive from a potential (that is, the force is the gradient of a scalar function, the potential, meaning that the forces are *conservative*), galpy also supports dissipative forces. Dissipative forces all inherit from the DissipativeForce class and they are required to take the velocity v=[vR, vT, vZ] in cylindrical coordinates as an argument to the force in addition to the standard (R, z, phi=0, t=0). The set of functions evaluateXforces (with X=R, z, r, phi, etc.) will evaluate the force due to Potential instances, DissipativeForce instances, or lists of combinations of these two.

Currently, the only dissipative force implemented in galpy is *ChandrasekharDynamicalFrictionForce*, an implementation of the classic Chandrasekhar dynamical-friction formula, with recent tweaks to better represent the results from *N*-body simulations.

Warning: Dissipative forces can currently only be used for 3D orbits in galpy. The code should throw an error when they are used for 2D orbits.

1.4.12 Adding potentials to the galpy framework

Potentials in galpy can be used in many places such as orbit integration, distribution functions, or the calculation of action-angle variables, and in most cases any instance of a potential class that inherits from the general Potential class (or a list of such instances) can be given. For example, all orbit integration routines work with any list of instances of the general Potential class. Adding new potentials to galpy therefore allows them to be used everywhere in galpy where general Potential instances can be used. Adding a new class of potentials to galpy consists of the following series of steps (for steps to add a new wrapper potential, also see *the next section*):

1. Implement the new potential in a class that inherits from galpy.potential.Potential (velocity-dependent forces should inherit from galpy.potential.DissipativeForce instead; see below for a brief discussion on differences in implementing such forces). The new class should have an __init__ method that sets up the necessary parameters for the class. An amplitude parameter amp= and two units parameters ro= and vo= should be taken as an argument for this class and before performing any other setup, the galpy.potential.Potential.__init__(self,amp=amp,ro=ro,vo=vo,amp_units=) method should be called to setup the amplitude and the system of units; the amp_units= keyword specifies the physical units of the amplitude parameter (e.g., amp_units='velocity2' when the

units of the amplitude are velocity-squared) To add support for normalizing the potential to standard galpy units, one can call the galpy.potential.Potential.normalize function at the end of the __init__ function.

The new potential class should implement some of the following functions:

- _evaluate(self,R,z,phi=0,t=0) which evaluates the potential itself (without the amp factor, which is added in the call method of the general Potential class).
- _Rforce(self, R, z, phi=0., t=0.) which evaluates the radial force in cylindrical coordinates (-d potential / dR).
- _zforce(self,R,z,phi=0.,t=0.) which evaluates the vertical force in cylindrical coordinates (-d potential / d z).
- _R2deriv(self, R, z, phi=0., t=0.) which evaluates the second (cylindrical) radial derivative of the potential (d^2 potential / d R^2).
- _z2deriv(self,R,z,phi=0.,t=0.) which evaluates the second (cylindrical) vertical derivative of the potential (d^2 potential / d z^2).
- _Rzderiv (self, R, z, phi=0., t=0.) which evaluates the mixed (cylindrical) radial and vertical derivative of the potential (d^2 potential / d R d z).
- _dens(self, R, z, phi=0., t=0.) which evaluates the density. If not given, the density is computed using the Poisson equation from the first and second derivatives of the potential (if all are implemented).
- _mass (self, R, z=0., t=0.) which evaluates the mass. For spherical potentials this should give the mass enclosed within the spherical radius; for axisymmetric potentials this should return the mass up to R and between -Z and Z. If not given, the mass is computed by integrating the density (if it is implemented or can be calculated from the Poisson equation).
- _phiforce(self,R,z,phi=0.,t=0.): the azimuthal force in cylindrical coordinates (assumed zero if not implemented).
- _phi2deriv (self, R, z, phi=0., t=0.): the second azimuthal derivative of the potential in cylindrical coordinates (d^2 potential / d phi^2; assumed zero if not given).
- _Rphideriv(self, R, z, phi=0., t=0.): the mixed radial and azimuthal derivative of the potential in cylindrical coordinates (d^2 potential / d R d phi; assumed zero if not given).
- OmegaP (self): returns the pattern speed for potentials with a pattern speed (used to compute the Jacobi integral for orbits).

If you want to be able to calculate the concentration for a potential, you also have to set self._scale to a scale parameter for your potential.

The code for galpy.potential.MiyamotoNagaiPotential gives a good template to follow for 3D axisymmetric potentials. Similarly, the code for galpy.potential. CosmphiDiskPotential provides a good template for 2D, non-axisymmetric potentials.

During development or if some of the forces or second derivatives are too tedious to implement, it is possible to numerically compute any non-implemented forces and second derivatives by inheriting from the *NumericalPotentialDerivativesMixin* class. Thus, a functioning potential can be implemented by simply implementing the _evaluate function and adding all forces and second derivatives using the NumericalPotentialDerivativesMixin.

After this step, the new potential will work in any part of galpy that uses pure python potentials. To get the potential to work with the C implementations of orbit integration or action-angle calculations, the potential also has to be implemented in C and the potential has to be passed from python to C (see below).

The __init__ method should be written in such a way that a relevant object can be initialized using Classname() (i.e., there have to be reasonable defaults given for all parameters, including the amplitude); doing this allows the nose tests for potentials to automatically check that your Potential's potential function, force functions, second derivatives, and density (through the Poisson equation) are correctly implemented (if they are implemented). The continuous-integration platform that builds the galpy codebase upon code pushes will then automatically test all of this, streamlining push requests of new potentials.

A few attributes need to be set depending on the potential: hasC=True for potentials for which the forces and potential are implemented in C (see below); self.hasC_dxdv=True for potentials for which the (planar) second derivatives are implemented in C; self.hasC_dens=True for potentials for which the density is implemented in C as well (necessary for them to work with dynamical friction in C); self.isNonAxi=True for non-axisymmetric potentials.

2. To add a C implementation of the potential, implement it in a .c file under potential/potential_c_ext. Look at potential/potential_c_ext/LogarithmicHaloPotential.c for the right format for 3D, axisymmetric potentials, or at potential/potential_c_ext/LopsidedDiskPotential.c for 2D, non-axisymmetric potentials.

For orbit integration, the functions such as:

- double LogarithmicHaloPotentialRforce(double R,double Z, double phi,double t,struct potentialArg * potentialArgs)
- double LogarithmicHaloPotentialzforce(double R,double Z, double phi,double t,struct potentialArg * potentialArgs)

are most important. For some of the action-angle calculations

double LogarithmicHaloPotentialEval(double R,double Z, double phi,double t,struct potentialArg * potentialArgs)

is most important (i.e., for those algorithms that evaluate the potential). If you want your potential to be able to be used as the density for the *ChandrasekharDynamicalFrictionForce* implementation in C, you need to implement the density in C as well

• double LogarithmicHaloPotentialDens(double R,double Z, double phi,double t,struct potentialArg * potentialArgs)

The arguments of the potential are passed in a potential Args structure that contains args, which are the arguments that should be unpacked. Again, looking at some example code will make this clear. The potential Args structure is defined in potential /potential _c_ext/galpy_potentials.h.

- $3. \ Add \ the \ potential's \ function \ declarations \ to \ potential/potential_c_ext/galpy_potentials.h$
- 4. (4. and 5. for planar orbit integration) Edit the code under orbit/orbit_c_ext/integratePlanarOrbit.c to set up your new potential (in the parse_leapFuncArgs function).
- 5. Edit the code in orbit/integratePlanarOrbit.py to set up your new potential (in the _parse_pot function).
- 6. Edit the code under orbit/orbit_c_ext/integrateFullOrbit.c to set up your new potential (in the parse_leapFuncArgs_Full function).
- 7. Edit the code in orbit/integrateFullOrbit.py to set up your new potential (in the _parse_pot function).
- 8. Finally, add self.hasC= True to the initialization of the potential in question (after the initialization of the super class, or otherwise it will be undone). If you have implemented the necessary second derivatives for integrating phase-space volumes, also add self.hasC_dxdv=True. If you have implemented the density in C, set self.hasC_dens=True.

After following the relevant steps, the new potential class can be used in any galpy context in which C is used to speed up computations.

Velocity-dependent forces (e.g., Chandrasekhar Dynamical Friction Force) should inherit from galpy.potential. Dissipative Force instead of from galpy.potential. Potential. Because such forces are not conservative, you only need to implement the forces themselves, in the same way as for a regular Potential. For dissipative forces, the force-evaluation functions (Rforce, etc.) need to take the velocity in cylindrical coordinates as a keyword argument: v = [vR, vT, vZ]. Implementing dissipative forces in C is similar: you only need to implement the forces themselves and the forces should take the velocity in cylindrical coordinates as an additional input, e.g.,

• double ChandrasekharDynamicalFrictionForceRforce(double R,double z, double phi,double t,struct potentialArg * potentialArgs,double vR,double vT,double vz)

1.4.13 Adding wrapper potentials to the galpy framework

Wrappers all inherit from the general WrapperPotential or planarWrapperPotential classes (which themselves inherit from the Potential and planarPotential classes and therefore all wrappers are Potentials or planarPotentials). Depending on the complexity of the wrapper, wrappers can be implemented much more economically in Python than new Potential instances as described above.

To add a Python implementation of a new wrapper, classes need to inherit from parentWrapperPotential, take the potentials to be wrapped as a pot= (a Potential, planarPotential, or a list thereof; automatically assigned to self._pot) input to __init__, and implement the _wrap(self,attribute,*args,**kwargs) function. This function modifies the Potential functions _evaluate, _Rforce, etc. (all of those listed above), with attribute the function that is being modified. Inheriting from parentWrapperPotential gives the class access to the self._wrap_pot_func(attribute) function which returns the relevant function for each attribute. For example, self._wrap_pot_func('_evaluate') returns the evaluatePotentials function that can then be called as self._wrap_pot_func('_evaluate') (self._pot,R,Z,phi=phi,t=t) to evaluate the potentials being wrapped. By making use of self._wrap_pot_func, wrapper potentials can be implemented in just a few lines. Your __init__ function should only initialize things in your wrapper; there is no need to manually assign self._pot or to call the superclass' __init__ (all automatically done for you!).

To correctly work with both 3D and 2D potentials, inputs to _wrap need to be specified as *args, **kwargs: grab the values you need for R, z, phi, t from these as R=args[0], z=0 if len(args) == 1 else args[1], phi=kwargs.get('phi',0.), t=kwargs.get('t',0.), where the complicated expression for z is to correctly deal with both 3D and 2D potentials (of course, if your wrapper depends on z, it probably doesn't make much sense to apply it to a 2D planarPotential; you could check the dimensionality of self._pot in your wrapper's __init__ function with from galpy.potential.Potential._dim and raise an error if it is not 3 in this case). Wrapping a 2D potential automatically results in a wrapper that is a subclass of planarPotential rather than Potential; this is done by the setup in parentWrapperPotential and hidden from the user. For wrappers of planar Potentials, self._wrap_pot_func(attribute) will return the evaluateplanarPotentials etc. functions instead, but this is again hidden from the user if you implement the _wrap function as explained above.

As an example, for the DehnenSmoothWrapperPotential, the _wrap function is

```
def _wrap(self,attribute,*args,**kwargs):
    return self._smooth(kwargs.get('t',0.))\
        *self._wrap_pot_func(attribute)(self._pot,*args,**kwargs)
```

where smooth(t) returns the smoothing function of the amplitude. When any of the basic Potential functions are called (_evaluate, _Rforce, etc.), _wrap gets called by the superclass WrapperPotential, and the _wrap function returns the corresponding function for the wrapped potentials with the amplitude modified by smooth(t). Therefore, one does not need to implement each of the _evaluate, _Rforce, etc. functions like for regular potential. The rest of the DehnenSmoothWrapperPotential is essentially (slightly simplified in non-crucial aspects)

```
def __init__(self,amp=1.,pot=None,tform=-4.,tsteady=None,ro=None,vo=None):
    # Note: (i) don't assign self._pot and (ii) don't run super.__init__
    self. tform= tform
    if tsteady is None:
        self._tsteady= self._tform/2.
    else:
        self._tsteady= self._tform+tsteady
    self.hasC= True
    self.hasC_dxdv= True
def _smooth(self,t):
    #Calculate relevant time
    if t < self._tform:</pre>
        smooth= 0.
    elif t < self._tsteady:</pre>
        deltat= t-self._tform
        xi= 2.*deltat/(self._tsteady-self._tform)-1.
        smooth= (3./16.*xi**5.-5./8*xi**3.+15./16.*xi+.5)
    else: #bar is fully on
        smooth= 1.
    return smooth
```

The source code for DehnenSmoothWrapperPotential potential may act as a guide to implementing new wrappers.

C implementations of potential wrappers can also be added in a similar way as C implementations of regular potentials (all of the steps listed in the *previous section* for adding a potential to C need to be followed). All of the necessary functions (...Rforce, ...zforce, ...phiforce, etc.) need to be implemented separately, but by including galpy_potentials.h calling the relevant functions of the wrapped potentials is easy. Look at DehnenSmoothWrapperPotential.c for an example that can be straightforwardly edited for other wrappers.

The glue between Python and C for wrapper potentials needs to glue both the wrapper and the wrapped potentials. This can be easily achieved by recursively calling the <code>_parse_pot</code> glue functions in Python (see the previous section; this needs to be done separately for each potential currently) and the <code>parse_leapFuncArgs</code> and <code>parse_leapFuncArgs_Full</code> functions in C (done automatically for all wrappers). Again, following the example of <code>DehnenSmoothWrapperPotential.py</code> should allow for a straightforward implementation of the glue for any new wrappers. Wrapper potentials should be given negative potential types in the glue to distinguish them from regular potentials.

1.4.14 Adding dissipative forces to the galpy framework

Dissipative forces are implemented in much the same way as forces that derive from potentials. Rather than inheriting from galpy.potential.Potential, dissipative forces inherit from galpy.potential. DissipativeForce. The procedure for implementing a new class of dissipative force is therefore very similar to that for *implementing a new potential*. The main differences are that (a) you only need to implement the forces and (b) the forces are required to take an extra keyword argument v= that gives the velocity in cylindrical coordinates (because dissipative forces will in general depend on the current velocity). Thus, the steps are:

1. Implement the new dissipative force in a class that inherits from galpy.potential. DissipativeForce. The new class should have an __init__ method that sets up the necessary parameters for the class. An amplitude parameter amp= and two units parameters ro= and vo= should be taken as an argument for this class and before performing any other setup, the galpy.potential. DissipativeForce.__init__ (self,amp=amp,ro=ro,vo=vo,amp_units=) method should be called to setup the amplitude and the system of units; the amp_units= keyword specifies the physical units of the amplitude parameter (e.g., amp_units='mass' when the units of the amplitude are mass)

The new dissipative-force class should implement the following functions:

- _Rforce(self, R, z, phi=0., t=0., v=None) which evaluates the radial force in cylindrical coordinates
- _phiforce(self,R,z,phi=0.,t=0.,v=None) which evaluates the azimuthal force in cylindrical coordinates
- _zforce(self,R,z,phi=0.,t=0.,v=None) which evaluates the vertical force in cylindrical coordinates

The code for galpy.potential.ChandrasekharDynamicalFrictionForce gives a good template to follow.

2. That's it, as for now there is no support for implementing a C version of dissipative forces.

1.5 A closer look at orbit integration

1.5.1 Orbit initialization

Standard initialization

Orbits can be initialized in various coordinate frames. The simplest initialization gives the initial conditions directly in the Galactocentric cylindrical coordinate frame (or in the rectangular coordinate frame in one dimension). Orbit () automatically figures out the dimensionality of the space from the initial conditions in this case. In three dimensions initial conditions are given either as [R, vR, vT, z, vz, phi] or one can choose not to specify the azimuth of the orbit and initialize with [R, vR, vT, z, vz]. Since potentials in galpy are easily initialized to have a circular velocity of one at a radius equal to one, initial coordinates are best given as a fraction of the radius at which one specifies the circular velocity, and initial velocities are best expressed as fractions of this circular velocity. For example,

```
>>> from galpy.orbit import Orbit
>>> o= Orbit([1.,0.1,1.1,0.,0.1,0.])
```

initializes a fully three-dimensional orbit, while

```
>>> o= Orbit([1.,0.1,1.1,0.,0.1])
```

initializes an orbit in which the azimuth is not tracked, as might be useful for axisymmetric potentials.

In two dimensions, we can similarly specify fully two-dimensional orbits o=Orbit([R, vR, vT, phi]) or choose not to track the azimuth and initialize with o=Orbit([R, vR, vT]).

In one dimension we simply initialize with o = Orbit([x, vx]).

Initialization with physical units

Orbits are normally used in galpy's *natural coordinates*. When Orbits are initialized using a distance scale ro= and a velocity scale vo=, then many Orbit methods return quantities in physical coordinates. Specifically, physical distance and velocity scales are specified as

```
>>> op= Orbit([1.,0.1,1.1,0.,0.1,0.],ro=8.,vo=220.)
```

All output quantities will then be automatically be specified in physical units: kpc for positions, km/s for velocities, (km/s)^2 for energies and the Jacobi integral, km/s kpc for the angular momentum o.L() and actions, 1/Gyr for frequencies, and Gyr for times and periods. See below for examples of this.

The actual initial condition can also be specified in physical units. For example, the Orbit above can be initialized as

In this case, it is unnecessary to specify the ro= and vo= scales; when they are not specified, ro and vo are set to the default values from the *configuration file*. However, if they are specified, then those values rather than the ones from the configuration file are used.

Tip: If you do input and output in physical units, the internal unit conversion specified by ro= and vo= does not matter!

Inputs to any Orbit method can also be specified with units as an astropy Quantity. galpy's natural units are still used under the hood, as explained in the section on *physical units in galpy*. For example, integration times can be specified in Gyr if you want to integrate for a specific time period.

If for any output you do *not* want the output in physical units, you can specify this by supplying the keyword argument use_physical=False.

Initialization from observed coordinates or astropy SkyCoord

For orbit integration and characterization of observed stars or clusters, initial conditions can also be specified directly as observed quantities when radec=True is set (see further down in this section on how to use an astropy SkyCoord instead). In this case a full three-dimensional orbit is initialized as o= Orbit ([RA,Dec,distance,pmRA,pmDec,Vlos],radec=True) where RA and Dec are expressed in degrees, the distance is expressed in kpc, proper motions are expressed in mas/yr (pmra = pmra' * cos[Dec]), and Vlos is the heliocentric line-of-sight velocity given in km/s. The observed epoch is currently assumed to be J2000.00. These observed coordinates are translated to the Galactocentric cylindrical coordinate frame by assuming a Solar motion that can be specified as either solarmotion='hogg' (2005ApJ...629..268H), solarmotion='dehnen' (1998MNRAS.298..387D) or solarmotion='schoenrich' (default; 2010MNRAS.403.1829S). A circular velocity can be specified as vo=220 in km/s and a value for the distance between the Galactic center and the Sun can be given as ro=8.0 in kpc (e.g., 2012ApJ...759..131B). While the inputs are given in physical units, the orbit is initialized assuming a circular velocity of one at the distance of the Sun (that is, the orbit's position and velocity is scaled to galpy's natural units after converting to the Galactocentric coordinate frame, using the specified ro= and vo=). The parameters of the coordinate transformations are stored internally, such that they are automatically used for relevant outputs (for example, when the RA of an orbit is requested). An example of all of this is:

```
>>> o= Orbit([20.,30.,2.,-10.,20.,50.],radec=True,ro=8.,vo=220.)
```

However, the internally stored position/velocity vector is

```
>>> print(o.vxvv)
# [1.1480792664061401, 0.1994859759019009, 1.8306295160508093, -0.13064400474040533,_

\( \to 0.58167185623715167, 0.14066246212987227 \)
```

and is therefore in *natural* units.

Tip: Initialization using observed coordinates can also use units. So, for example, proper motions can be specified as 2*units.mas/units.yr.

Similarly, one can also initialize orbits from Galactic coordinates using o= Orbit([glon,glat,distance, pmll,pmbb,Vlos],lb=True), where glon and glat are Galactic longitude and latitude expressed in degrees, and

the proper motions are again given in mas/yr ((pmll = pmll' * cos[glat]):

```
>>> o= Orbit([20.,30.,2.,-10.,20.,50.],lb=True,ro=8.,vo=220.)
>>> print(o.vxvv)
# [0.79959714332811838, 0.073287283885367677, 0.5286278286083651, 0.12748861331872263,

$\to$ 0.89074407199364924, 0.0927414387396788]
```

When radec=True or lb=True is set, velocities can also be specified in Galactic coordinates if UVW=True is set. The input is then [RA, Dec, distance, U, V, W], where the velocities are expressed in km/s. U is, as usual, defined as -vR (minus vR).

Finally, orbits can also be initialized using an astropy.coordinates.SkyCoord object. For example, the (ra,dec) example from above can also be initialized as:

In this case, you can still specify the properties of the transformation to Galactocentric coordinates using the standard ro, vo, zo, and solarmotion keywords, or you can use the SkyCoord Galactocentric frame specification and these are propagated to the Orbit instance. For example,

A subtlety here is that the <code>galcen_distance</code> and ro keywords are not interchangeable, because the former is the distance between the Sun and the Galactic center and ro is the projection of this distance onto the Galactic midplane. Another subtlety is that the <code>astropy</code> Galactocentric frame is a right-handed frame, while galpy normally uses a left-handed frame, so the sign of the x component of <code>galcen_v_sun</code> is the opposite of what it would be in <code>solarmotion</code>. Because the Galactocentric frame in <code>astropy</code> does not specify the circular velocity, but only the Sun's velocity, you still need to specify vo to use a non-default circular velocity.

When orbits are initialized using radec=True, lb=True, or using a SkyCoord physical scales ro= and vo= are automatically specified (because they have defaults of ro=8 and vo=220). Therefore, all output quantities will be specified in physical units (see above). If you do want to get outputs in galpy's natural coordinates, you can turn this behavior off by doing

```
>>> o.turn_physical_off()
```

All outputs will then be specified in galpy's natural coordinates.

Initializing multiple objects at once

In all of the examples above, the Orbit instance corresponds to a single object, but Orbit instances can also contain and analyze multiple objects at once. This makes handling Orbit instances highly convenient and also allows for efficient handling of multiple objects. Many of the most computationally-intense methods have been parallelized (orbit integration; analytic eccentricity, zmax, etc. calculation; action-angle calculations) and some other methods switch to more efficient algorithms for larger numbers of objects (e.g., rguiding).

All of the methods for initializing Orbit instances above work for multiple objects. Specifically, the initial conditions can be:

- Array of arbitrary shape (shape, phasedim); needs to be in internal units (for Quantity input; see 'list' option below or use
 - in Galactocentric cylindrical coordinates with phase-space coordinates arranged as [R,vR,vT(,z,vz,phi)];
 - [ra,dec,d,mu_ra, mu_dec,vlos] or [l,b,d,mu_l, mu_b, vlos] in [deg,deg,kpc,mas/yr,mas/yr,km/s], or [ra,dec,d,U,V,W] or [l,b,d,U,V,W] in [deg,deg,kpc,km/s,km/s,kms] (ICRS where relevant; mu_ra = mu_ra * cos dec and mu_l = mu_l * cos); use the radec=, lb=, and UVW= keywords as before
- astropy (>v3.0) SkyCoord with arbitrary shape, including velocities;
- · lists of initial conditions, entries can be
 - individual Orbit instances (of single objects)
 - Quantity arrays arranged as in the first bullet above (so things like [R,vR,vT,z,vz,phi], where R, vR,
 ... can be arbitrary shape Quantity arrays)
 - list of Quantities (so things like [R1,vR1,...], where R1, vR1,... are scalar Quantities
 - None: assumed to be the Sun; if None occurs in a list it is assumed to be the Sun and all other items in the list are assumed to be [ra,dec,...]; cannot be combined with Quantity lists
 - lists of scalar phase-space coordinates arranged as in the first bullet above (so things like [R,vR,...] where R.vR are scalars in internal units

Tip: For multiple object initialization using an array or SkyCoord, arbitrary input shapes are supported.

An example initialization with an array is:

```
>>> vxvvs= numpy.array([[1.,0.1,1.,0.1,-0.2,1.5],[0.1,0.3,1.1,-0.3,0.4,2.]])
>>> orbits= Orbit(vxvvs)
>>> print(orbits.R())
# [1. 0.1]
```

and with a SkyCoord:

```
>>> numpy.random.seed(1)
>>> nrand= 30
>>> ras= numpy.random.uniform(size=nrand) *360.*u.deg
>>> decs= 90.*(2.*numpy.random.uniform(size=nrand)-1.)*u.deg
>>> dists= numpy.random.uniform(size=nrand) *10.*u.kpc
>>> pmras= 20.*(2.*numpy.random.uniform(size=nrand)-1.)*20.*u.mas/u.yr
>>> pmdecs= 20.*(2.*numpy.random.uniform(size=nrand)-1.)*20.*u.mas/u.yr
>>> vloss= 200.*(2.*numpy.random.uniform(size=nrand)-1.)*u.km/u.s
# Without any custom coordinate-transformation parameters
>>> co= SkyCoord(ra=ras,dec=decs,distance=dists,
                 pm_ra_cosdec=pmras,pm_dec=pmdecs,
                 radial_velocity=vloss,
                 frame='icrs')
>>> orbits= Orbit(co)
>>> print(orbits.ra()[:3],ras[:3])
                                       4.11749371e-02] deg [ 1.50127922e+02
    1.50127922e+02 2.59316818e+02
→59316818e+02
                4.11749342e-02] deg
```

As before, you can use the SkyCoord Galactocentric frame specification here.

Orbit instances containing multiple objects act like numpy arrays in many ways, but have some subtly different behaviors for some functions. For example, one can do:

```
>>> print(len(orbits))
# 30
>>> print(orbits.shape)
# (30,)
>>> print(orbits.size)
# 30
>>> orbits.reshape((6,5)) # reshape is done inplace
>>> print(len(orbits))
# 6
>>> print(orbits.shape)
# (6,5)
>>> print(orbits.size)
# 30
>>> sliced_orbits= orbits[:3,1:5] # Extract a subset using numpy's slicing rules
>>> print(sliced_orbits.shape)
# (3,4)
>>> single_orbit= orbits[1,3] # Extract a single object
>>> print(single_orbit.shape)
# ()
```

Slicing creates a new Orbit instance. When slicing an Orbit instance that has been integrated, the integrated orbit will be transferred to the new instance.

The shape of the Orbit instances is retained for all relevant outputs. Continuing on from the previous example (where orbits has shape (6,5) after we reshaped it), we have:

```
>>> print(orbits.R().shape)
# (6,5)
>>> print(orbits.L().shape)
# (6,5,3)
```

After orbit integration, evaluating orbits.R(times) would return an array with shape (6,5, ntimes) here.

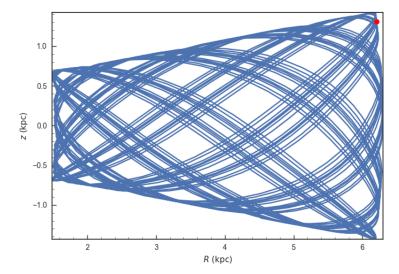
Initialization from an object's name

A convenience method, Orbit.from_name, is also available to initialize orbits from the name of an object. For example, for the star Lacaille 8760:

```
>>> o= Orbit.from_name('Lacaille 8760', ro=8., vo=220.)
>>> [o.ra(), o.dec(), o.dist(), o.pmra(), o.pmdec(), o.vlos()]
# [319.31362023999276, -38.86736390000036, 0.003970940656277758, -3258.5529999996584, -1145.3959999996205, 20.5600000000006063]
```

but this also works for globular clusters, e.g., to obtain Omega Cen's orbit and current location in the Milky Way do:

```
>>> o= Orbit.from_name('Omega Cen')
>>> from galpy.potential import MWPotential2014
>>> ts= numpy.linspace(0.,100.,2001)
>>> o.integrate(ts,MWPotential2014)
>>> o.plot()
>>> plot([o.R()],[o.z()],'ro')
```



We see that Omega Cen is currently close to its maximum distance from both the Galactic center and from the Galactic midplane (note that Omega Cen's phase-space coordinates were updated internally in galpy after this plot was made and the orbit is now slightly different).

Similarly, you can do:

```
>>> o= Orbit.from_name('LMC')
>>> [o.ra(), o.dec(), o.dist(), o.pmra(), o.pmdec(), o.vlos()]
# [80.89420000000055, -69.756099999999947, 49.999999999999, 1.90999999999, 0.

$\times 2290000000000037, 262.199999999999]
```

It is also possible to initialize using multiple names, for example:

```
>>> o= Orbit.from_name(['LMC','SMC'])
>>> print(o.ra(),o.dec(),o.dist())
# [ 80.8942 13.1583] deg [-69.7561 -72.8003] deg [ 50. 60.] kpc
```

The names are stored in the name attribute:

```
>>> print(o.name)
# ['LMC', 'SMC']
```

The Orbit.from_name method attempts to resolve the name of the object in SIMBAD, and then use the observed coordinates found there to generate an Orbit instance. In order to query SIMBAD, Orbit.from_name requires the astroquery package to be installed. A small number of objects, mainly Milky Way globular clusters and dwarf satellite galaxies, have their phase-space coordinates stored in a file that is part of galpy and for these objects the values from this file are used rather than querying SIMBAD. Orbit.from_name supports tab completion in IPython/Jupyter for this list of objects

```
In [1]: from galpy.orbit import Orbit
```

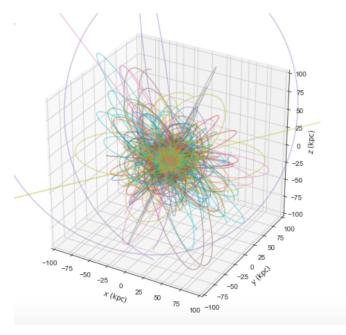
In [2]: Orbit.fro	m_name('			
ro=	47Tuc	BH229	Djorg2	
solarmotion=	Arp2	BH261	E1	
vo=	BH176	Crater	E3	>
zo=	BH184	Djorg1	ES028006	

The Orbit.from_name method also allows you to load some collections of objects in a simple manner. Currently, three collections are supported: 'MW globular clusters', 'MW satellite galaxies', and 'solar system'. Specifying 'MW globular clusters' loads all of the Milky-Way globular clusters with data from Gaia DR2 (using the Vasiliev 2019 catalog):

```
>>> o= Orbit.from_name('MW globular clusters')
>>> print(len(o))
# 150
>>> print(o.name)
# ['NGC5286', 'Terzan12', 'Arp2', 'NGC5024', ...]
>>> print(o.r())
# [ 8.86999028  3.33270877  21.42173795  18.41411889, ...]
```

It is then easy to, for example, integrate the orbits of all Milky-Way globular clusters in MWPotential2014 and plot them in 3D:

```
>>> ts= numpy.linspace(0.,300.,1001)
>>> o.integrate(ts,MWPotential2014)
>>> o.plot3d(alpha=0.4)
>>> xlim(-100.,100.)
>>> ylim(-100.,100)
>>> gca().set_zlim3d(-100.,100);
```



Similarly, 'MW satellite galaxies' loads all of the Milky-Way satellite galaxies from Fritz et al. (2018):

(continues on next page)

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```
79.06498854 23.70062857 364.87901007 141.29780146 116.28700298
128.81345239 83.47228672 147.90512269 25.68800918 272.93146472
227.39435987 154.7574384 173.77516411 49.60813235 418.76813979
181.9540996 32.92030664 19.06561359 84.81046251 27.67609888
42.13122436 60.28760354 88.86197382 34.58139798 53.79147743
21.05413655 101.85224099 40.70060809 77.72601419 42.65853777] kpc
```

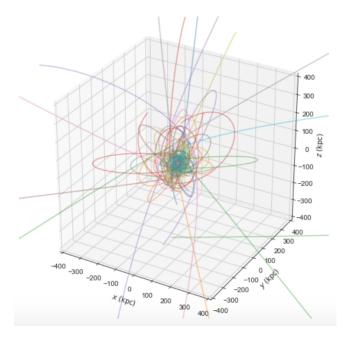
and we can integrate and plot them in 3D as above:

```
>>> o.plot3d(alpha=0.4)

>>> xlim(-400.,400.)

>>> ylim(-400.,400)

>>> gca().set_zlim3d(-400.,400)
```



Because MWPotential2014 has a relatively low-mass dark-matter halo, a bunch of the satellites are unbound (to make them bound, you can increase the mass of the halo by, for example, multiplying it by 1.5, as in MWPotential2014[2] *= 1.5).

Finally, for illustrative purposes, the solar system is included as a collection as well. The solar system is set up such that the center of what is normally the Galactocentric coordinate frame in galpy is now the solar system barycenter and the coordinate frame is a heliocentric one. The solar system data are taken from Bovy et al. (2010) and they represent the positions and planets on April 1, 2009. To load the solar system do:

```
>>> o= Orbit.from_name('solar system')
```

Giving for example:

```
>>> print(o.name)
# ['Mercury', 'Venus', 'Earth', 'Mars', 'Jupiter', 'Saturn', 'Uranus', 'Neptune']
```

You can then, for example, integrate the solar system for 10 years as follows

```
>>> import astropy.units as u
>>> from galpy.potential import KeplerPotential
```

(continues on next page)

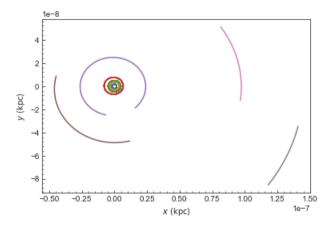
(continued from previous page)

```
>>> from galpy.util.bovy_conversion import get_physical
>>> kp= KeplerPotential(amp=1.*u.Msun,**get_physical(o)) # Need to use **get_physical_

→ to get the ro= and vo= parameters, which differ from the default for the solar_

→ system
>>> ts= numpy.linspace(0.,10.,1001)*u.yr
>>> o.integrate(ts,kp)
>>> o.plot(d1='x',d2='y')
```

which gives



Note that, as usual, physical outputs are in kpc, leading to very small numbers!

Tip: Setting up an Orbit instance *without* arguments will return an Orbit instance representing the Sun: o= Orbit(). This instance has physical units *turned on by default*, so methods will return outputs in physical units unless you o.turn_physical_off().

Warning: Orbits initialized using Orbit.from_name have physical output *turned on by default*, so methods will return outputs in physical units unless you o.turn_physical_off().

1.5.2 Orbit integration

After an orbit is initialized, we can integrate it for a set of times ts, given as a numpy array. For example, in a simple logarithmic potential we can do the following

```
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> o= Orbit([1.,0.1,1.1,0.,0.1,0.])
>>> import numpy
>>> ts= numpy.linspace(0,100,10000)
>>> o.integrate(ts,lp)
```

to integrate the orbit from t=0 to t=100, saving the orbit at 10000 instances. In physical units, we can integrate for 10 Gyr as follows

```
>>> from astropy import units
>>> ts= numpy.linspace(0,10.,10000)*units.Gyr
>>> o.integrate(ts,lp)
```

Warning: When the integration times are not specified using a Quantity, they are assumed to be in natural units.

If we initialize the Orbit using a distance scale ro= and a velocity scale vo=, then Orbit plots and outputs will use physical coordinates (currently, times, positions, and velocities)

```
>>> op= Orbit([1.,0.1,1.1,0.,0.1,0.],ro=8.,vo=220.) #Use Vc=220 km/s at R= 8 kpc as 

the normalization
>>> op.integrate(ts,lp)
```

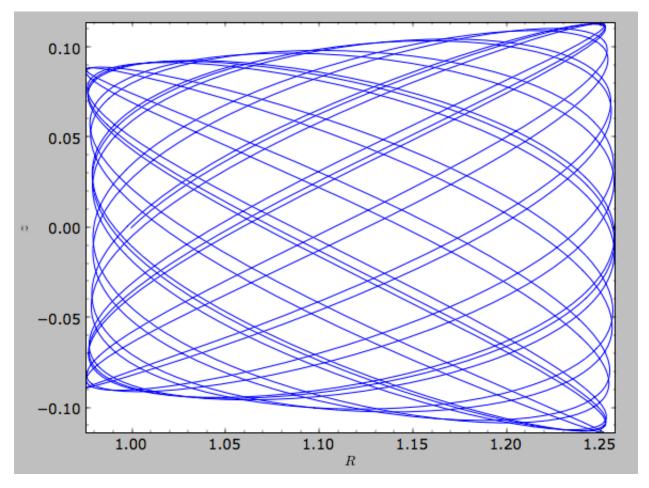
An Orbit instance containing multiple objects can be integrated in the same way and the orbit integration will be performed in parallel on machines with multiple cores. For the fast C integrators (*see below*), this parallelization is done using OpenMP in C and requires one to set the OMP_NUM_THREADS environment variable to control the number of cores used. The Python integrators are parallelized in Python and by default also use the OMP_NUM_THREADS variable to set the number of cores (but for the Python integrators this can be overwritten). A simple example is

1.5.3 Displaying the orbit

After integrating the orbit, it can be displayed by using the plot () function. The quantities that are plotted when plot () is called depend on the dimensionality of the orbit: in 3D the (R,z) projection of the orbit is shown; in 2D either (X,Y) is plotted if the azimuth is tracked and (R,vR) is shown otherwise; in 1D (x,vx) is shown. E.g., for the example given above,

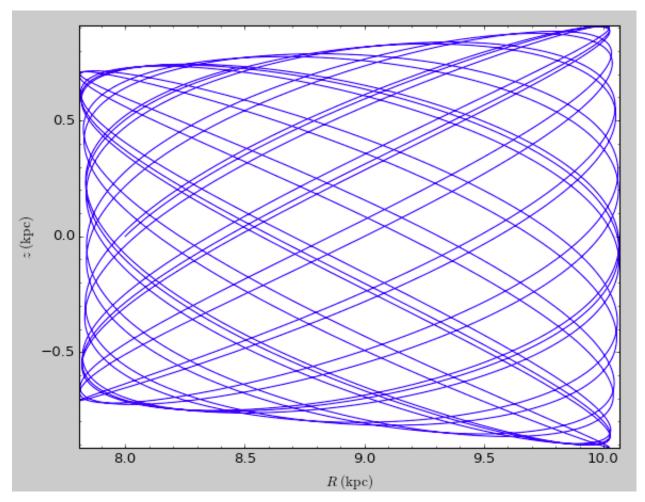
```
>>> o.plot()
```

gives



If we do the same for the Orbit that has physical distance and velocity scales associated with it, we get the following

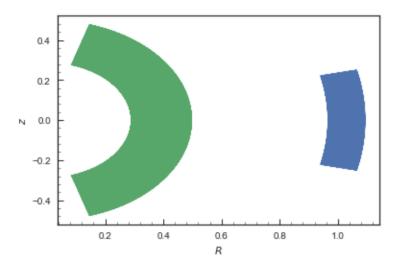
>>> op.plot()



If we call op.plot (use_physical=False), the quantities will be displayed in natural galpy coordinates. Plotting an Orbit instance that consists of multiple objects plots all objects at once, e.g.,

```
>>> orbits.plot()
```

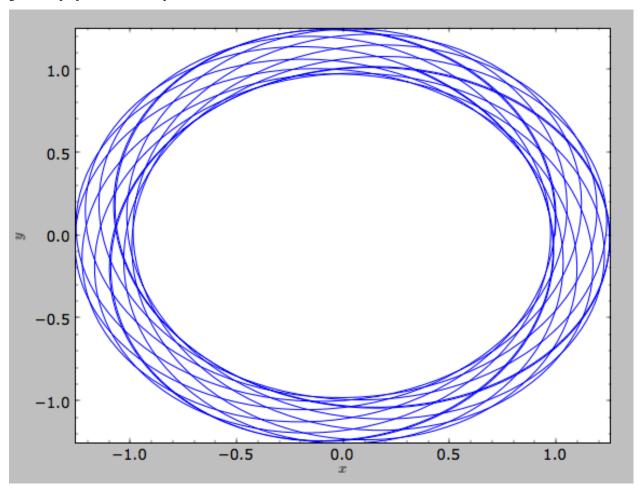
gives



Other projections of the orbit can be displayed by specifying the quantities to plot. E.g.,

```
>>> o.plot(d1='x',d2='y')
```

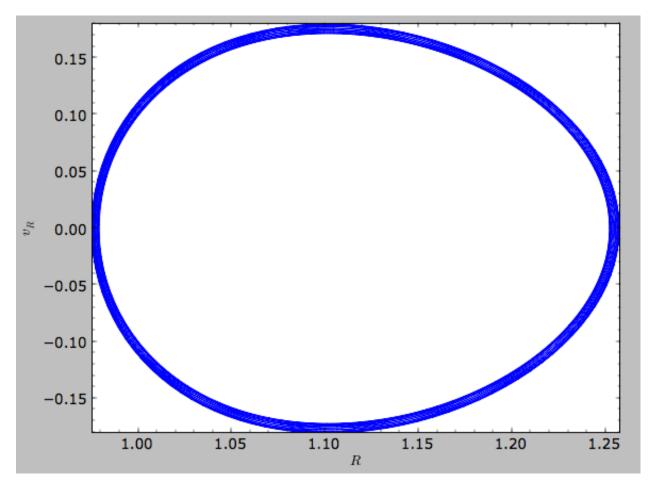
gives the projection onto the plane of the orbit:



while

```
>>> o.plot(d1='R',d2='vR')
```

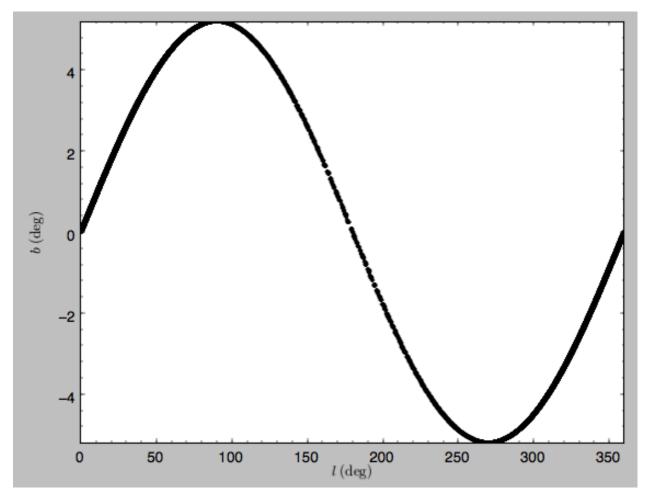
gives the projection onto (R,vR):



We can also plot the orbit in other coordinate systems such as Galactic longitude and latitude

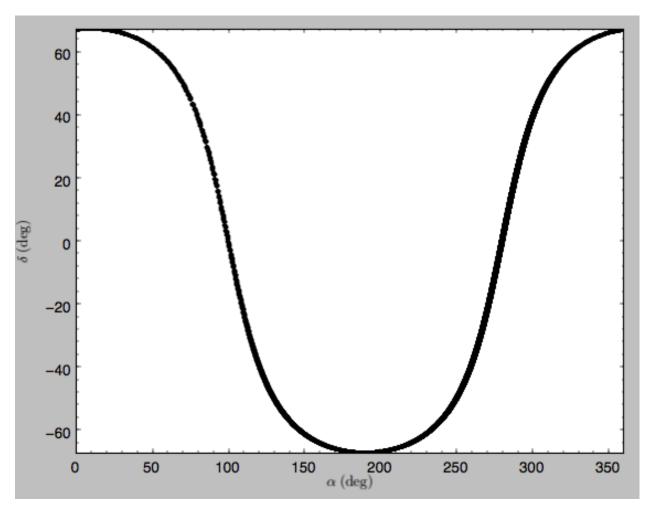
```
>>> o.plot('k.',d1='ll',d2='bb')
```

which shows



or RA and Dec

```
>>> o.plot('k.',d1='ra',d2='dec')
```

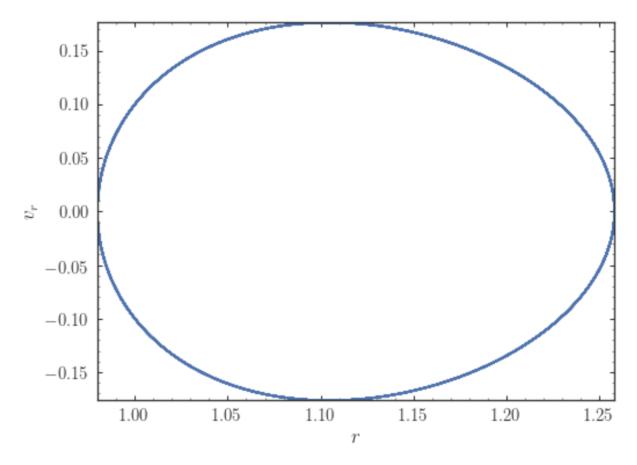


See the documentation of the o.plot function and the o.ra(), o.ll(), etc. functions on how to provide the necessary parameters for the coordinate transformations.

It is also possible to plot quantities computed from the basic Orbit outputs like o.x(), o.r(), etc. For this to work, the numexpr module needs to be installed; this can be done using pip or conda. Then you can ask for plots like

```
>>> o.plot(d1='r',d2='vR*R/r+vz*z/r')
```

where d2= converts the velocity to spherical coordinates (this is currently not a pre-defined option). This gives the following orbit (which is closed in this projection, because we are using a spherical potential):



You can also do more complex things like

which shows the orbit in the regular (x,y) frame as well as in a (x,y) frame that is rotating at the angular frequency of the orbit. When doing more complex calculations like this, you need to make sure that you are getting the units right: parameters param in the expression you provide are directly evaluated as o.param(), which depending on how you setup the object may or may not return output in physical units. The expression above is safe, because o.Op evaluated like this will be in a consistent unit system with the rest of the expression. Expressions cannot contain astropy Quantities (these cannot be parsed by the parser), which is why quantity=False is specified; this is also used internally.

Finally, it is also possible to plot arbitrary functions of time with Orbit.plot, by specifying d1= or d2= as a function. For example, to display the orbital velocity in the spherical radial direction, which we also did with the expression above, you can do the following

For a function like this, just specifying it as the expression d2="vR*R/r+vz*z/r" is much more convenient, but expressions that cannot be parsed automatically could be directly given as a function.

1.5.4 Animating the orbit

Warning: Animating orbits is a new, experimental feature at this time that may be changed in later versions. It has only been tested in a limited fashion. If you are having problems with it, please open an Issue and list all relevant details about your setup (python version, jupyter version, browser, any error message in full). It may also be helpful to check the javascript console for any errors.

In a jupyter notebook or in jupyterlab (jupyterlab versions >= 0.33) you can also create an animation of an orbit *after* you have integrated it. For example, to do this for the op orbit from above (but only integrated for 2 Gyr to create a shorter animation as an example here), do

```
>>> op.animate()
```

This will create the following animation

Tip: There is currently no option to save the animation within galpy, but you could use screen capture software (for example, QuickTime's Screen Recording feature) to record your screen while the animation is running and save it as a video.

animate has options to specify the width and height of the resulting animation, and it can also animate up to three projections of an orbit at the same time. For example, we can look at the orbit in both (x,y) and (R,z) at the same time with

```
>>> op.animate(d1=['x','R'],d2=['y','z'],width=800)
```

which gives

If you want to embed the animation in a webpage, you can obtain the necessary HTML using the <code>_repr_html_()</code> function of the IPython.core.display.HTML object returned by <code>animate</code>. By default, the HTML includes the entire orbit's data, but <code>animate</code> also has an option to store the orbit in a separate <code>JSON</code> file that will then be loaded by the output HTML code.

animate also works in principle for Orbit instances containing multiple objects, but in practice the resulting animation is very slow once more than a few orbits/projections are used.

1.5.5 Orbit characterization

The properties of the orbit can also be found using galpy. For example, we can calculate the peri- and apocenter radii of an orbit, its eccentricity, and the maximal height above the plane of the orbit

```
>>> o.rap(), o.rperi(), o.e(), o.zmax()
# (1.2581455175173673,0.97981663263371377,0.12436710999105324,0.11388132751079502)
```

or for multiple objects at once

```
>>> orbits.rap(), orbits.rperi(), orbits.e(), orbits.zmax()
# (array([ 1.0918143 ,  0.49557137]),
# array([ 0.96779816,  0.29150873]),
# array([ 0.06021334,  0.2592654 ]),
# array([ 0.24734084,  0.47327396]))
```

These four quantities can also be computed using analytical means (exact or approximations depending on the potential) by specifying analytic=True

or for multiple objects at once (this calculation is done in parallel on systems that support it)

We can also calculate the energy of the orbit, either in the potential that the orbit was integrated in, or in another potential:

```
>>> o.E(), o.E(pot=mp)
# (0.615000000000001, -0.6739062500000015)
```

where mp is the Miyamoto-Nagai potential of Introduction: Rotation curves.

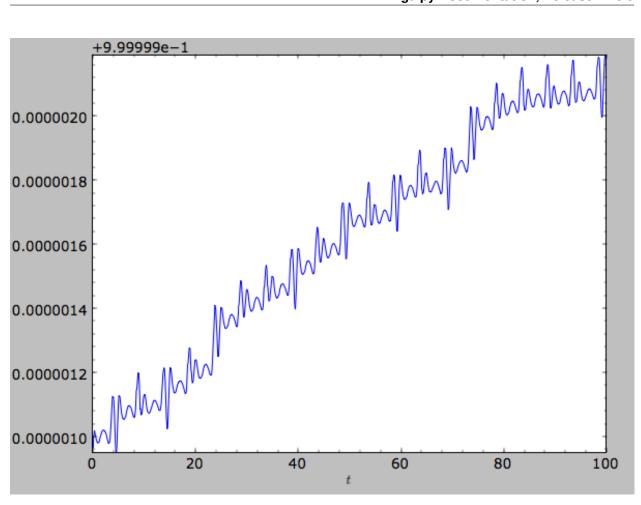
For the Orbit op that was initialized above with a distance scale ro= and a velocity scale vo=, these outputs are all in physical units

```
>>> op.rap(), op.rperi(), op.e(), op.zmax()
# (10.065158988860341,7.8385312810643057,0.12436696983841462,0.91105035688072711) #kpc
>>> op.E(), op.E(pot=mp)
# (29766.000000000004, -32617.062500000007) #(km/s)^2
```

We can also show the energy as a function of time (to check energy conservation)

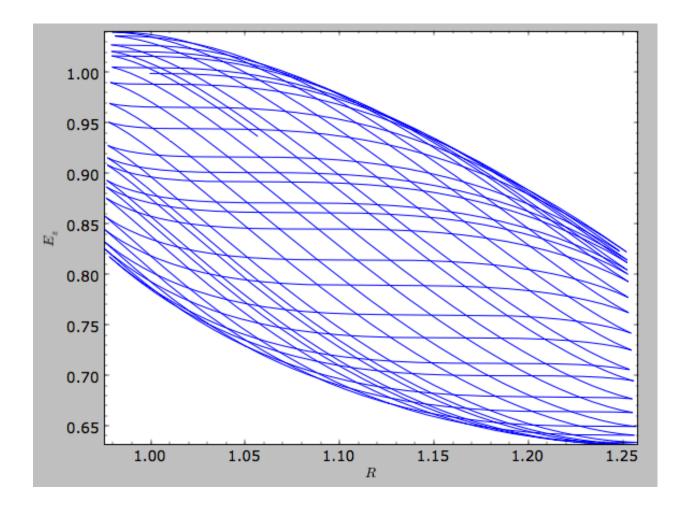
```
>>> o.plotE(normed=True)
```

gives



We can specify another quantity to plot the energy against by specifying d1=. We can also show the vertical energy, for example, as a function of R

```
>>> o.plotEz(d1='R', normed=True)
```



1.5.6 Fast orbit characterization

It is also possible to use galpy for the fast estimation of orbit parameters as demonstrated in Mackereth & Bovy (2018) via the Staeckel approximation (originally used by Binney (2012) for the appoximation of actions in axisymmetric potentials), without performing any orbit integration. The method uses the geometry of the orbit tori to estimate the orbit parameters. After initialising an Orbit instance, the method is applied by specifying analytic=True and selecting type='staeckel'.

```
>>> o.e(analytic=True, type='staeckel')
```

if running the above without integrating the orbit, the potential should also be specified in the usual way

```
>>> o.e(analytic=True, type='staeckel', pot=mp)
```

This interface automatically estimates the necessary delta parameter based on the initial condition of the Orbit object. (delta is the focal-length parameter of the prolate spheroidal coordinate system used in the approximation, see *the documentation of the actionAngleStaeckel class*).

While this is useful and fast for individual Orbit objects, it is likely that users will want to rapidly evaluate the orbit parameters of large numbers of objects. The easiest way to do this is by setting up an Orbit instance that contains all objects and call the same functions as above (in this case, the necessary delta parameter will be automatically determined for each object in the instance based on its initial condition)

```
>>> os= Orbit([R, vR, vT, z, vz, phi])
>>> os.e(analytic=True, type='staeckel', pot=mp)
```

In this case, the returned array has the same shape as the input R, vR, . . . arrays.

Rather than automatically estimating delta, you can specify an array for delta when calling os .e (or zmax, rperi, and rap), for example by first estimating good delta parameters as follows:

```
>>> from galpy.actionAngle import estimateDeltaStaeckel
>>> delta= estimateDeltaStaeckel(mp, R, z, no_median=True)
```

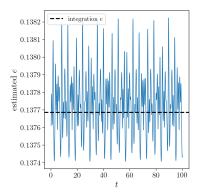
where no_median=True specifies that the function return the delta parameter at each given point rather than the median of the calculated deltas (which is the default option). Then one can compute the eccentricity etc. using individual delta values as:

```
>>> os.e(analytic=True,type='staeckel',pot=mp,delta=delta)
```

We can test the speed of this method in iPython by finding the parameters at 100000 steps along an orbit in MWPotential2014, like this

you can see that in this potential, each phase space point is calculated in roughly 60µs. further speed-ups can be gained by using the galpy.actionAngle.actionAngleStaeckelGrid module, which first calculates the parameters using a grid-based interpolation

where interpecc=True is required to perform the interpolation of the orbit parameter grid. Looking at how the eccentricity estimation varies along the orbit, and comparing to the calculation using the orbit integration, we see that the estimation good job



1.5.7 Accessing the raw orbit

The value of R, vR, vT, z, vz, x, vx, y, vy, phi, and vphi at any time can be obtained by calling the corresponding function with as argument the time (the same holds for other coordinates ra, dec, pmra, pmdec, vra, vdec, ll, bb, pmll, pmbb, vll, vbb, vlos, dist, helioX, helioY, helioZ, U, V, and W). If no time is given the initial condition is returned, and if a time is requested at which the orbit was not saved spline interpolation is used to return the value. Examples include

```
>>> o.R(1.)
# 1.1545076874679474
>>> o.phi(99.)
# 88.105603035901169
>>> o.ra(2.,obs=[8.,0.,0.],ro=8.)
# array([ 285.76403985])
>>> o.helioX(5.)
# array([ 1.24888927])
>>> o.pmll(10.,obs=[8.,0.,0.,0.,245.,0.],ro=8.,vo=230.)
# array([-6.45263888])
```

For the Orbit op that was initialized above with a distance scale ro= and a velocity scale vo=, the first of these would be

```
>>> op.R(1.)
# 9.2360614837829225 #kpc
```

which we can also access in natural coordinates as

```
>>> op.R(1.,use_physical=False)
# 1.1545076854728653
```

We can also specify a different distance or velocity scale on the fly, e.g.,

```
>>> op.R(1.,ro=4.) #different velocity scale would be vo=
# 4.6180307418914612
```

For Orbit instances that contain multiple objects, the functions above return arrays with the shape of the Orbit.

We can also initialize an Orbit instance using the phase-space position of another Orbit instance evaulated at time t. For example,

```
>>> newOrbit= o(10.)
```

will initialize a new Orbit instance with as initial condition the phase-space position of orbit o at time=10.. If multiple times are given, an Orbit instance with one object for each time will be instantiated (this works even if the original Orbit instance contained multiple objects already).

The whole orbit can also be obtained using the function getOrbit

```
>>> o.getOrbit()
```

which returns a matrix of phase-space points with dimensions [ntimes,nphasedim] or [shape,ntimes,nphasedim] for Orbit instances with multiple objects.

1.5.8 Fast orbit integration and available integrators

The standard orbit integration is done purely in python using standard scipy integrators. When fast orbit integration is needed for batch integration of a large number of orbits, a set of orbit integration routines are written in C that can

be accessed for most potentials, as long as they have C implementations, which can be checked by using the attribute hasC

```
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,amp=1.,normalize=1.)
>>> mp.hasC
# True
```

Fast C integrators can be accessed through the method= keyword of the orbit.integrate method. Currently available integrators are

- rk4 c
- rk6 c
- dopr54_c
- dop853_c

which are Runge-Kutta and Dormand-Prince methods. There are also a number of symplectic integrators available

- leapfrog_c
- symplec4_c
- symplec6_c

The higher order symplectic integrators are described in Yoshida (1993). In pure Python, the available integrators are

- · leapfrog
- · odeint
- dop853

For most applications I recommend symplec4_c or dop853_c, which are speedy and reliable. For example, compare

```
>>> o= Orbit([1.,0.1,1.1,0.,0.1])
>>> timeit(o.integrate(ts,mp,method='leapfrog'))
# 1.34 s ± 41.8 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
>>> timeit(o.integrate(ts,mp,method='leapfrog_c'))
# galpyWarning: Using C implementation to integrate orbits
# 91 ms ± 2.42 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)
>>> timeit(o.integrate(ts,mp,method='symplec4_c'))
# galpyWarning: Using C implementation to integrate orbits
# 9.67 ms ± 48.3 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
>>> timeit(o.integrate(ts,mp,method='dop853_c'))
# 4.65 ms ± 86.8 µs per loop (mean ± std. dev. of 7 runs, 100 loops each)
```

If the C extensions are unavailable for some reason, I recommend using the odeint pure-Python integrator, as it is the fastest. Using the same example as above

```
>>> o= Orbit([1.,0.1,1.1,0.,0.1])
>>> timeit(o.integrate(ts,mp,method='leapfrog'))
# 2.62 s ± 128 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
>>> timeit(o.integrate(ts,mp,method='odeint'))
# 153 ms ± 2.59 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)
>>> timeit(o.integrate(ts,mp,method='dop853'))
# 1.61 s ± 218 ms per loop (mean ± std. dev. of 7 runs, 1 loop each)
```

1.5.9 Integration of the phase-space volume

galpy further supports the integration of the phase-space volume through the method integrate_dxdv, although this is currently only implemented for two-dimensional orbits (planarOrbit). As an example, we can check Liouville's theorem explicitly. We initialize the orbit

```
>>> o= Orbit([1.,0.1,1.1,0.])
```

and then integrate small deviations in each of the four phase-space directions

```
>>> ts= numpy.linspace(0.,28.,1001) #~1 Gyr at the Solar circle
>>> o.integrate_dxdv([1.,0.,0.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dx= o.getOrbit_dxdv()[-1,:] # evolution of dxdv[0] along the orbit
>>> o.integrate_dxdv([0.,1.,0.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dy= o.getOrbit_dxdv()[-1,:]
>>> o.integrate_dxdv([0.,0.,1.,0.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dvx= o.getOrbit_dxdv()[-1,:]
>>> o.integrate_dxdv([0.,0.,0.,1.],ts,mp,method='dopr54_c',rectIn=True,rectOut=True)
>>> dvy= o.getOrbit_dxdv()[-1,:]
```

We can then compute the determinant of the Jacobian of the mapping defined by the orbit integration from time zero to the final time

```
>>> tjac= numpy.linalg.det(numpy.array([dx,dy,dvx,dvy]))
```

This determinant should be equal to one

```
>>> print(tjac)
# 0.99999991189
>>> numpy.fabs(tjac-1.) < 10.**-8.
# True
```

The calls to integrate_dxdv above set the keywords rectIn= and rectOut= to True, as the default input and output uses phase-space volumes defined as (dR,dvR,dvT,dphi) in cylindrical coordinates. When rectIn or rectOut is set, the in- or output is in rectangular coordinates ([x,y,vx,vy]] in two dimensions).

Implementing the phase-space integration for three-dimensional FullOrbit instances is straightforward and is part of the longer term development plan for galpy. Let the main developer know if you would like this functionality, or better yet, implement it yourself in a fork of the code and send a pull request!

1.5.10 Example: The eccentricity distribution of the Milky Way's thick disk

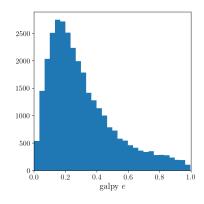
A straightforward application of galpy's orbit initialization and integration capabilities is to derive the eccentricity distribution of a set of thick disk stars. We start by downloading the sample of SDSS SEGUE (2009AJ....137.4377Y) thick disk stars compiled by Dierickx et al. (2010arXiv1009.1616D) from CDS at this link. Downloading the table and the ReadMe will allow you to read in the data using astropy.io.ascii like so

After reading in the data (RA,Dec,distance,pmRA,pmDec,vlos; see above) as a vector vxvv with dimensions [6,ndata] we (a) define the potential in which we want to integrate the orbits, and (b) integrate all orbits and compute their eccentricity numerically from the orbit integration and analytically following the *Staeckel approximation method* (the

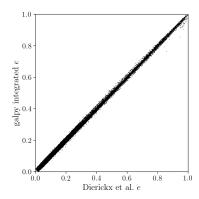
following takes lots of memory; you might want to slice the orbits object to a smaller number to run this code faster)

```
>>> ts= np.linspace(0.,20.,10000)
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> orbits= Orbit(vxvv,radec=True,ro=8.,vo=220.,solarmotion='hogg')
>>> e_ana= orbits.e(analytic=True,pot=lp,delta=1e-6)
>>> orbits.integrate(ts,lp)
>>> e_int= orbits.e()
```

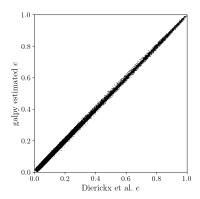
We then find the following eccentricity distribution (from the numerical eccentricities)



The eccentricity calculated by integration in galpy compare well with those calculated by Dierickx et al., except for a few objects

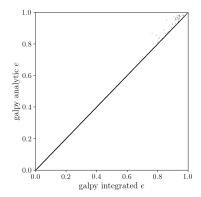


and the analytical estimates are equally as good:



In comparing the analytic and integrated eccentricity estimates - one can see that in this case the estimation is almost

exact, due to the spherical symmetry of the chosen potential:



A script that calculates and plots everything can be downloaded here. To generate the plots just run:

```
python dierickx_eccentricities.py ../path/to/folder
```

specifiying the location you want to put the plots and data.

Alternatively - one can transform the observed coordinates into spherical coordinates and perform the estimations in one batch using the actionAngle interface, which takes considerably less time:

The above code calculates the parameters in roughly 100ms on a single core.

1.5.11 Example: The orbit of the Large Magellanic Cloud in the presence of dynamical friction

As a further example of what you can do with galpy, we investigate the Large Magellanic Cloud's (LMC) past and future orbit. Because the LMC is a massive satellite of the Milky Way, its orbit is affected by dynamical friction, a frictional force of gravitational origin that occurs when a massive object travels through a sea of low-mass objects (halo stars and dark matter in this case). First we import all the necessary packages:

```
>>> from astropy import units
>>> from galpy.potential import MWPotential2014, ChandrasekharDynamicalFrictionForce
>>> from galpy.orbit import Orbit
```

(also do %pylab inline if running this in a jupyter notebook or turn on the pylab option in ipython for plotting). We can load the current phase-space coordinates for the LMC using the Orbit.from_name function described above:

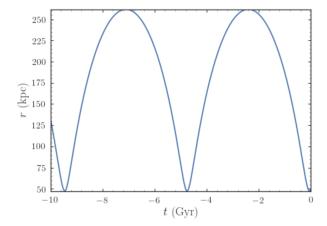
```
>>> o= Orbit.from_name('LMC')
```

We will use MWPotential2014 as our Milky-Way potential model. Because the LMC is in fact unbound in MWPotential2014, we increase the halo mass by 50% to make it bound (this corresponds to a Milky-Way halo mass of $\approx 1.2 \times 10^{12}\,M_{\odot}$, a not unreasonable value). We can adjust a galpy Potential's amplitude simply by multiplying the potential by a number, so to increase the mass by 50% we do

```
>>> MWPotential2014[2]*= 1.5
```

Let us now integrate the orbit backwards in time for 10 Gyr and plot it:

```
>>> ts= numpy.linspace(0.,-10.,1001)*units.Gyr
>>> o.integrate(ts,MWPotential2014)
>>> o.plot(d1='t',d2='r')
```



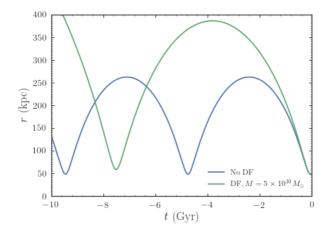
We see that the LMC is indeed bound, with an apocenter just over 250 kpc. Now let's add dynamical friction for the LMC, assuming that its mass is $5 \times 10^{10} M_{\odot}$. We setup the dynamical-friction object:

Dynamical friction depends on the velocity distribution of the halo, which is assumed to be an isotropic Gaussian distribution with a radially-dependent velocity dispersion. If the velocity dispersion is not given (like in the example above), it is computed from the spherical Jeans equation. We have set the half-mass radius to 5 kpc for definiteness. We now make a copy of the orbit instance above and integrate it in the potential that includes dynamical friction:

```
>>> odf= o()
>>> odf.integrate(ts,MWPotential2014+cdf)
```

Overlaying the orbits, we can see the difference in the evolution:

```
>>> o.plot(d1='t',d2='r',label=r'$\mathrm{No\ DF}$')
>>> odf.plot(d1='t',d2='r',overplot=True,label=r'$\mathrm{DF}, M=5\times10^{10}\,M_
$\leftrightarrow\codot$')
>>> ylim(0.,400.)
>>> legend()
```



We see that dynamical friction removes energy from the LMC's orbit, such that its past apocenter is now around 400 kpc rather than 250 kpc! The period of the orbit is therefore also much longer. Clearly, dynamical friction has a big impact on the orbit of the LMC.

Recent observations have suggested that the LMC may be even more massive than what we have assumed so far, with masses over $10^{11}\,M_\odot$ seeming in good agreement with various observations. Let's see how a mass of $10^{11}\,M_\odot$ changes the past orbit of the LMC. We can change the mass of the LMC used in the dynamical-friction calculation as

```
>>> cdf.GMs= 10.**11.*units.Msun
```

This way of changing the mass is preferred over re-initializing the ChandrasekharDynamicalFrictionForce object, because it avoids having to solve the Jeans equation again to obtain the velocity dispersion. Then we integrate the orbit and overplot it on the previous results:

```
>>> odf2= o()
>>> odf2.integrate(ts,MWPotential2014+cdf)
```

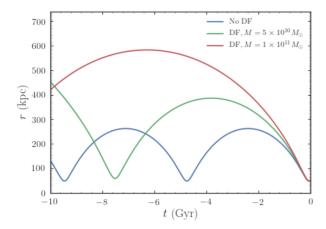
and

```
>>> o.plot(d1='t',d2='r',label=r'$\mathrm{No\ DF}$')
>>> odf.plot(d1='t',d2='r',overplot=True,label=r'$\mathrm{DF}, M=5\times10^{10}\,M_

\leftarrow\odot$')
>>> odf2.plot(d1='t',d2='r',overplot=True,label=r'$\mathrm{DF}, M=1\times10^{11}\,M_

\leftarrow\odot$')
>>> ylim(0.,740.)
>>> legend()
```

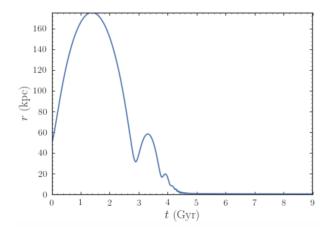
which gives



Now the apocenter increases to about 600 kpc and the LMC doesn't perform a full orbit over the last 10 Gyr.

Finally, let's see what will happen in the future if the LMC is as massive as $10^{11} M_{\odot}$. We simply flip the sign of the integration times to get the future trajectory:

```
>>> odf2.integrate(-ts[-ts < 9*units.Gyr],MWPotential2014+cdf)
>>> odf2.plot(d1='t',d2='r')
```



Because of the large effect of dynamical friction, the LMC will merge with the Milky-Way in about 4 Gyr after a few more pericenter passages. Note that we have not taken any mass-loss into account. Because mass-loss would lead to a smaller dynamical-friction force, this would somewhat increase the merging timescale, but dynamical friction will inevitably lead to the merger of the LMC with the Milky Way.

Warning: When using dynamical friction, if the radius gets very small, the integration sometimes becomes very erroneous, which can lead to a big, unphysical kick (even though we turn off friction at very small radii); this is the reason why we have limited the future integration to 9 Gyr in the example above. When using dynamical friction, inspect the full orbit to make sure to catch whether a merger has happened.

1.6 Two-dimensional disk distribution functions

galpy contains various disk distribution functions, both in two and three dimensions. This section introduces the twodimensional distribution functions, useful for studying the dynamics of stars that stay relatively close to the mid-plane of a galaxy. The vertical motions of these stars may be approximated as being entirely decoupled from the motion in the plane.

1.6.1 Types of disk distribution functions

galpy contains the following distribution functions for razor-thin disks: galpy.df.dehnendf, galpy.df.shudf, and galpy.df.schwarzschilddf. These are the distribution functions of Dehnen (1999AJ....118.1201D), Shu (1969ApJ...158..505S), and Schwarzschild (the Shu DF in the epicycle approximation, see Binney & Tremaine 2008). Everything shown below for dehnendf can also be done for shudf and schwarzschilddf. The Schwarzschild DF is primarily included as an educational tool; it is *not* a true steady-state DF, because it uses the approximate energy from the epicycle approximation rather than the true energy, and is fully superseded by the Shu DF, which *is* a good steady-state DF.

These disk distribution functions are functions of the energy and the angular momentum alone. They can be evaluated for orbits, or for a given energy and angular momentum. At this point, only power-law rotation curves are supported. A dehnendf instance is initialized as follows

```
>>> from galpy.df import dehnendf
>>> dfc= dehnendf(beta=0.)
```

This initializes a dehnendf instance based on an exponential surface-mass profile with scale-length 1/3 and an exponential radial-velocity-dispersion profile with scale-length 1 and a value of 0.2 at R=1. Different parameters for these profiles can be provided as an initialization keyword. For example,

```
>>> dfc= dehnendf(beta=0.,profileParams=(1./4.,1.,0.2))
```

initializes the distribution function with a radial scale length of 1/4 instead.

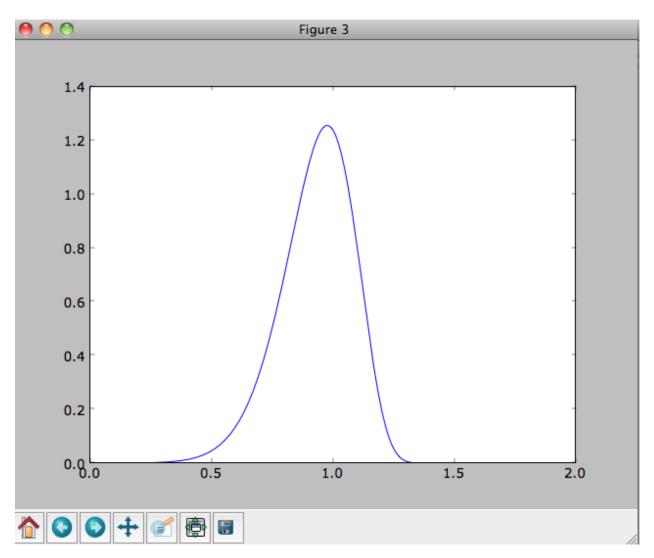
We can show that these distribution functions have an asymmetric drift built-in by evaluating the DF at R=1. We first create a set of orbit-instances and then evaluate the DF at them

```
>>> from galpy.orbit import Orbit

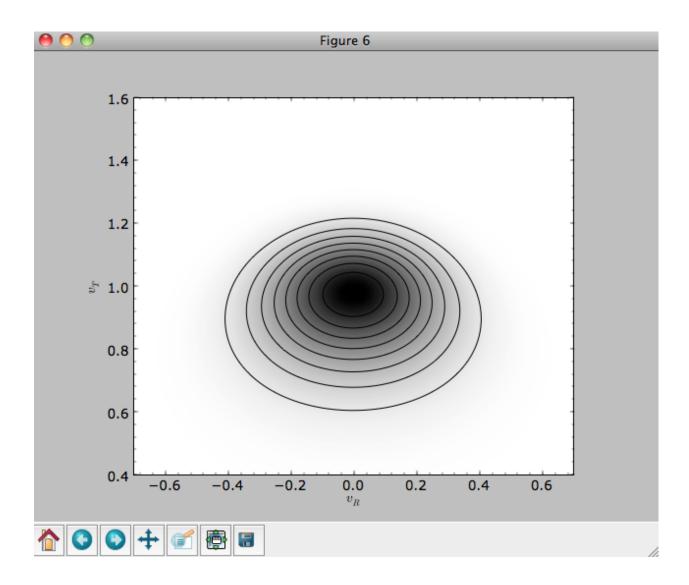
>>> os= [Orbit([1.,0.,1.+-0.9+1.8/1000*ii]) for ii in range(1001)]

>>> dfro= [dfc(o) for o in os]

>>> plot([1.+-0.9+1.8/1000*ii for ii in range(1001)],dfro)
```



Or we can plot the two-dimensional density at R=1.



1.6.2 Evaluating moments of the DF

galpy can evaluate various moments of the disk distribution functions. For example, we can calculate the mean velocities (for the DF with a scale length of 1/3 above)

```
>>> dfc.meanvT(1.)

# 0.91715276979447324

>>> dfc.meanvR(1.)

# 0.0
```

and the velocity dispersions

```
>>> numpy.sqrt(dfc.sigmaR2(1.))
# 0.19321086259083936
>>> numpy.sqrt(dfc.sigmaT2(1.))
# 0.15084122011271159
```

and their ratio

```
>>> dfc.sigmaR2(1.)/dfc.sigmaT2(1.)
# 1.6406766813028968
```

In the limit of zero velocity dispersion (the epicycle approximation) this ratio should be equal to 2, which we can check as follows

```
>>> dfccold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.02))
>>> dfccold.sigmaR2(1.)/dfccold.sigmaT2(1.)
# 1.9947493895454664
```

We can also calculate higher order moments

```
>>> dfc.skewvT(1.)

# -0.48617143862047763

>>> dfc.kurtosisvT(1.)

# 0.13338978593181494

>>> dfc.kurtosisvR(1.)

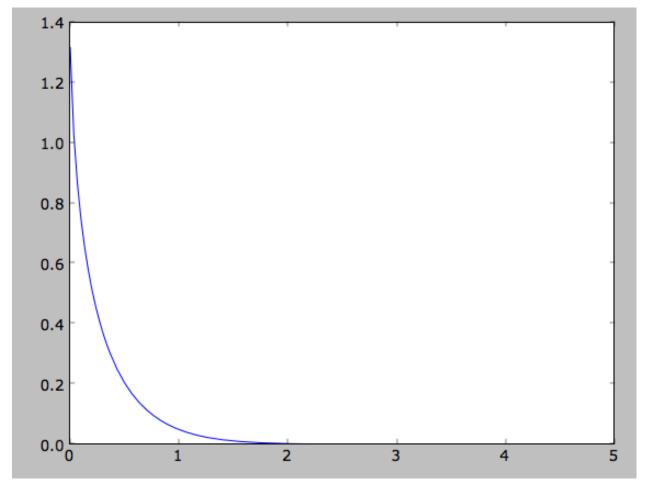
# -0.12159407676394096
```

We already saw above that the velocity dispersion at R=1 is not exactly equal to the input velocity dispersion (0.19321086259083936 vs. 0.2). Similarly, the whole surface-density and velocity-dispersion profiles are not quite equal to the exponential input profiles. We can calculate the resulting surface-mass density profile using surfacemass, sigmaR2, and sigma2surfacemass. The latter calculates the product of the velocity dispersion squared and the surface-mass density. E.g.,

```
>>> dfc.surfacemass(1.)
# 0.050820867101511534
```

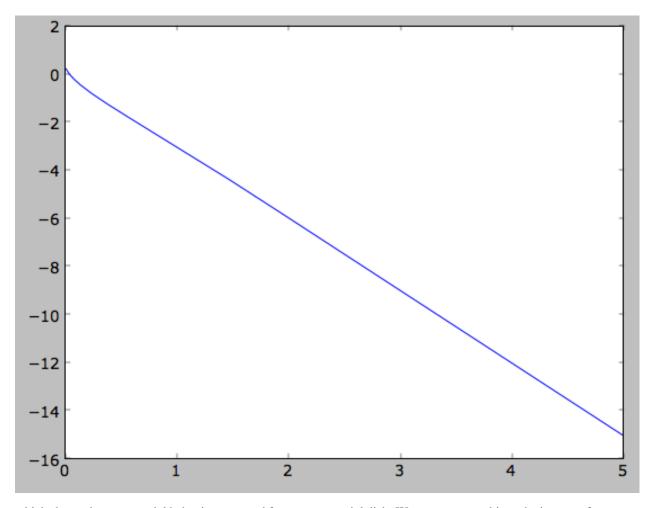
We can plot the surface-mass density as follows

```
>>> Rs= numpy.linspace(0.01,5.,151)
>>> out= [dfc.surfacemass(r) for r in Rs]
>>> plot(Rs, out)
```



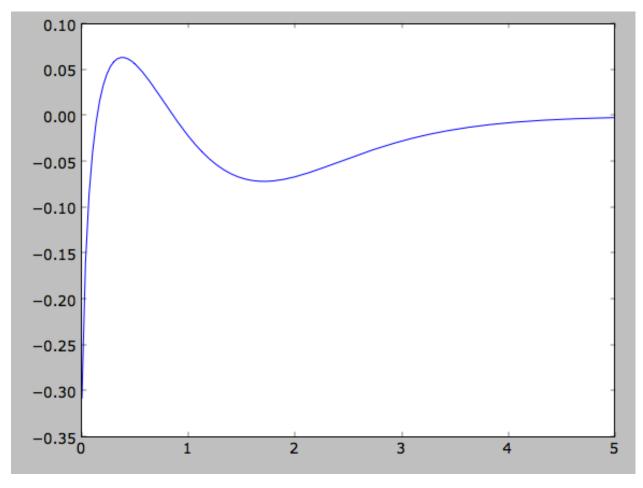
or

>>> plot(Rs, numpy.log(out))



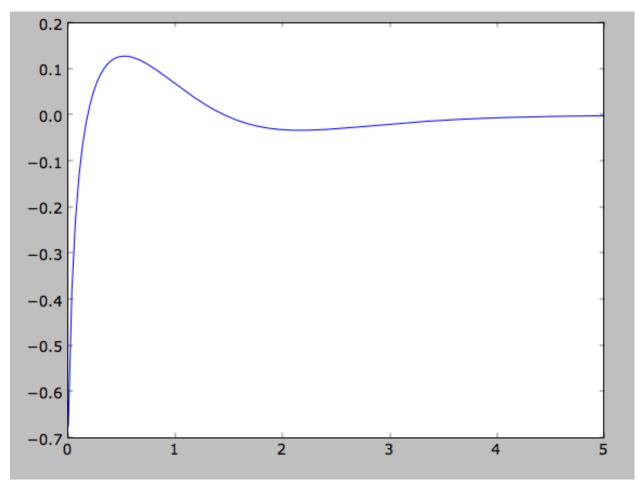
which shows the exponential behavior expected for an exponential disk. We can compare this to the input surface-mass density

```
>>> input_out= [dfc.targetSurfacemass(r) for r in Rs]
>>> plot(Rs,numpy.log(input_out)-numpy.log(out))
```



which shows that there are significant differences between the desired surface-mass density and the actual surface-mass density. We can do the same for the velocity-dispersion profile

```
>>> out= [dfc.sigmaR2(r) for r in Rs]
>>> input_out= [dfc.targetSigma2(r) for r in Rs]
>>> plot(Rs,numpy.log(input_out)-numpy.log(out))
```



That the input surface-density and velocity-dispersion profiles are not the same as the output profiles, means that estimates of DF properties based on these profiles will not be quite correct. Obviously this is the case for the surface-density and velocity-dispersion profiles themselves, which have to be explicitly calculated by integration over the DF rather than by evaluating the input profiles. This also means that estimates of the asymmetric drift based on the input profiles will be wrong. We can calculate the asymmetric drift at R=1 using the asymmetric drift equation derived from the Jeans equation (eq. 4.228 in Binney & Tremaine 2008), using the input surface-density and velocity dispersion profiles

```
>>> dfc.asymmetricdrift(1.)
# 0.09000000000000024
```

which should be equal to the circular velocity minus the mean rotational velocity

```
>>> 1.-dfc.meanvT(1.)
# 0.082847230205526756
```

These are not the same in part because of the difference between the input and output surface-density and velocity-dispersion profiles (and because the asymmetricdrift method assumes that the ratio of the velocity dispersions squared is two using the epicycle approximation; see above).

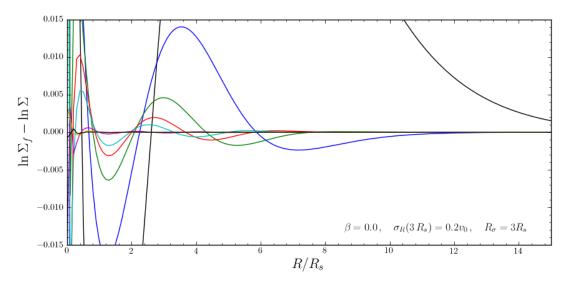
1.6.3 Using corrected disk distribution functions

As shown above, for a given surface-mass density and velocity dispersion profile, the two-dimensional disk distribution functions only do a poor job of reproducing the desired profiles. We can correct this by calculating a

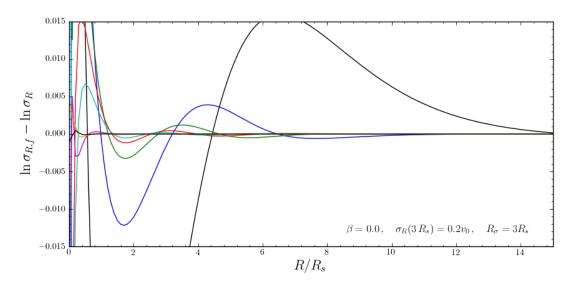
set of *corrections* to the input profiles such that the output profiles more closely resemble the desired profiles (see 1999AJ....118.1201D). galpy supports the calculation of these corrections, and comes with some pre-calculated corrections (these can be found here). For example, the following initializes a dehnendf with corrections up to 20th order (the default)

```
>>> dfc= dehnendf(beta=0.,correct=True)
```

The following figure shows the difference between the actual surface-mass density profile and the desired profile for 1, 2, 3, 4, 5, 10, 15, and 20 iterations



and the same for the velocity-dispersion profile



galpy will automatically save any new corrections that you calculate.

All of the methods for an uncorrected disk DF can be used for the corrected DFs as well. For example, the velocity dispersion is now

```
>>> numpy.sqrt(dfc.sigmaR2(1.))
# 0.19999985069451526
```

and the mean rotation velocity is

```
>>> dfc.meanvT(1.)
# 0.90355161181498711
```

and (correct) asymmetric drift

```
>>> 1.-dfc.meanvT(1.)
# 0.09644838818501289
```

That this still does not agree with the simple dfc.asymmetricdrift estimate is because of the latter's using the epicycle approximation for the ratio of the velocity dispersions.

1.6.4 Oort constants and functions

galpy also contains methods to calculate the Oort functions for two-dimensional disk distribution functions. These are known as the *Oort constants* when measured in the solar neighborhood. They are combinations of the mean velocities and derivatives thereof. galpy calculates these by direct integration over the DF and derivatives of the DF. Thus, we can calculate

```
>>> dfc= dehnendf(beta=0.)
>>> dfc.oortA(1.)
# 0.43190780889218749
>>> dfc.oortB(1.)
# -0.48524496090228575
```

The *K* and *C* Oort constants are zero for axisymmetric DFs

```
>>> dfc.oortC(1.)
# 0.0
>>> dfc.oortK(1.)
# 0.0
```

In the epicycle approximation, for a flat rotation curve A = -B = 0.5. The explicit calculates of A and B for warm DFs quantify how good (or bad) this approximation is

```
>>> dfc.oortA(1.)+dfc.oortB(1.)
# -0.053337152010098254
```

For the cold DF from above the approximation is much better

```
>>> dfccold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.02))
>>> dfccold.oortA(1.), dfccold.oortB(1.)
# (0.49917556666144003, -0.49992824742490816)
```

1.6.5 Sampling data from the DF

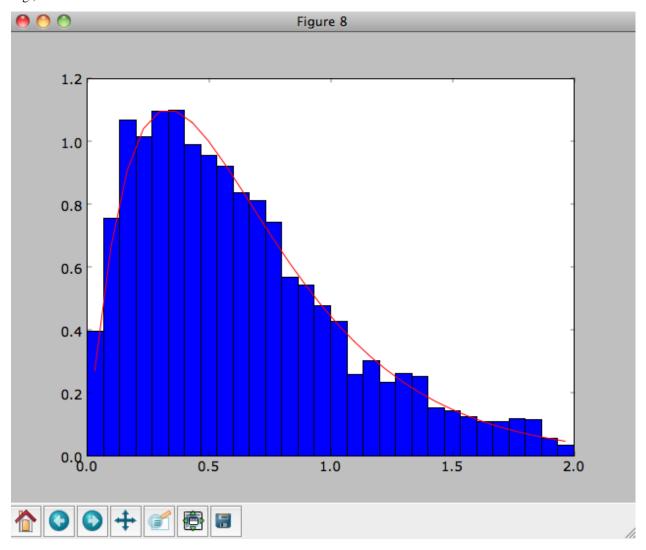
We can sample from the disk distribution functions using sample. sample can return either an energy-angular-momentum pair, or a full orbit initialization. We can sample 4000 orbits for example as (could take two minutes)

```
>>> o= dfc.sample(n=4000,returnOrbit=True,nphi=1)
```

We can then plot the histogram of the sampled radii and compare it to the input surface-mass density profile

```
>>> Rs= [e.R() for e in o]
>>> hists, bins, edges= hist(Rs,range=[0,2],normed=True,bins=30)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> plot(xs, xs*exp(-xs*3.)*9.,'r-')
```

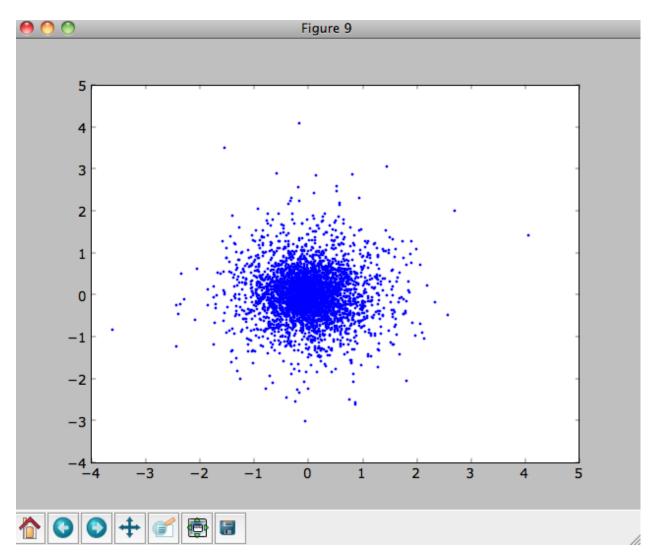
E.g.,



We can also plot the spatial distribution of the sampled disk

```
>>> xs= [e.x() for e in o]
>>> ys= [e.y() for e in o]
>>> figure()
>>> plot(xs,ys,',')
```

E.g.,

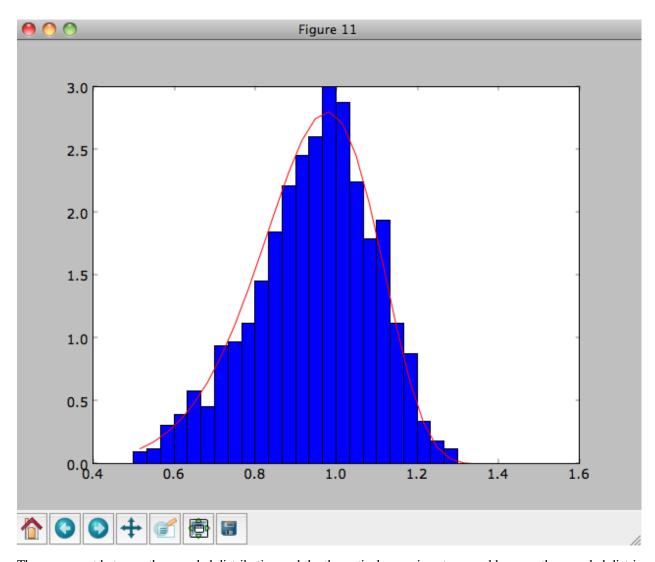


We can also sample points in a specific radial range (might take a few minutes)

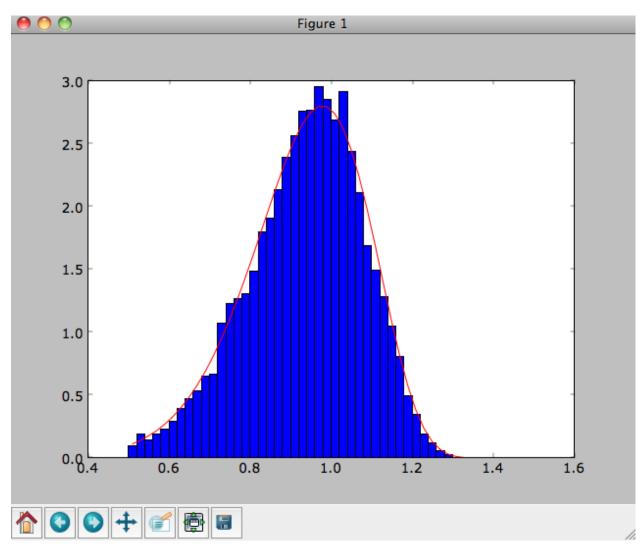
```
>>> o= dfc.sample(n=1000,returnOrbit=True,nphi=1,rrange=[0.8,1.2])
```

and we can plot the distribution of tangential velocities

```
>>> vTs= [e.vxvv[2] for e in o]
>>> hists, bins, edges= hist(vTs,range=[.5,1.5],normed=True,bins=30)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> dfro= [dfc(Orbit([1.,0.,x]))/9./numpy.exp(-3.) for x in xs]
>>> plot(xs,dfro,'r-')
```

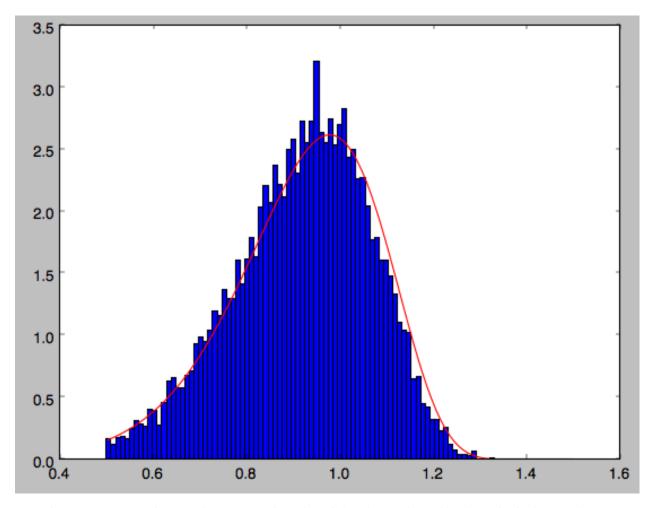


The agreement between the sampled distribution and the theoretical curve is not as good because the sampled distribution has a finite radial range. If we sample 10,000 points in rrange=[0.95, 1.05] the agreement is better (this takes a long time):



We can also directly sample velocities at a given radius rather than in a range of radii. Doing this for a correct DF gives

```
>>> dfc= dehnendf(beta=0.,correct=True)
>>> vrvt= dfc.sampleVRVT(1.,n=10000)
>>> hists, bins, edges= hist(vrvt[:,1],range=[.5,1.5],normed=True,bins=101)
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> dfro= [dfc(Orbit([1.,0.,x])) for x in xs]
>>> plot(xs,dfro/numpy.sum(dfro)/(xs[1]-xs[0]),'r-')
```



galpy further has support for sampling along a given line of sight in the disk, which is useful for interpreting surveys consisting of a finite number of pointings. For example, we can sampled distances along a given line of sight

```
>>> ds= dfc.sampledSurfacemassLOS(30./180.*numpy.pi,n=10000)
```

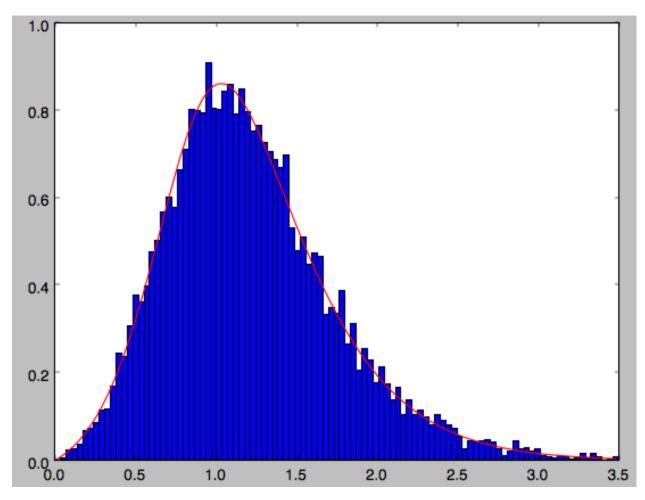
which is very fast. We can histogram these

```
>>> hists, bins, edges= hist(ds,range=[0.,3.5],normed=True,bins=101)
```

and compare it to the predicted distribution, which we can calculate as

```
>>> xs= numpy.array([(bins[ii+1]+bins[ii])/2. for ii in range(len(bins)-1)])
>>> fd= numpy.array([dfc.surfacemassLOS(d,30.) for d in xs])
>>> plot(xs,fd/numpy.sum(fd)/(xs[1]-xs[0]),'r-')
```

which shows very good agreement with the sampled distances



galpy can further sample full 4D phase-space coordinates along a given line of sight through dfc.sampleLOS.

1.6.6 Non-axisymmetric, time-dependent disk distribution functions

galpy also supports the evaluation of non-axisymmetric, time-dependent two-dimensional DFs. These specific DFs are constructed by assuming an initial axisymmetric steady state, described by a DF of the family discussed above, that is then acted upon by a non-axisymmetric, time-dependent perturbation. The DF at a given time and phase-space position is evaluated by integrating the orbit backwards in time in the non-axisymmetric potential until the time of the initial DF is reached. From Liouville's theorem, which states that phase-space volume is conserved along the orbit, we then know that we can evaluate the non-axisymmetric DF today as the initial DF at the initial point on the orbit. This procedure was first used by Dehnen (2000).

This is implemented in galpy as galpy.df.evolveddiskdf. Such a DF is setup by specifying the initial DF, the non-axisymmetric potential, and the time of the initial state. For example, we can look at the effect of an elliptical perturbation to the potential like that described by Kuijken & Tremaine. To do this, we set up an elliptical perturbation to a logarithmic potential that is grown slowly to minimize non-adiabatic effects

```
>>> from galpy.potential import LogarithmicHaloPotential, EllipticalDiskPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> ep= EllipticalDiskPotential(twophio=0.05,phib=0.,p=0.,tform=-150.,tsteady=125.)
```

This perturbation starts to be grown at tform=-150 over a time period of tsteady=125 time units. We will consider the effect of this perturbation on a very cold disk (velocity dispersion $\sigma_R=0.0125\,v_c$) and a warm disk ($\sigma_R=0.15\,v_c$). We set up these two initial DFs

```
>>> idfcold= dehnendf(beta=0.,profileParams=(1./3.,1.,0.0125))
>>> idfwarm= dehnendf(beta=0.,profileParams=(1./3.,1.,0.15))
```

and then set up the evolveddiskdf

```
>>> from galpy.df import evolveddiskdf
>>> edfcold= evolveddiskdf(idfcold,[lp,ep],to=-150.)
>>> edfwarm= evolveddiskdf(idfwarm,[lp,ep],to=-150.)
```

where we specify that the initial state is at to=-150.

We can now use these evolveddiskdf instances in much the same way as diskdf instances. One difference is that there is much more support for evaluating the DF on a grid (to help speed up the rather slow computations involved). Thus, we can evaluate the mean radial velocity at R=0.9, phi=22.5 degree, and t=0 by using a grid

```
>>> mvrcold, gridcold= edfcold.meanvR(0.9,phi=22.5,deg=True,t=0.,grid=True,

→returnGrid=True,gridpoints=51,nsigma=6.)
>>> mvrwarm, gridwarm= edfwarm.meanvR(0.9,phi=22.5,deg=True,t=0.,grid=True,

→returnGrid=True,gridpoints=51)
>>> print(mvrcold, mvrwarm)
# -0.0358753028951 -0.0294763627935
```

The cold response agrees well with the analytical calculation, which predicts that this is $-0.05/\sqrt{2}$:

```
>>> print(mvrcold+0.05/sqrt(2.))
# -0.000519963835811
```

The warm response is slightly smaller in amplitude

```
>>> print(mvrwarm/mvrcold)
# 0.821633837619
```

although the numerical uncertainty in mvrwarm is large, because the grid is not sufficiently fine.

We can then re-use this grid in calculations of other moments of the DF, e.g.,

```
>>> print(edfcold.meanvT(0.9,phi=22.5,deg=True,t=0.,grid=gridcold))
# 0.965058551359
>>> print(edfwarm.meanvT(0.9,phi=22.5,deg=True,t=0.,grid=gridwarm))
# 0.915397094614
```

which returns the mean rotational velocity, and

```
>>> print(edfcold.vertexdev(0.9,phi=22.5,deg=True,t=0.,grid=gridcold))
# 0.0560531474616
>>> print(edfwarm.vertexdev(0.9,phi=22.5,deg=True,t=0.,grid=gridwarm))
# 0.0739164830253
```

which gives the vertex deviation in rad. The reason we have to calculate the grid out to 6nsigma for the cold response is that the response is much bigger than the velocity dispersion of the population. This velocity dispersion is used to automatically to set the grid edges, but sometimes has to be adjusted to contain the full DF.

evolveddiskdf can also calculate the Oort functions, by directly calculating the spatial derivatives of the DF. These can also be calculated on a grid, such that we can do

It is clear that these are quite different. The cold calculation is again close to the analytical prediction, which says that $A = A_{\rm axi} + 0.05/(2\sqrt{2})$ where $A_{\rm axi} = 1/(2 \times 0.9)$ in this case:

```
>>> print(oortacold-(0.5/0.9+0.05/2./sqrt(2.)))
# 0.0022613349141670236
```

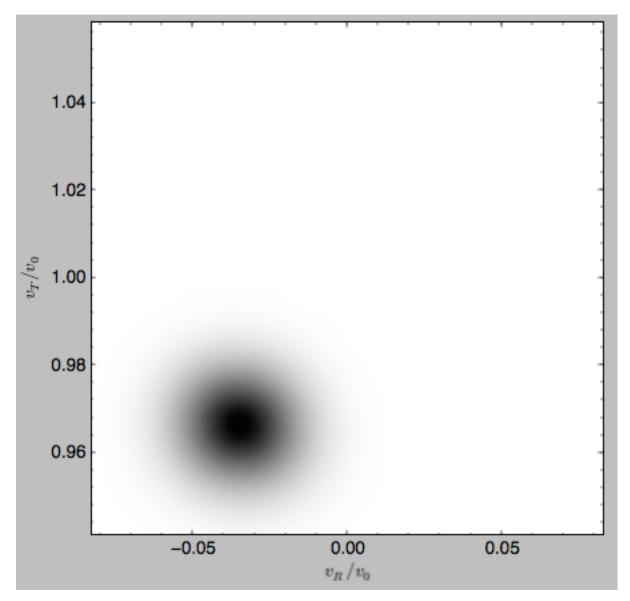
These grids can then be re-used for the other Oort functions, for example,

and similar for oortC and oortK. These warm results should again be considered for illustration only, as the grid is not sufficiently fine to have a small numerical error.

The grids that have been calculated can also be plotted to show the full velocity DF. For example,

```
>>> gridcold.plot()
```

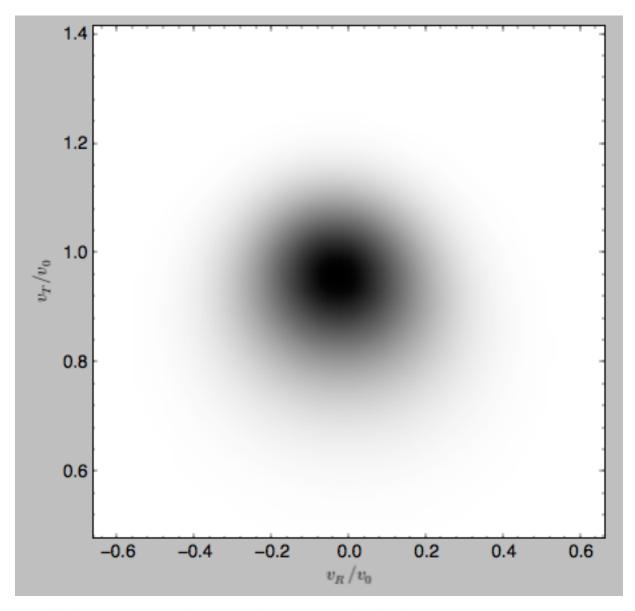
gives



which demonstrates that the DF is basically the initial DF that has been displaced (by a significant amount compared to the velocity dispersion). The warm velocityd distribution is given by

```
>>> gridwarm.plot()
```

which returns



The shift of the smooth DF here is much smaller than the velocity dispersion.

1.6.7 Example: The Hercules stream in the Solar neighborhood as a result of the Galactic bar

We can combine the orbit integration capabilities of galpy with the provided distribution functions and see the effect of the Galactic bar on stellar velocities. By backward integrating orbits starting at the Solar position in a potential that includes the Galactic bar we can evaluate what the velocity distribution is that we should see today if the Galactic bar stirred up a steady-state disk. For this we initialize a flat rotation curve potential and Dehnen's bar potential

```
>>> from galpy.potential import LogarithmicHaloPotential, DehnenBarPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> dp= DehnenBarPotential()
```

The Dehnen bar potential is initialized to start bar formation four bar periods before the present day and to have completely formed the bar two bar periods ago. We can integrate back to the time before bar-formation:

```
>>> ts= numpy.linspace(0,dp.tform(),1000)
```

where dp.tform() is the time of bar-formation (in the usual time-coordinates).

We initialize orbits on a grid in velocity space and integrate them

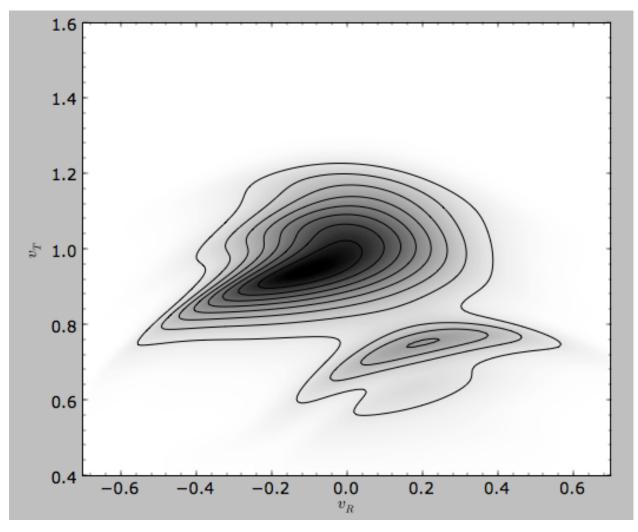
We can then evaluate the weight of these orbits by assuming that the disk was in a steady-state before bar-formation with a Dehnen distribution function. We evaluate the Dehnen distribution function at dp.tform() for each of the orbits (evaluating the distribution function only works for an Orbit with a single object, so we need to unpack the Orbit instance that contains all orbits)

```
>>> dfc= dehnendf(beta=0.,correct=True)
>>> out= [[dfc(o(dp.tform())) for o in j] for j in ins]
>>> out= numpy.array(out)
```

This gives

```
>>> from galpy.util.bovy_plot import bovy_dens2d
>>> bovy_dens2d(out,origin='lower',cmap='gist_yarg',contours=True,xrange=[-0.7,0.7],

-yrange=[0.4,1.6],xlabel=r'$v_R$',ylabel=r'$v_T$')
```



Now that galpy contains the evolveddiskdf described above, this whole calculation is encapsulated in this module and can be done much more easily as

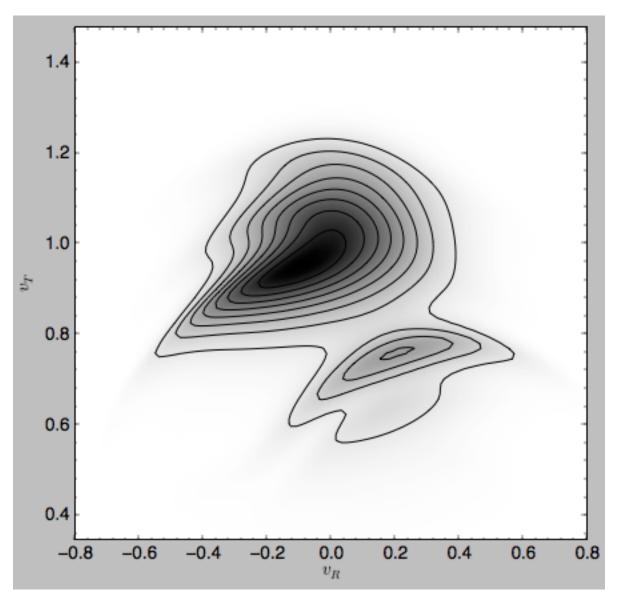
```
>>> edf= evolveddiskdf(dfc,[lp,dp],to=dp.tform())
>>> mvr, grid= edf.meanvR(1.,grid=True,gridpoints=101,returnGrid=True)
```

The gridded DF can be accessed as grid.df, which we can plot as before

```
>>> bovy_dens2d(grid.df.T,origin='lower',cmap='gist_yarg',contours=True,xrange=[grid.

-vRgrid[0],grid.vRgrid[-1]],yrange=[grid.vTgrid[0],grid.vTgrid[-1]],xlabel=r'$v_R$',

-ylabel=r'$v_T$')
```



For more information see 2000AJ....119..800D and 2010ApJ...725.1676B. Note that the x-axis in the Figure above is defined as minus the x-axis in these papers.

1.7 Action-angle coordinates

galpy can calculate actions and angles for a large variety of potentials (any time-independent potential in principle). These are implemented in a separate module <code>galpy.actionAngle</code>, and the preferred method for accessing them is through the routines in this module. There is also some support for accessing the actionAngle routines as methods of the <code>Orbit</code> class.

Tip: If you want to quickly and easily compute actions, angles, or frequencies using the Staeckel approximation, using the Orbit interface as described in *this section* is recommended. Especially if you are starting from observed coordinates, as Orbit instances can easily be initialized using these.

Since v1.2, galpy can also compute positions and velocities corresponding to a given set of actions and angles for

axisymmetric potentials using the TorusMapper code of Binney & McMillan (2016). This is described in *this section* below. The interface for this is different than for the other action-angle classes, because the transformations are generally different.

Action-angle coordinates can be calculated for the following potentials/approximations:

- Isochrone potential
- · Spherical potentials
- · Adiabatic approximation
- · Staeckel approximation
- A general orbit-integration-based technique

There are classes corresponding to these different potentials/approximations and actions, frequencies, and angles can typically be calculated using these three methods:

- __call__: returns the actions
- actionsFreqs: returns the actions and the frequencies
- actionsFreqsAngles: returns the actions, frequencies, and angles

These are not all implemented for each of the cases above yet.

The adiabatic and Staeckel approximation have also been implemented in C and using grid-based interpolation, for extremely fast action-angle calculations (see below).

1.7.1 Action-angle coordinates for the isochrone potential

The isochrone potential is the only potential for which all of the actions, frequencies, and angles can be calculated analytically. We can do this in galpy by doing

```
>>> from galpy.potential import IsochronePotential
>>> from galpy.actionAngle import actionAngleIsochrone
>>> ip= IsochronePotential(b=1.,normalize=1.)
>>> aAI= actionAngleIsochrone(ip=ip)
```

aAI is now an instance that can be used to calculate action-angle variables for the specific isochrone potential ip. Calling this instance returns (J_R, L_Z, J_Z)

```
>>> aAI(1.,0.1,1.1,0.1,0.) #inputs R, vR, vT, z, vz
# (array([ 0.00713759]), array([ 1.1]), array([ 0.00553155]))
```

or for a more eccentric orbit

```
>>> aAI(1.,0.5,1.3,0.2,0.1)
# (array([ 0.13769498]), array([ 1.3]), array([ 0.02574507]))
```

Note that we can also specify phi, but this is not necessary

```
>>> aAI(1.,0.5,1.3,0.2,0.1,0.)
# (array([ 0.13769498]), array([ 1.3]), array([ 0.02574507]))
```

We can likewise calculate the frequencies as well

```
>>> aAI.actionsFreqs(1.,0.5,1.3,0.2,0.1,0.)

# (array([ 0.13769498]),

# array([ 1.3]),

# array([ 0.02574507]),

# array([ 1.29136096]),

# array([ 0.79093738]),

# array([ 0.79093738]))
```

The output is $(J_R, L_Z, J_Z, \Omega_R, \Omega_\phi, \Omega_Z)$. For any spherical potential, $\Omega_\phi = \operatorname{sgn}(L_Z)\Omega_Z$, such that the last two frequencies are the same.

We obtain the angles as well by calling

```
>>> aAI.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)

# (array([ 0.13769498]),

# array([ 1.3]),

# array([ 0.02574507]),

# array([ 1.29136096]),

# array([ 0.79093738]),

# array([ 0.79093738]),

# array([ 0.57101518]),

# array([ 5.96238847]),

# array([ 1.24999949]))
```

The output here is $(J_R, L_Z, J_Z, \Omega_R, \Omega_{\phi}, \Omega_Z, \theta_R, \theta_{\phi}, \theta_Z)$.

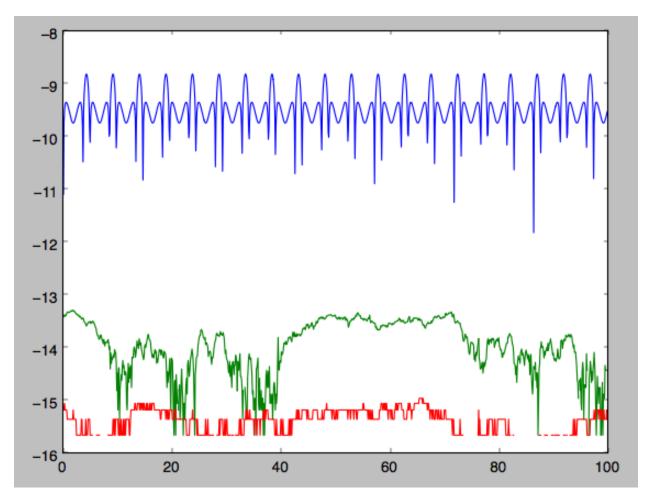
To check that these are good action-angle variables, we can calculate them along an orbit

```
>>> from galpy.orbit import Orbit
>>> o= Orbit([1.,0.5,1.3,0.2,0.1,0.])
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,ip)
>>> jfa= aAI.actionsFreqsAngles(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),o.phi(ts))
```

which works because we can provide arrays for the R etc. inputs.

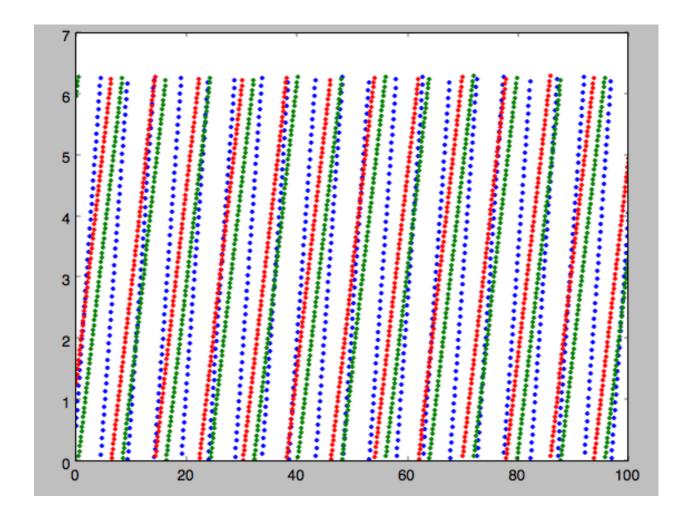
We can then check that the actions are constant over the orbit

```
>>> plot(ts,numpy.log10(numpy.fabs((jfa[0]-numpy.mean(jfa[0])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[1]-numpy.mean(jfa[1])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[2]-numpy.mean(jfa[2]))))))
```



The actions are all conserved. The angles increase linearly with time

```
>>> plot(ts, jfa[6], 'b.')
>>> plot(ts, jfa[7], 'g.')
>>> plot(ts, jfa[8], 'r.')
```



1.7.2 Action-angle coordinates for spherical potentials

Action-angle coordinates for any spherical potential can be calculated using a few orbit integrations. These are implemented in galpy in the actionAngleSpherical module. For example, we can do

```
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> from galpy.actionAngle import actionAngleSpherical
>>> aAS= actionAngleSpherical(pot=lp)
```

For the same eccentric orbit as above we find

```
>>> aAS(1.,0.5,1.3,0.2,0.1,0.)
# (array([ 0.22022112]), array([ 1.3]), array([ 0.02574507]))
>>> aAS.actionsFreqs(1.,0.5,1.3,0.2,0.1,0.)
# (array([ 0.22022112]),
# array([ 1.3]),
# array([ 0.02574507]),
# array([ 0.87630459]),
# array([ 0.60872881]),
# array([ 0.60872881]))
>>> aAS.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
# (array([ 0.22022112]),
```

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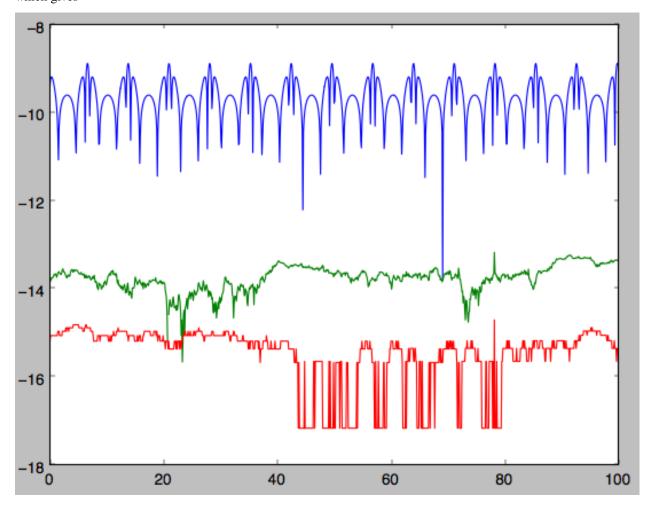
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```
# array([ 1.3]),
# array([ 0.02574507]),
# array([ 0.87630459]),
# array([ 0.60872881]),
# array([ 0.60872881]),
# array([ 0.40443857]),
# array([ 5.85965048]),
# array([ 1.1472615]))
```

We can again check that the actions are conserved along the orbit and that the angles increase linearly with time:

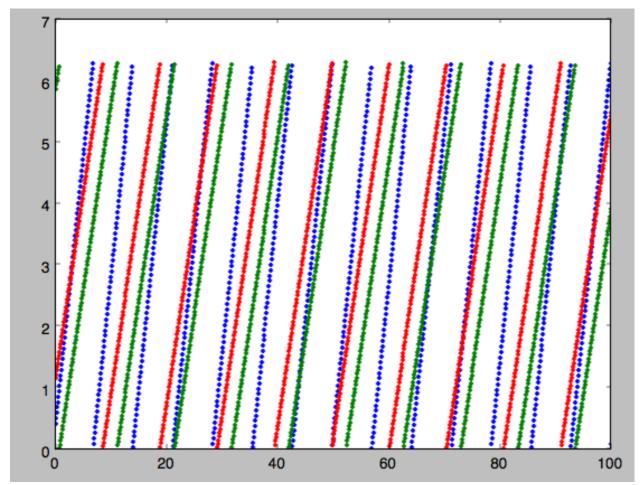
where we use fixed_quad=True for a faster evaluation of the required one-dimensional integrals using Gaussian quadrature. We then plot the action fluctuations

```
>>> plot(ts,numpy.log10(numpy.fabs((jfa[0]-numpy.mean(jfa[0])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[1]-numpy.mean(jfa[1])))))
>>> plot(ts,numpy.log10(numpy.fabs((jfa[2]-numpy.mean(jfa[2])))))
```



showing that the actions are all conserved. The angles again increase linearly with time

```
>>> plot(ts, jfa[6], 'b.')
>>> plot(ts, jfa[7], 'g.')
>>> plot(ts, jfa[8], 'r.')
```



We can check the spherical action-angle calculations against the analytical calculations for the isochrone potential. Starting again from the isochrone potential used in the previous section

```
>>> ip= IsochronePotential(b=1.,normalize=1.)
>>> aAI= actionAngleIsochrone(ip=ip)
>>> aAS= actionAngleSpherical(pot=ip)
```

we can compare the actions, frequencies, and angles computed using both

```
>>> aAI.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)

# (array([ 0.13769498]),

# array([ 1.3]),

# array([ 0.02574507]),

# array([ 1.29136096]),

# array([ 0.79093738]),

# array([ 0.79093738]),

# array([ 0.57101518]),

# array([ 5.96238847]),

# array([ 1.24999949]))
```

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```
>>> aAS.actionsFreqsAngles(1.,0.5,1.3,0.2,0.1,0.)
# (array([ 0.13769498]),
# array([ 1.3]),
# array([ 0.02574507]),
# array([ 1.29136096]),
# array([ 0.79093738]),
# array([ 0.79093738]),
# array([ 0.57101518]),
# array([ 5.96238838]),
# array([ 1.2499994]))
```

or more explicitly comparing the two

1.7.3 Action-angle coordinates using the adiabatic approximation

For non-spherical, axisymmetric potentials galpy contains multiple methods for calculating approximate action—angle coordinates. The simplest of those is the adiabatic approximation, which works well for disk orbits that do not go too far from the plane, as it assumes that the vertical motion is decoupled from that in the plane (e.g., 2010MN-RAS.401.2318B).

Setup is similar as for other actionAngle objects

```
>>> from galpy.potential import MWPotential2014
>>> from galpy.actionAngle import actionAngleAdiabatic
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014)
```

and evaluation then proceeds similarly as before

```
>>> aAA(1.,0.1,1.1,0.,0.05)
# (0.01351896260559274, 1.1, 0.0004690133479435352)
```

We can again check that the actions are conserved along the orbit

```
>>> from galpy.orbit import Orbit
>>> ts=numpy.linspace(0.,100.,1001)
>>> o= Orbit([1.,0.1,1.1,0.,0.05])
>>> o.integrate(ts,MWPotential2014)
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
```

This takes a while. The adiabatic approximation is also implemented in C, which leads to great speed-ups. Here is how to use it

```
>>> timeit(aAA(1.,0.1,1.1,0.,0.05))
# 10 loops, best of 3: 73.7 ms per loop
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
>>> timeit(aAA(1.,0.1,1.1,0.,0.05))
# 1000 loops, best of 3: 1.3 ms per loop
```

or about a 50 times speed-up. For arrays the speed-up is even more impressive

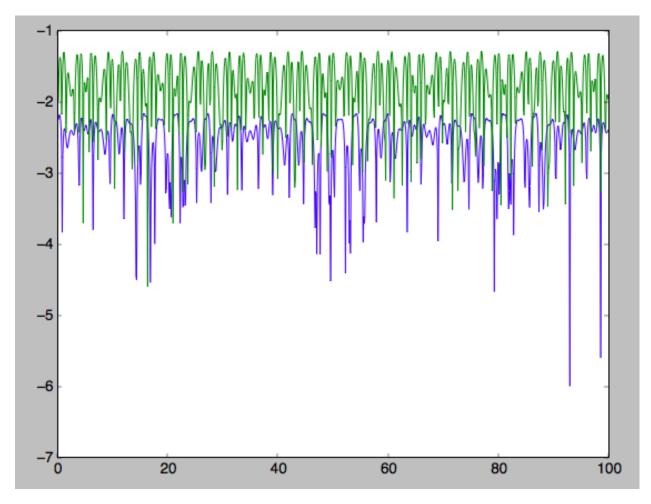
```
>>> s= numpy.ones(100)
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 10 loops, best of 3: 37.8 ms per loop
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014) #back to no C
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 1 loops, best of 3: 7.71 s per loop
```

or a speed-up of 200! Back to the previous example, you can run it with c=True to speed up the computation

```
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
```

We can plot the radial- and vertical-action fluctuation as a function of time

```
>>> plot(ts,numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts,numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2])))/numpy.mean(js[2]))))
```



The radial action is conserved to about half a percent, the vertical action to two percent.

Another way to speed up the calculation of actions using the adiabatic approximation is to tabulate the actions on a grid in (approximate) integrals of the motion and evaluating new actions by interpolating on this grid. How this is done in practice is described in detail in the galpy paper. To setup this grid-based interpolation method, which is contained in actionAngleAdiabaticGrid, do

```
>>> from galpy.actionAngle import actionAngleAdiabaticGrid
>>> aAG= actionAngleAdiabaticGrid(pot=MWPotential2014, nR=31, nEz=31, nEr=51, nLz=51,

G=True)
```

where c=True specifies that we use the C implementation of actionAngleAdiabatic for speed. We can now evaluate in the same was as before, for example

```
>>> aAA(1.,0.1,1.1,0.,0.05), aAG(1.,0.1,1.1,0.,0.05)
# ((array([ 0.01352523]), array([ 1.1]), array([ 0.00046909])),
# (0.013527010324238781, 1.1, 0.00047747359874375148))
```

which agree very well. To look at the timings, we first switch back to not using C and then list all of the relevant timings:

```
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=False)

# Not using C, direct calculation
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))

# 1 loops, best of 3: 9.05 s per loop
```

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```
>>> aAA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
# Using C, direct calculation
>>> timeit(aAA(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 10 loops, best of 3: 39.7 ms per loop
# Grid-based calculation
>>> timeit(aAG(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 1000 loops, best of 3: 1.09 ms per loop
```

Thus, in this example (and more generally) the grid-based calculation is significantly faster than even the direct implementation in C. The overall speed up between the direct Python version and the grid-based version is larger than 8,000; the speed up between the direct C version and the grid-based version is 36. For larger arrays of input phase-space positions, the latter speed up can increase to 150. For simpler, fully analytical potentials the speed up will be slightly less, but for MWPotential2014 and other more complicated potentials (such as those involving a double-exponential disk), the overhead of setting up the grid is worth it when evaluating more than a few thousand actions.

The adiabatic approximation works well for orbits that stay close to the plane. The orbit we have been considering so far only reaches a height two percent of R_0 , or about 150 pc for $R_0 = 8$ kpc.

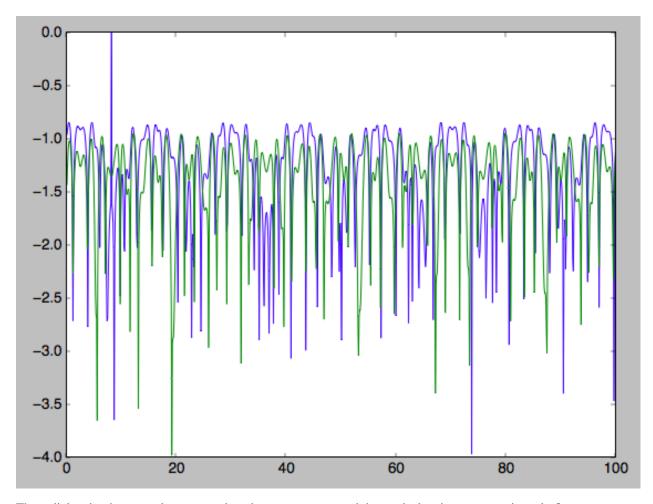
```
>>> o.zmax()*8.
# 0.17903686455491979
```

For orbits that reach distances of a kpc and more from the plane, the adiabatic approximation does not work as well. For example,

```
>>> o= Orbit([1.,0.1,1.1,0.,0.25])
>>> o.integrate(ts,MWPotential2014)
>>> o.zmax()*8.
# 1.3506059038621048
```

and we can again calculate the actions along the orbit

```
>>> js= aAA(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts))
>>> plot(ts,numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts,numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2]))/numpy.mean(js[2]))))
```



The radial action is now only conserved to about ten percent and the vertical action to approximately five percent.

Warning: Frequencies and angles using the adiabatic approximation are not implemented at this time.

1.7.4 Action-angle coordinates using the Staeckel approximation

A better approximation than the adiabatic one is to locally approximate the potential as a Staeckel potential, for which actions, frequencies, and angles can be calculated through numerical integration. galpy contains an implementation of the algorithm of Binney (2012; 2012MNRAS.426.1324B), which accomplishes the Staeckel approximation for disk-like (i.e., oblate) potentials without explicitly fitting a Staeckel potential. For all intents and purposes the adiabatic approximation is made obsolete by this new method, which is as fast and more precise. The only advantage of the adiabatic approximation over the Staeckel approximation is that the Staeckel approximation requires the user to specify a *focal length* Δ to be used in the Staeckel approximation. However, this focal length can be easily estimated from the second derivatives of the potential (see Sanders 2012; 2012MNRAS.426..128S).

Starting from the second orbit example in the adiabatic section above, we first estimate a good focal length of the MWPotential2014 to use in the Staeckel approximation. We do this by averaging (through the median) estimates at positions around the orbit (which we integrated in the example above)

```
>>> from galpy.actionAngle import estimateDeltaStaeckel
>>> estimateDeltaStaeckel(MWPotential2014, o.R(ts), o.z(ts))
# 0.40272708556203662
```

We will use $\Delta=0.4$ in what follows. We set up the actionAngleStaeckel object

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False) #c=True is the_
default
```

and calculate the actions

```
>>> aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz())
# (0.019212848866725911, 1.100000000000001, 0.015274597971510892)
```

The adiabatic approximation from above gives

```
>>> aAA(o.R(),o.vR(),o.vT(),o.z(),o.vz())
# (array([ 0.01686478]), array([ 1.1]), array([ 0.01590001]))
```

The actionAngleStaeckel calculations are sped up in two ways. First, the action integrals can be calculated using Gaussian quadrature by specifying fixed_quad=True

```
>>> aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz(),fixed_quad=True)
# (0.01922167296633687, 1.100000000000001, 0.015276825017286706)
```

which in itself leads to a ten times speed up

```
>>> timeit(aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz(),fixed_quad=False))
# 10 loops, best of 3: 129 ms per loop
>>> timeit(aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz(),fixed_quad=True))
# 100 loops, best of 3: 10.3 ms per loop
```

Second, the actionAngleStaeckel calculations have also been implemented in C, which leads to even greater speed-ups, especially for arrays

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=True)
>>> s= numpy.ones(100)
>>> timeit(aAS(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 10 loops, best of 3: 35.1 ms per loop
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False) #back to no C
>>> timeit(aAS(1.*s,0.1*s,1.1*s,0.*s,0.05*s,fixed_quad=True))
# 1 loops, best of 3: 496 ms per loop
```

or a fifteen times speed up. The speed up is not that large because the bulge model in MWPotential2014 requires expensive special functions to be evaluated. Computations could be sped up ten times more when using a simpler bulge model.

Similar to actionAngleAdiabaticGrid, we can also tabulate the actions on a grid of (approximate) integrals of the motion and interpolate over this look-up table when evaluating new actions. The details of how this look-up table is setup and used are again fully explained in the galpy paper. To use this grid-based Staeckel approximation, contained in actionAngleStaeckelGrid, do

```
>>> from galpy.actionAngle import actionAngleStaeckelGrid
>>> aASG= actionAngleStaeckelGrid(pot=MWPotential2014, delta=0.4, nE=51, npsi=51, nLz=61, Ge=True)
```

where c=True makes sure that we use the C implementation of the Staeckel method to calculate the grid. Because this is a fully three-dimensional grid, setting up the grid takes longer than it does for the adiabatic method (which only uses two two-dimensional grids). We can then evaluate actions as before

```
>>> aAS(o.R(),o.vR(),o.vT(),o.z(),o.vz()), aASG(o.R(),o.vR(),o.vT(),o.z(),o.vz())
# ((0.019212848866725911, 1.100000000000001, 0.015274597971510892),
# (0.019221119033345408, 1.10000000000001, 0.015022528662310393))
```

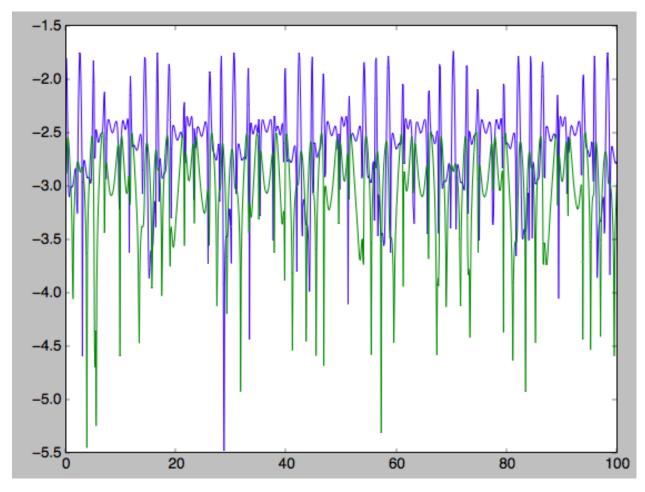
These actions agree very well. We can compare the timings of these methods as above

```
>>> timeit(aAS(1.*s,0.1*s,1.1*s,0.*s,0.05*s,fixed_quad=True))
# 1 loops, best of 3: 576 ms per loop # Not using C, direct calculation
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=True)
>>> timeit(aAS(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 100 loops, best of 3: 17.8 ms per loop # Using C, direct calculation
>>> timeit(aASG(1.*s,0.1*s,1.1*s,0.*s,0.05*s))
# 100 loops, best of 3: 3.45 ms per loop # Grid-based calculation
```

This demonstrates that the grid-based interpolation again leeds to a significant speed up, even over the C implementation of the direct calculation. This speed up becomes more significant for larger array input, although it saturates at about 25 times (at least for MWPotential2014).

We can now go back to checking that the actions are conserved along the orbit (going back to the c=False version of actionAngleStaeckel)

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=False)
>>> js= aAS(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),fixed_quad=True)
>>> plot(ts,numpy.log10(numpy.fabs((js[0]-numpy.mean(js[0]))/numpy.mean(js[0]))))
>>> plot(ts,numpy.log10(numpy.fabs((js[2]-numpy.mean(js[2])))/numpy.mean(js[2]))))
```



The radial action is now conserved to better than a percent and the vertical action to only a fraction of a percent. Clearly, this is much better than the five to ten percent errors found for the adiabatic approximation above.

For the Staeckel approximation we can also calculate frequencies and angles through the actionsFreqs and actionsFreqsAngles methods.

Warning: Frequencies and angles using the Staeckel approximation are *only* implemented in C. So use c=True in the setup of the actionAngleStaeckel object.

Warning: Angles using the Staeckel approximation in galpy are such that (a) the radial angle starts at zero at pericenter and increases then going toward apocenter; (b) the vertical angle starts at zero at z=0 and increases toward positive zmax. The latter is a different convention from that in Binney (2012), but is consistent with that in actionAngleIsochrone and actionAngleSpherical.

```
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.4,c=True)
>>> o= Orbit([1.,0.1,1.1,0.,0.25,0.]) #need to specify phi for angles
>>> aAS.actionsFreqsAngles(o.R(),o.vR(),o.vT(),o.z(),o.vz(),o.phi())
# (array([ 0.01922167]),
# array([ 1.1]),
# array([ 1.11317796]),
```

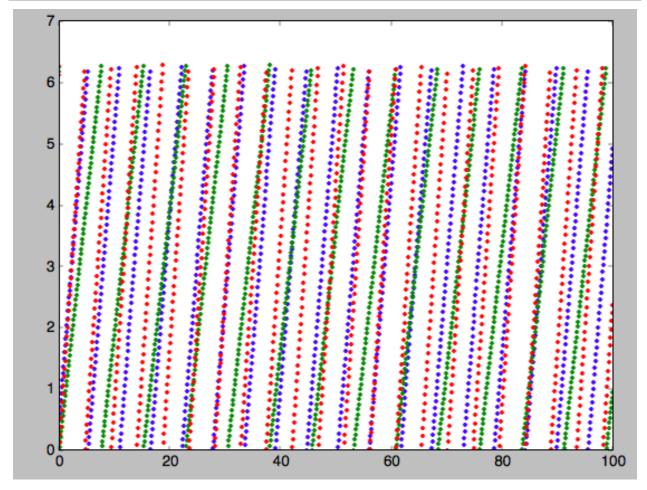
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```
# array([ 0.82538032]),
# array([ 1.34126138]),
# array([ 0.37758087]),
# array([ 6.17833493]),
# array([ 6.13368239]))
```

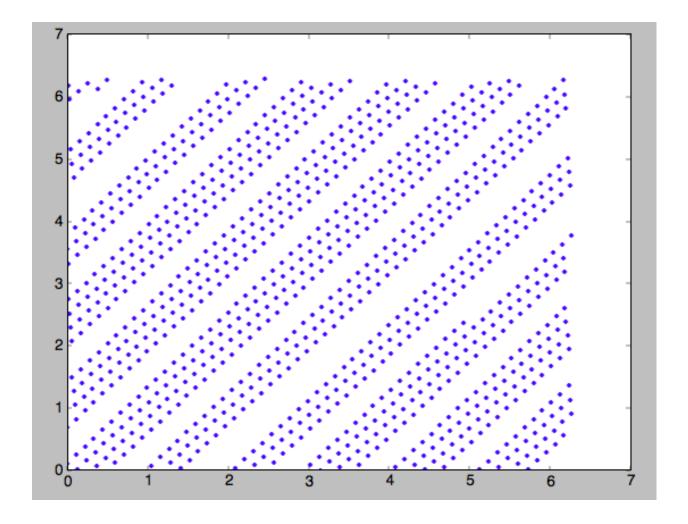
and we can check that the angles increase linearly along the orbit

```
>>> o.integrate(ts,MWPotential2014)
>>> jfa= aAS.actionsFreqsAngles(o.R(ts),o.vR(ts),o.vT(ts),o.z(ts),o.vz(ts),o.phi(ts))
>>> plot(ts,jfa[6],'b.')
>>> plot(ts,jfa[7],'g.')
>>> plot(ts,jfa[8],'r.')
```



or

```
>>> plot(jfa[6],jfa[8],'b.')
```



1.7.5 Action-angle coordinates using an orbit-integration-based approximation

The adiabatic and Staeckel approximations used above are good for stars on close-to-circular orbits, but they break down for more eccentric orbits (specifically, orbits for which the radial and/or vertical action is of a similar magnitude as the angular momentum). This is because the approximations made to the potential in these methods (that it is separable in R and z for the adiabatic approximation and that it is close to a Staeckel potential for the Staeckel approximation) break down for such orbits. Unfortunately, these methods cannot be refined to provide better approximations for eccentric orbits.

galpy contains a new method for calculating actions, frequencies, and angles that is completely general for any static potential. It can calculate the actions to any desired precision for any orbit in such potentials. The method works by employing an auxiliary isochrone potential and calculates action-angle variables by arithmetic operations on the actions and angles calculated in the auxiliary potential along an orbit (integrated in the true potential). Full details can be found in Appendix A of Bovy (2014).

We setup this method for a logarithmic potential as follows

```
>>> from galpy.actionAngle import actionAngleIsochroneApprox
>>> from galpy.potential import LogarithmicHaloPotential
>>> lp= LogarithmicHaloPotential(normalize=1.,q=0.9)
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=0.8)
```

b=0.8 here sets the scale parameter of the auxiliary isochrone potential; this potential can also be specified as an

IsochronePotential instance through ip=). We can now calculate the actions for an orbit similar to that of the GD-1 stream

```
>>> obs= numpy.array([1.56148083,0.35081535,-1.15481504,0.88719443,-0.47713334,0.

\display12019596]) #orbit similar to GD-1

>>> aAIA(*obs)

# (array([ 0.16605011]), array([-1.80322155]), array([ 0.50704439]))
```

An essential requirement of this method is that the angles calculated in the auxiliary potential go through the full range $[0, 2\pi]$. If this is not the case, galpy will raise a warning

Therefore, some care should be taken to choosing a good auxiliary potential. galpy contains a method to estimate a decent scale parameter for the auxiliary scale parameter, which works similar to estimateDeltaStaeckel above except that it also gives a minimum and maximum b if multiple R and z are given

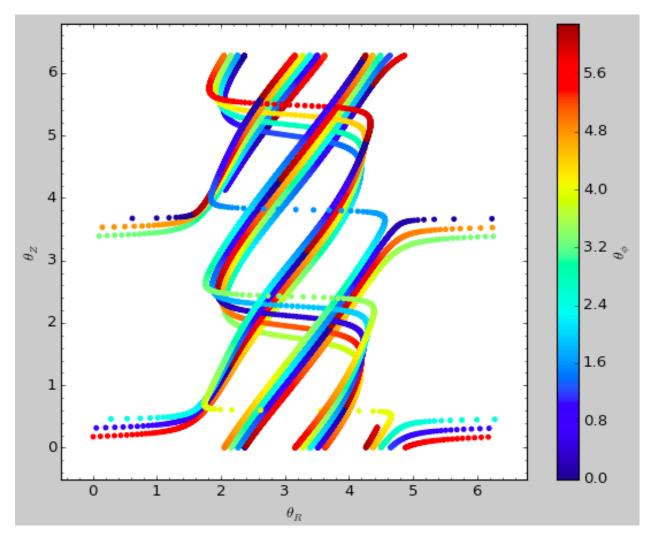
```
>>> from galpy.actionAngle import estimateBIsochrone
>>> from galpy.orbit import Orbit
>>> o= Orbit(obs)
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,lp)
>>> estimateBIsochrone(lp,o.R(ts),o.z(ts))
# (0.78065062339131952, 1.2265541473461612, 1.4899326335155412) #bmin,bmedian,bmax_
over the orbit
```

Experience shows that a scale parameter somewhere in the range returned by this function makes sure that the angles go through the full $[0, 2\pi]$ range. However, even if the angles go through the full range, the closer the angles increase to linear, the better the converenge of the algorithm is (and especially, the more accurate the calculation of the frequencies and angles is, see below). For example, for the scale parameter at the upper and of the range

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=1.5)
>>> aAIA(*obs)
# (array([ 0.01120145]), array([-1.80322155]), array([ 0.50788893]))
```

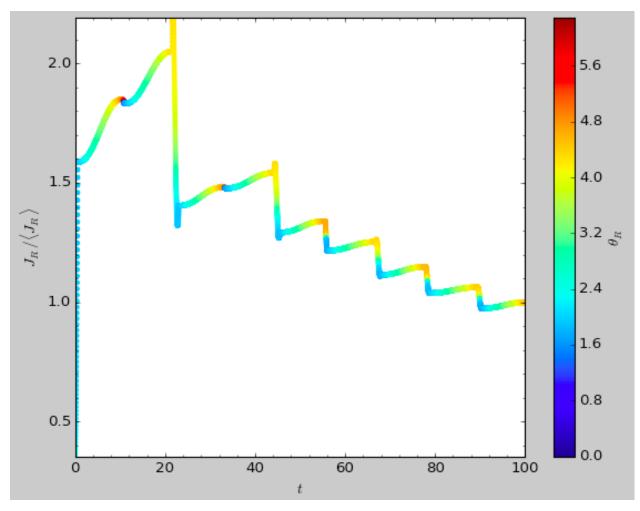
which does not agree with the previous calculation. We can inspect how the angles increase and how the actions converge by using the aAIA.plot function. For example, we can plot the radial versus the vertical angle in the auxiliary potential

```
>>> aAIA.plot(*obs,type='araz')
```



and this clearly shows that the angles increase *very* non-linearly, because the auxiliary isochrone potential used is too far from the real potential. This causes the actions to converge only very slowly. For example, for the radial action we can plot the converge as a function of integration time

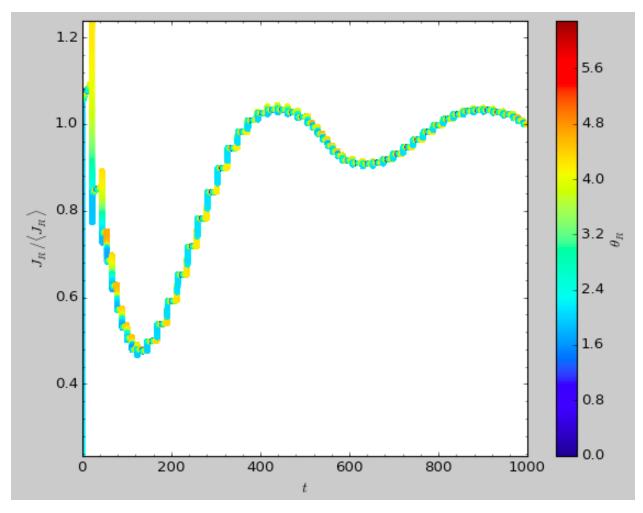
```
>>> aAIA.plot(*obs,type='jr')
```



This Figure clearly shows that the radial action has not converged yet. We need to integrate *much* longer in this auxiliary potential to obtain convergence and because the angles increase so non-linearly, we also need to integrate the orbit much more finely:

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=1.5,tintJ=1000,ntintJ=800000)
>>> aAIA(*obs)
# (array([ 0.01711635]), array([-1.80322155]), array([ 0.51008058]))
>>> aAIA.plot(*obs,type='jr')
```

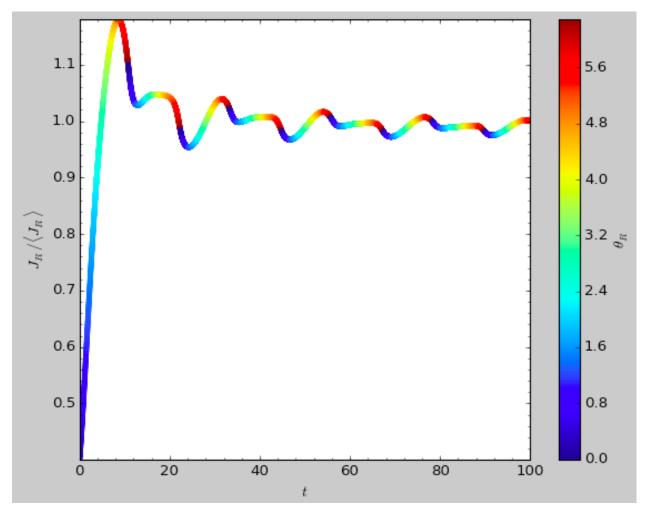
which shows slow convergence



Finding a better auxiliary potential makes convergence *much* faster and also allows the frequencies and the angles to be calculated by removing the small wiggles in the auxiliary angles vs. time (in the angle plot above, the wiggles are much larger, such that removing them is hard). The auxiliary potential used above had b=0.8, which shows very quick converenge and good behavior of the angles

```
>>> aAIA= actionAngleIsochroneApprox(pot=lp,b=0.8)
>>> aAIA.plot(*obs,type='jr')
```

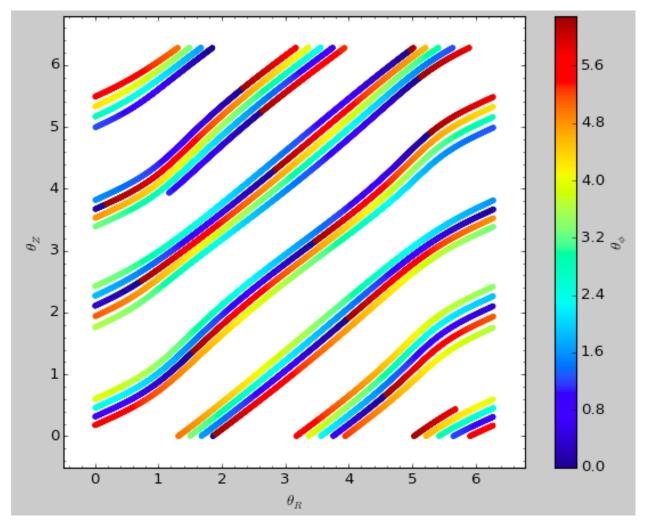
gives



and

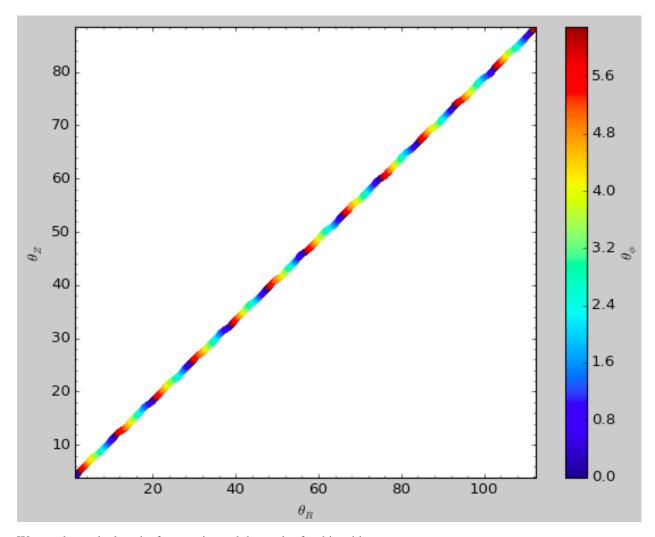
>>> aAIA.plot(*obs,type='araz')

gives



We can remove the periodic behavior from the angles, which clearly shows that they increase close-to-linear with time

```
>>> aAIA.plot(*obs,type='araz',deperiod=True)
```



We can then calculate the frequencies and the angles for this orbit as

```
>>> aAIA.actionsFreqsAngles(*obs)

# (array([ 0.16392384]),

# array([-1.80322155]),

# array([ 0.55999882]),

# array([ 0.55808933]),

# array([ 0.38475753]),

# array([ 0.42199713]),

# array([ 0.18739688]),

# array([ 0.3131815]),

# array([ 2.18425661]))
```

This function takes as an argument maxn = the maximum n for which to remove sinusoidal wiggles. So we can raise this, for example to 4 from 3

```
>>> aAIA.actionsFreqsAngles(*obs,maxn=4)

# (array([ 0.16392384]),

# array([-1.80322155]),

# array([ 0.50999882]),

# array([ 0.55808776]),

# array([-0.38475733]),

# array([ 0.4219968]),
```

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```
# array([ 0.18732009]),
# array([ 0.31318534]),
# array([ 2.18421296]))
```

Clearly, there is very little change, as most of the wiggles are of low n.

This technique also works for triaxial potentials, but using those requires the code to also use the azimuthal angle variable in the auxiliary potential (this is unnecessary in axisymmetric potentials as the z component of the angular momentum is conserved). We can calculate actions for triaxial potentials by specifying that nonaxi=True:

```
>>> aAIA(*obs,nonaxi=True)
# (array([ 0.16605011]), array([-1.80322155]), array([ 0.50704439]))
```

1.7.6 Action-angle coordinates using the TorusMapper code

All of the methods described so far allow one to compute the actions, angles, and frequencies for a given phase-space location. galpy also contains some support for computing the inverse transformation by using an interface to the TorusMapper code. Currently, this is limited to axisymmetric potentials, because the TorusMapper code is limited to such potentials.

The basic use of this part of galpy is to compute an orbit $(R, v_R, v_T, z, v_z, \phi)$ for a given torus, specified by three actions (J_R, L_Z, J_Z) and as many angles along a torus as you want. First we set up an actionAngleTorus object

```
>>> from galpy.actionAngle import actionAngleTorus
>>> from galpy.potential import MWPotential2014
>>> aAT= actionAngleTorus(pot=MWPotential2014)
```

To compute an orbit, we first need to compute the frequencies, which we do as follows

```
>>> jr,lz,jz= 0.1,1.1,0.2
>>> Om= aAT.Freqs(jr,lz,jz)
```

This set consists of $(\Omega_R, \Omega_{\phi}, \Omega_Z, \text{TMerr})$, where the last entry is the exit code of the TorusMapper code (will be printed as a warning when it is non-zero). Then we compute a set of angles that fall along an orbit as $\theta(t) = \theta_0 + \Omega t$ for a set of times t

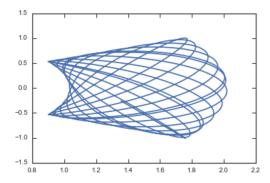
```
>>> times= numpy.linspace(0.,100.,10001)
>>> init_angle= numpy.array([1.,2.,3.])
>>> angles= numpy.tile(init_angle,(len(times),1))+Om[:3]*numpy.tile(times,(3,1)).T
```

Then we can compute the orbit by transforming the orbit in action-angle coordinates to configuration space as follows

```
>>> RvR,_,_,_= aAT.xvFreqs(jr,lz,jz,angles[:,0],angles[:,1],angles[:,2])
```

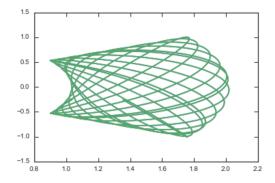
Note that the frequency is also always computed and returned by this method, because it can be obtained at zero cost. The RvR array has shape (ntimes, 6) and the six phase-space coordinates are arranged in the usual (R, vR, vT, z, vz, phi) order. The orbit in (R, Z) is then given by

```
>>> plot(RvR[:,0],RvR[:,3])
```



We can compare this to the direct numerical orbit integration. We integrate the orbit, starting at the position and velocity of the initial angle RvR[0]

```
>>> from galpy.orbit import Orbit
>>> orb= Orbit(RvR[0])
>>> orb.integrate(times, MWPotential2014)
>>> orb.plot(overplot=True)
```

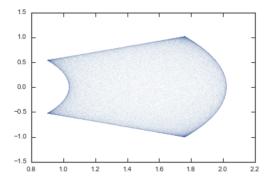


The two orbits are exactly the same.

Of course, we do not have to follow the path of an orbit to map the entire orbital torus and thus reveal the orbital building blocks of galaxies. To directly map a torus, we can do (don't worry, this doesn't take very long)

```
>>> nangles= 200001
>>> angler= numpy.random.uniform(size=nangles) *2.*numpy.pi
>>> anglep= numpy.random.uniform(size=nangles) *2.*numpy.pi
>>> anglez= numpy.random.uniform(size=nangles) *2.*numpy.pi
>>> RvR,_,_,_ = aAT.xvFreqs(jr,lz,jz,angler,anglep,anglez)
>>> plot(RvR[:,0],RvR[:,3],',',alpha=0.02)
```

which directly shows where the orbit spends most of its time:



actionAngleTorus has additional methods documented on the action-angle API page for computing Hessians and Jacobians of the transformation between action-angle and configuration space coordinates.

1.7.7 Accessing action-angle coordinates for Orbit instances

While the most flexible way to access the actionAngle routines is through the methods in the galpy.actionAngle modules, action-angle coordinates can also be calculated for galpy.orbit.Orbit instances and this is often more convenient. This is illustrated here briefly. We initialize an Orbit instance

```
>>> from galpy.orbit import Orbit
>>> from galpy.potential import MWPotential2014
>>> o= Orbit([1.,0.1,1.1,0.,0.25,0.])
```

and we can then calculate the actions (default is to use the staeckel approximation with an automatically-estimated delta parameter, but this can be adjusted)

```
>>> o.jr(pot=MWPotential2014), o.jp(pot=MWPotential2014), o.jz(pot=MWPotential2014) # (0.018194068808944613,1.1,0.01540155584446606)
```

 \circ . jp here gives the azimuthal action (which is the z component of the angular momentum for axisymmetric potentials). We can also use the other methods described above or adjust the parameters of the approximation (see above):

These two methods give very precise actions for this orbit (both are converged to about 1%) and they agree very well

We can also calculate the frequencies and the angles. This requires using the Staeckel or Isochrone approximations, because frequencies and angles are currently not supported for the adiabatic approximation. For example, the radial frequency

```
>>> o.Or(pot=MWPotential2014,type='staeckel',delta=0.4)
# 1.1131779637307115
>>> o.Or(pot=MWPotential2014,type='isochroneApprox',b=0.8)
# 1.1134635974560649
```

and the radial angle

```
>>> o.wr(pot=MWPotential2014,type='staeckel',delta=0.4)
# 0.37758086786371969
>>> o.wr(pot=MWPotential2014,type='isochroneApprox',b=0.8)
# 0.38159809018175395
```

which again agree to 1%. We can also calculate the other frequencies, angles, as well as periods using the functions o.Op, o.oz, o.wp, o.wz, o.Tr, o.Tp, o.Tz.

All of the functions above also work for Orbit instances that contain multiple objects. This is particularly convenient if you have data in observed coordinates (e.g., RA, Dec, etc.), for example,

```
>>> numpy.random.seed(1)
>>> nrand= 30
>>> ras= numpy.random.uniform(size=nrand) *360.*u.deg
>>> decs= 90.*(2.*numpy.random.uniform(size=nrand)-1.)*u.deg
>>> dists= numpy.random.uniform(size=nrand) *10.*u.kpc
>>> pmras= 2.*(2.*numpy.random.uniform(size=nrand)-1.)*u.mas/u.yr
>>> pmdecs= 2.*(2.*numpy.random.uniform(size=nrand)-1.)*u.mas/u.yr
>>> vloss= 200.*(2.*numpy.random.uniform(size=nrand)-1.)*u.km/u.s
>>> co= SkyCoord(ra=ras,dec=decs,distance=dists,
                 pm_ra_cosdec=pmras,pm_dec=pmdecs,
                 radial_velocity=vloss,
                 frame='icrs')
>>> orbits= Orbit(co)
>>> orbits.jr(pot=MWPotential2014)
# [2363.7957, 360.12445, 690.32238, 1046.2924, 132.9572, 86.989812, 272.06487, 360.
→73566, 55.568238, 698.18447, 24.783574, 21.889352, 16.148216, 3870.4286, 743.63456, □
→317.66551, 325.93816, 183.86429, 56.087796, 180.42838, 1121.8019, 8700.8335, 977.
→8525, 7.569396, 8.2847477, 210.72127, 160.9785, 680.63864, 1093.7413, 87.
→629873]kmkpcs
```

1.7.8 Example: Evidence for a Lindblad resonance in the Solar neighborhood

We can use galpy to calculate action-angle coordinates for a set of stars in the Solar neighborhood and look for unexplained features. For this we download the data from the Geneva-Copenhagen Survey (2009A&A...501..941H; data available at viZier). Since the velocities in this catalog are given as U,V, and W, we use the radec and UVW keywords to initialize the orbits from the raw data. For each object ii

```
>>> o= Orbit(vxvv[ii,:],radec=True,uvw=True,vo=220.,ro=8.)
```

We then calculate the actions and angles for each object in a flat rotation curve potential

```
>>> lp= LogarithmicHaloPotential(normalize=1.)
>>> myjr[ii]= o.jr(lp)
```

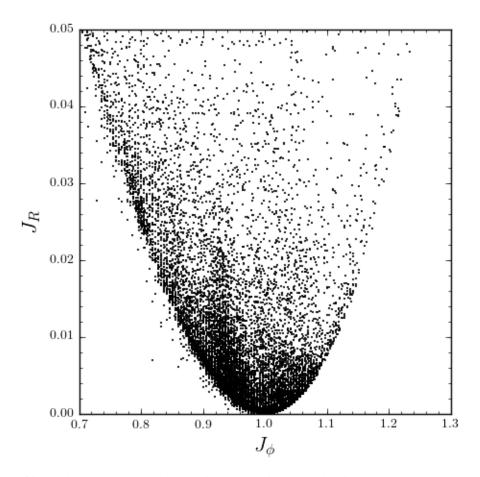
etc.

Plotting the radial action versus the angular momentum

```
>>> plot.bovy_plot(myjp,myjr,'k.',ms=2.,xlabel=r'$J_{\phi}$',ylabel=r'$J_R$',

\to xrange=[0.7,1.3],yrange=[0.,0.05])
```

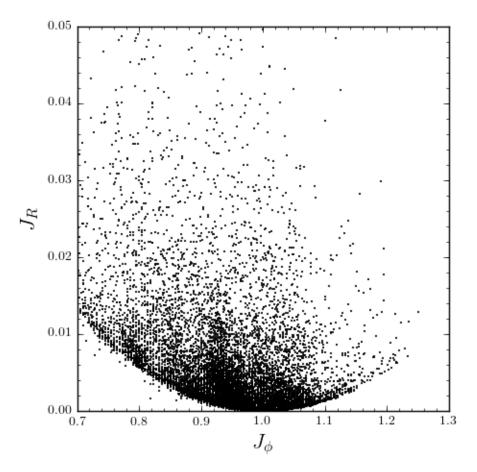
shows a feature in the distribution



If instead we use a power-law rotation curve with power-law index 1

```
>>> pp= PowerSphericalPotential(normalize=1.,alpha=-2.)
>>> myjr[ii]= o.jr(pp)
```

We find that the distribution is stretched, but the feature remains



Code for this example can be found here (note that this code uses a particular download of the GCS data set; if you use your own version, you will need to modify the part of the code that reads the data). For more information see 2010MNRAS.409..145S.

1.7.9 Example: actions in an N-body simulation

To illustrate how we can use galpy to calculate actions in a snapshot of an N-body simulation, we again look at the g15784 snapshot in the pynbody test suite, discussed in *The potential of N-body simulations*. Please look at that section for information on how to setup the potential of this snapshot in galpy. One change is that we should set enable_c=True in the instantiation of the InterpSnapshotRZPotential object

where we again normalize the potential to use galpy's natural units.

We first load a pristine copy of the simulation (because the normalization above leads to some inconsistent behavior in pynbody)

```
>>> sc = pynbody.load('Repos/pynbody-testdata/g15784.lr.01024.gz'); hc = sc.halos(); hc1= hc[1]; pynbody.analysis.halo.center(hc1,mode='hyb'); pynbody.analysis.angmom. faceon(hc1, cen=(0,0,0),mode='ssc'); sc.physical_units()
```

and then select particles near R=8 kpc by doing

These have physical units, so we normalize them (the velocity normalization is the circular velocity at R=10 kpc, see here).

```
>>> ro, vo= 10., 294.62723076942245

>>> R/= ro

>>> z/= ro

>>> vR/= vo

>>> vT/= vo

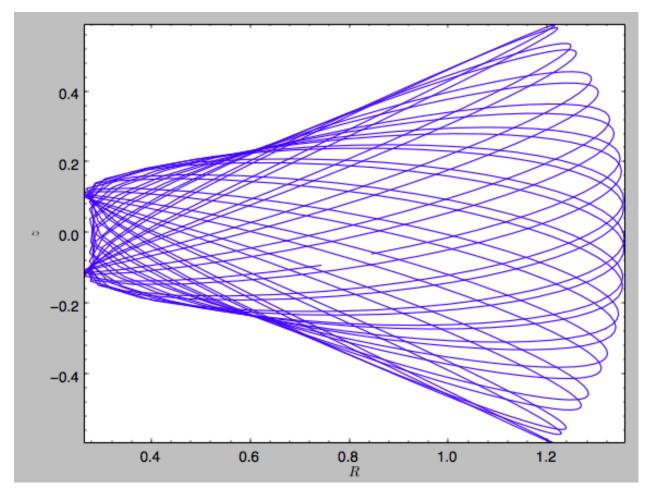
>>> vz/= vo
```

We will calculate actions using actionAngleStaeckel above. We can first integrate a random orbit in this potential

```
>>> from galpy.orbit import Orbit
>>> numpy.random.seed(1)
>>> ii= numpy.random.permutation(len(R))[0]
>>> o= Orbit([R[ii],vR[ii],vT[ii],z[ii],vz[ii]])
>>> ts= numpy.linspace(0.,100.,1001)
>>> o.integrate(ts,spi)
```

This orbit looks like this

```
>>> o.plot()
```



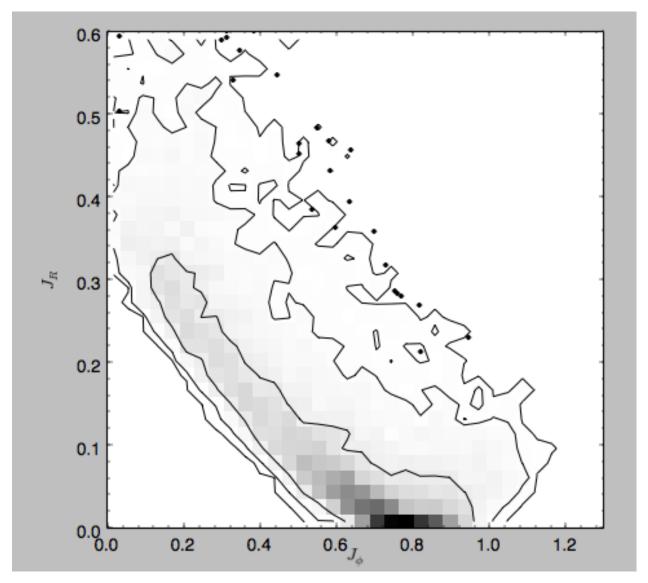
We can now calculate the actions by doing

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=spi,delta=0.45,c=True)
>>> jr,lz,jz= aAS(R,vR,vT,z,vz)
```

These actions are also in *natural units*; you can obtain physical units by multiplying with ro*vo. We can now plot these actions

```
>>> from galpy.util import bovy_plot
>>> bovy_plot.scatterplot(lz,jr,'k.',xlabel=r'$J_\phi$',ylabel=r'$J_R$',xrange=[0.,1.

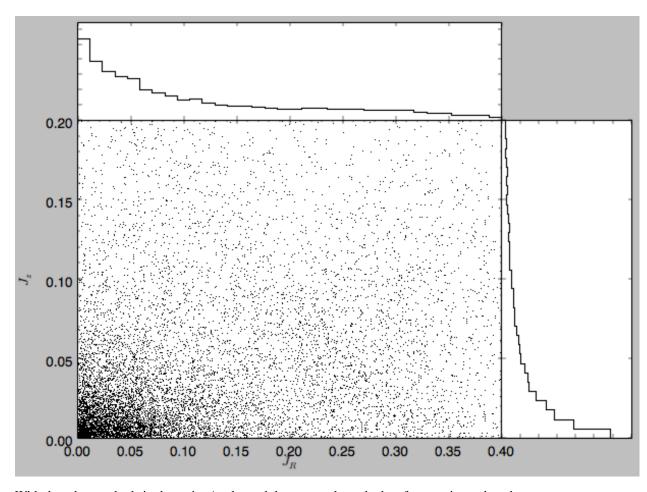
\[ \text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\texi\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{\text{
```



Note the similarity between this figure and the GCS figure above. The curve shape is due to the selection (low angular momentum stars can only enter the selected radial ring if they are very elliptical and therefore have large radial action) and the density gradient in angular momentum is due to the falling surface density of the disk. We can also look at the distribution of radial and vertical actions.

```
>>> bovy_plot.bovy_plot(jr,jz,'k,',xlabel=r'$J_R$',ylabel=r'$J_z$',xrange=[0.,.4],

\( \text{syrange} = [0.,0.2], \text{onedhists} = \text{True} \)
```



With the other methods in the actionAngle module we can also calculate frequencies and angles.

1.8 Three-dimensional disk distribution functions

galpy contains a fully three-dimensional disk distribution: galpy.df.quasiisothermaldf, which is an approximately isothermal distribution function expressed in terms of action—angle variables (see 2010MNRAS.401.2318B and 2011MNRAS.413.1889B). Recent research shows that this distribution function provides a good model for the DF of mono-abundance sub-populations (MAPs) of the Milky Way disk (see 2013MNRAS.434..652T and 2013ApJ...779..115B). This distribution function family requires action-angle coordinates to evaluate the DF, so galpy.df.quasiisothermaldf makes heavy use of the routines in galpy.actionAngle (in particular those in galpy.actionAngleAdiabatic and galpy.actionAngle.actionAngleStaeckel).

1.8.1 Setting up the DF and basic properties

The quasi-isothermal DF is defined by a gravitational potential and a set of parameters describing the radial surfacedensity profile and the radial and vertical velocity dispersion as a function of radius. In addition, we have to provide an instance of a galpy.actionAngle class to calculate the actions for a given position and velocity. For example, for a galpy.potential.MWPotential2014 potential using the adiabatic approximation for the actions, we import and define the following

```
>>> from galpy.potential import MWPotential2014
>>> from galpy.actionAngle import actionAngleAdiabatic
>>> from galpy.df import quasiisothermaldf
>>> aA= actionAngleAdiabatic(pot=MWPotential2014,c=True)
```

and then setup the quasiisothermaldf instance

```
>>> qdf= quasiisothermaldf(1./3.,0.2,0.1,1.,1.,pot=MWPotential2014,aA=aA, 

cutcounter=True)
```

which sets up a DF instance with a radial scale length of $R_0/3$, a local radial and vertical velocity dispersion of $0.2\,V_c(R_0)$ and $0.1\,V_c(R_0)$, respectively, and a radial scale lengths of the velocity dispersions of R_0 . Cutcounter=True specifies that counter-rotating stars are explicitly excluded (normally these are just exponentially suppressed). As for the two-dimensional disk DFs, these parameters are merely input (or target) parameters; the true density and velocity dispersion profiles calculated by evaluating the relevant moments of the DF (see below) are not exactly exponential and have scale lengths and local normalizations that deviate slightly from these input parameters. We can estimate the DF's actual radial scale length near R_0 as

```
>>> qdf.estimate_hr(1.)
# 0.32908034635647182
```

which is quite close to the input value of 1/3. Similarly, we can estimate the scale lengths of the dispersions

```
>>> qdf.estimate_hsr(1.)

# 1.1913935820372923

>>> qdf.estimate_hsz(1.)

# 1.0506918075359255
```

The vertical profile is fully specified by the velocity dispersions and radial density / dispersion profiles under the assumption of dynamical equilibrium. We can estimate the scale height of this DF at a given radius and height as follows

```
>>> qdf.estimate_hz(1.,0.125)
# 0.021389597757156088
```

Near the mid-plane this vertical scale height becomes very large because the vertical profile flattens, e.g.,

```
>>> qdf.estimate_hz(1.,0.125/100.)
# 1.006386030587223
```

or even

```
>>> qdf.estimate_hz(1.,0.)
# 187649.98447377066
```

which is basically infinity.

1.8.2 Evaluating moments

We can evaluate various moments of the DF giving the density, mean velocities, and velocity dispersions. For example, the mean radial velocity is again everywhere zero because the potential and the DF are axisymmetric

```
>>> qdf.meanvR(1.,0.)
# 0.0
```

Likewise, the mean vertical velocity is everywhere zero

```
>>> qdf.meanvz(1.,0.)
# 0.0
```

The mean rotational velocity has a more interesting dependence on position. Near the plane, this is the same as that calculated for a similar two-dimensional disk DF (see *Evaluating moments of the DF*)

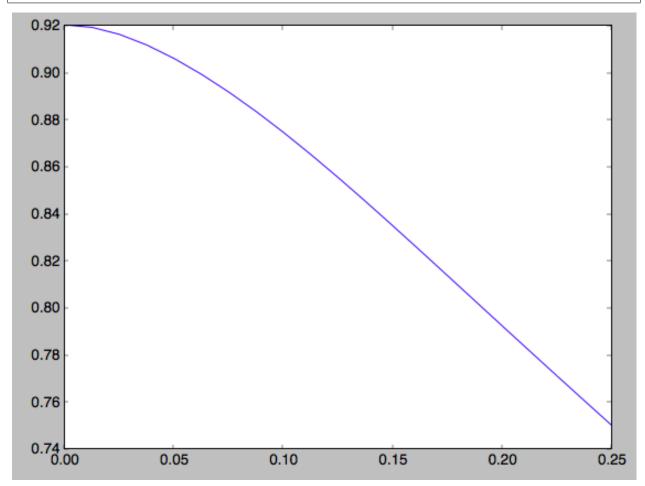
```
>>> qdf.meanvT(1.,0.)
# 0.91988346380781227
```

However, this value decreases as one moves further from the plane. The quasiisothermaldf allows us to calculate the average rotational velocity as a function of height above the plane. For example,

```
>>> zs= numpy.linspace(0.,0.25,21)
>>> mvts= numpy.array([qdf.meanvT(1.,z) for z in zs])
```

which gives

```
>>> plot(zs, mvts)
```



We can also calculate the second moments of the DF. We can check whether the radial and velocity dispersions at R_0 are close to their input values

```
>>> numpy.sqrt(qdf.sigmaR2(1.,0.))
# 0.20807112565801389
```

(continues on next page)

(continued from previous page)

```
>>> numpy.sqrt(qdf.sigmaz2(1.,0.))
# 0.090453510526130904
```

and they are pretty close. We can also calculate the mixed R and z moment, for example,

```
>>> qdf.sigmaRz(1.,0.125)
# 0.0
```

or expressed as an angle (the tilt of the velocity ellipsoid)

```
>>> qdf.tilt(1.,0.125)
# 0.0
```

This tilt is zero because we are using the adiabatic approximation. As this approximation assumes that the motions in the plane are decoupled from the vertical motions of stars, the mixed moment is zero. However, this approximation is invalid for stars that go far above the plane. By using the Staeckel approximation to calculate the actions, we can model this coupling better. Setting up a quasiisothermaldf instance with the Staeckel approximation

```
>>> from galpy.actionAngle import actionAngleStaeckel
>>> aAS= actionAngleStaeckel(pot=MWPotential2014,delta=0.45,c=True)
>>> qdfS= quasiisothermaldf(1./3.,0.2,0.1,1.,1.,pot=MWPotential2014,aA=aAS,

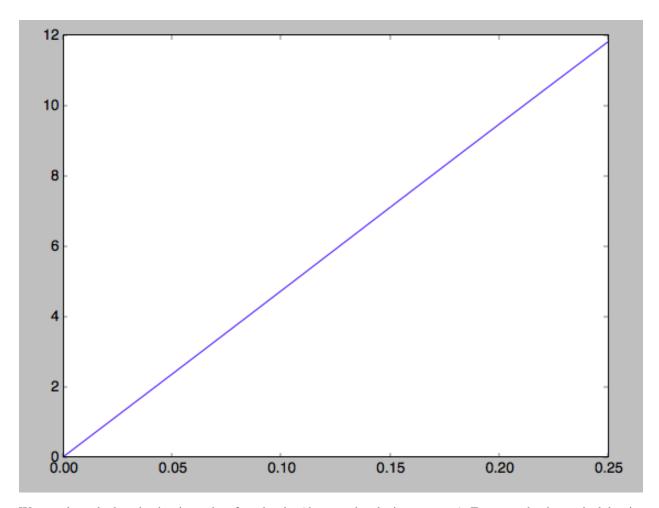
cutcounter=True)
```

we can similarly calculate the tilt

```
>>> qdfS.tilt(1.,0.125)
# 0.10314272868452541
```

or about 5 degrees (the returned value has units of rad). As a function of height, we find

```
>>> tilts= numpy.array([qdfS.tilt(1.,z) for z in zs])
>>> plot(zs,tilts*180./numpy.pi)
```



We can also calculate the density and surface density (the zero-th velocity moments). For example, the vertical density

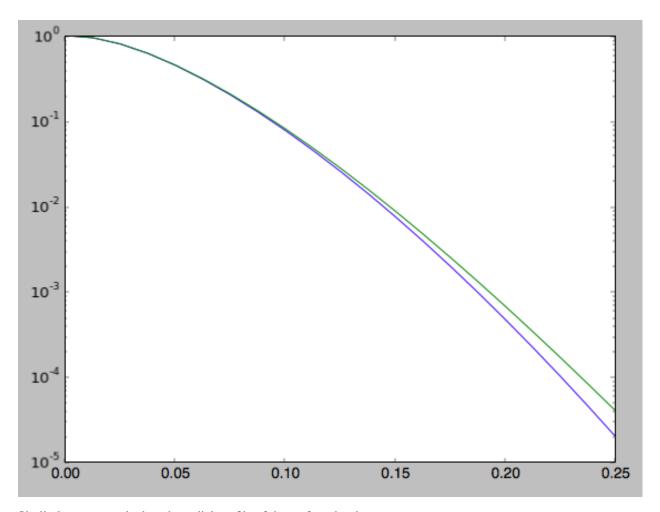
```
>>> densz= numpy.array([qdf.density(1.,z) for z in zs])
```

and

```
>>> denszS= numpy.array([qdfS.density(1.,z) for z in zs])
```

We can compare the vertical profiles calculated using the adiabatic and Staeckel action-angle approximations

```
>>> semilogy(zs,densz/densz[0])
>>> semilogy(zs,denszS/denszS[0])
```

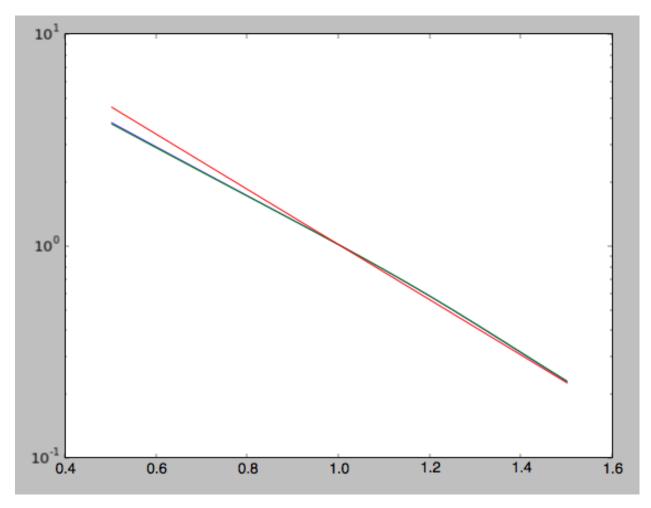


Similarly, we can calculate the radial profile of the surface density

```
>>> rs= numpy.linspace(0.5,1.5,21)
>>> surfr= numpy.array([qdf.surfacemass_z(r) for r in rs])
>>> surfrS= numpy.array([qdfS.surfacemass_z(r) for r in rs])
```

and compare them with each other and an exponential with scale length 1/3

```
>>> semilogy(rs,surfr/surfr[10])
>>> semilogy(rs,surfrS/surfrS[10])
>>> semilogy(rs,numpy.exp(-(rs-1.)/(1./3.)))
```



The two radial profiles are almost indistinguishable and are very close, if somewhat shallower, than the pure exponential profile.

General velocity moments, including all higher order moments, are implemented in quasiisothermaldf. vmomentdensity.

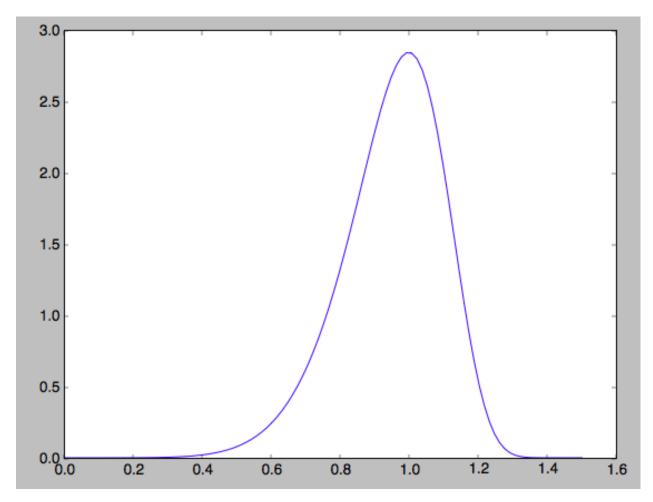
1.8.3 Evaluating and sampling the full probability distribution function

We can evaluate the distribution itself by calling the object, e.g.,

```
>>> qdf(1.,0.1,1.1,0.1,0.) #input: R, vR, vT, z, vz
# array([ 16.86790643])
```

or as a function of rotational velocity, for example in the mid-plane

```
>>> vts= numpy.linspace(0.,1.5,101)
>>> pvt= numpy.array([qdfS(1.,0.,vt,0.,0.) for vt in vts])
>>> plot(vts,pvt/numpy.sum(pvt)/(vts[1]-vts[0]))
```



This is, however, not the true distribution of rotational velocities at R = 0 and z = 0, because it is conditioned on zero radial and vertical velocities. We can calculate the distribution of rotational velocities marginalized over the radial and vertical velocities as

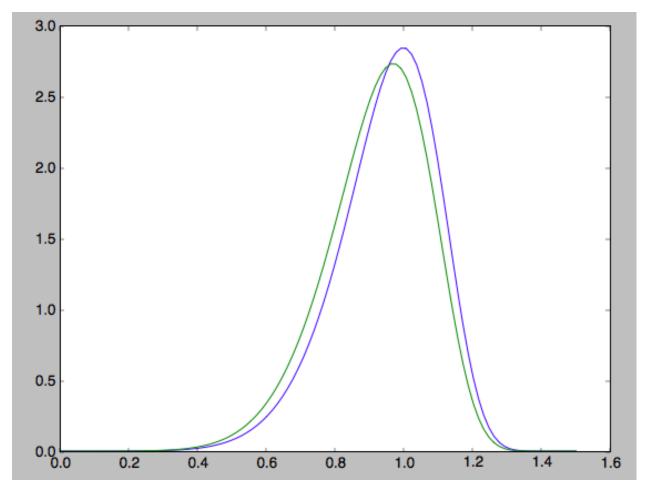
```
>>> qdfS.pvT(1.,1.,0.) #input vT,R,z
# 14.677231196899195
```

or as a function of rotational velocity

```
>>> pvt= numpy.array([qdfS.pvT(vt,1.,0.) for vt in vts])
```

overplotting this over the previous distribution gives

```
>>> plot(vts,pvt/numpy.sum(pvt)/(vts[1]-vts[0]))
```

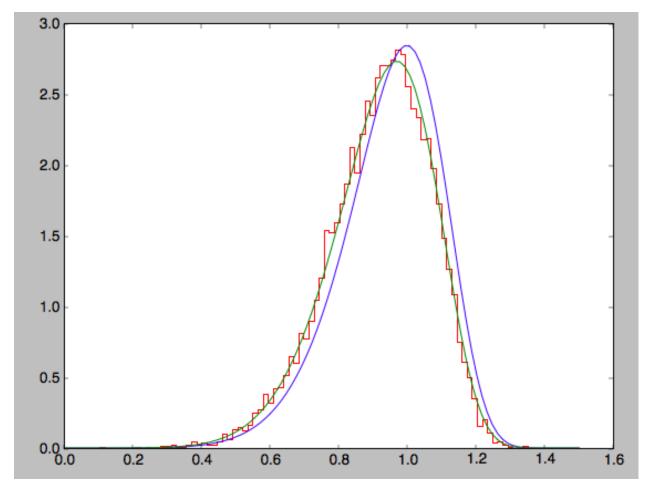


which is slightly different from the conditioned distribution. Similarly, we can calculate marginalized velocity probabilities pvR, pvZ, pvRvT, pvRvz, and pvTvz. These are all multiplied with the density, such that marginalizing these over the remaining velocity component results in the density.

We can sample velocities at a given location using quasiisothermaldf.sampleV (there is currently no support for sampling locations from the density profile, although that is rather trivial):

```
>>> vs= qdfS.sampleV(1.,0.,n=10000)
>>> hist(vs[:,1],normed=True,histtype='step',bins=101,range=[0.,1.5])
```

gives



which shows very good agreement with the green (marginalized over vR and vz) curve (as it should).

1.9 Dynamical modeling of tidal streams

galpy contains tools to model the dynamics of tidal streams, making extensive use of action-angle variables. As an example, we can model the dynamics of the following tidal stream (that of Bovy 2014; 2014ApJ...795...95B). This movie shows the disruption of a cluster on a GD-1-like orbit around the Milky Way:

The blue line is the orbit of the progenitor cluster and the black points are cluster members. The disruption is shown in an approximate orbital plane and the movie is comoving with the progenitor cluster.

Streams can be represented by simple dynamical models in action-angle coordinates. In action-angle coordinates, stream members are stripped from the progenitor cluster onto orbits specified by a set of actions (J_R, J_ϕ, J_Z) , which remain constant after the stars have been stripped. This is shown in the following movie, which shows the generation of the stream in action space

The color-coding gives the angular momentum J_{ϕ} and the black dot shows the progenitor orbit. These actions were calculated using <code>galpy.actionAngle.actionAngleIsochroneApprox</code>. The points move slightly because of small errors in the action calculation (these are correlated, so the cloud of points moves coherently because of calculation errors). The same movie that also shows the actions of stars in the cluster can be found here. This shows that the actions of stars in the cluster are not conserved (because the self-gravity of the cluster is important), but that the actions of stream members freeze once they are stripped. The angle difference between stars in a stream and the progenitor increases linearly with time, which is shown in the following movie:

where the radial and vertical angle difference with respect to the progenitor (co-moving at $(\theta_R, \theta_\phi, \theta_Z) = (\pi, \pi, \pi)$) is shown for each snapshot (the color-coding gives θ_ϕ).

One last movie provides further insight in how a stream evolves over time. The following movie shows the evolution of the stream in the two dimensional plane of frequency and angle along the stream (that is, both are projections of the three dimensional frequencies or angles onto the angle direction along the stream). The points are color-coded by the time at which they were removed from the progenitor cluster.

It is clear that disruption happens in bursts (at pericenter passages) and that the initial frequency distribution at the time of removal does not change (much) with time. However, stars removed at larger frequency difference move away from the cluster faster, such that the end of the stream is primarily made up of stars with large frequency differences with respect to the progenitor. This leads to a gradient in the typical orbit in the stream, and the stream is on average *not* on a single orbit.

1.9.1 Modeling streams in galpy

In galpy we can model streams using the tools in galpy.df.streamdf. We setup a streamdf instance by specifying the host gravitational potential pot=, an actionAngle method (typically galpy.actionAngle.actionAngleIsochroneApprox), a galpy.orbit.Orbit instance with the position of the progenitor, a parameter related to the velocity dispersion of the progenitor, and the time since disruption began. We first import all of the necessary modules

```
>>> from galpy.df import streamdf
>>> from galpy.orbit import Orbit
>>> from galpy.potential import LogarithmicHaloPotential
>>> from galpy.actionAngle import actionAngleIsochroneApprox
>>> from galpy.util import bovy_conversion #for unit conversions
```

setup the potential and actionAngle instances

```
>>> lp= LogarithmicHaloPotential(normalize=1.,q=0.9)
>>> aAI= actionAngleIsochroneApprox(pot=1p,b=0.8)
```

define a progenitor Orbit instance

```
>>> obs= Orbit([1.56148083,0.35081535,-1.15481504,0.88719443,-0.47713334,0.12019596])
```

and instantiate the streamdf model

```
>>> sigv= 0.365 #km/s
>>> sdf= streamdf(sigv/220.,progenitor=obs,pot=lp,aA=aAI,leading=True,nTrackChunks=11,

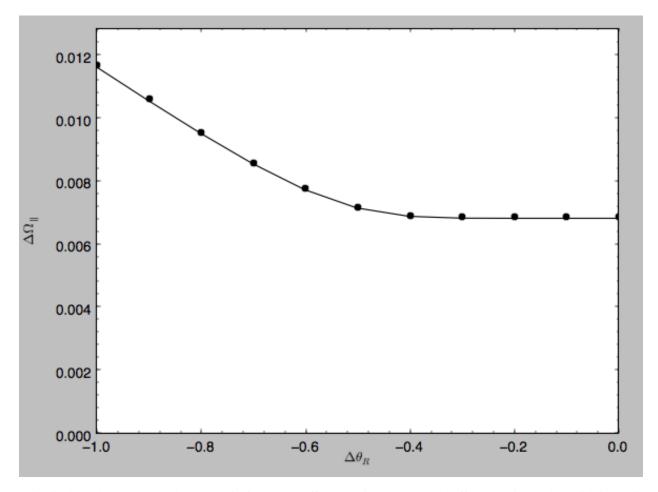
+tdisrupt=4.5/bovy_conversion.time_in_Gyr(220.,8.))
```

for a leading stream. This runs in about half a minute on a 2011 Macbook Air.

Bovy (2014) discusses how the calculation of the track needs to be iterated for potentials where there is a large offset between the track and a single orbit. One can increase the default number of iterations by specifying nTrackIterations= in the streamdf initialization (the default is set based on the angle between the track's frequency vector and the progenitor orbit's frequency vector; you can access the number of iterations used as sdf.nTrackIterations). To check whether the track is calculated accurately, one can use the following

```
>>> sdf.plotCompareTrackAAModel()
```

which in this case gives



This displays the stream model's track in frequency offset (y axis) versus angle offset (x axis) as the solid line; this is the track that the model should have if it is calculated correctly. The points are the frequency and angle offset calculated from the calculated track's (x, v). For a properly computed track these should line up, as they do in this figure. If they do not line up, increasing nTrackIterations is necessary.

We can calculate some simple properties of the stream, such as the ratio of the largest and second-to-largest eigenvalue of the Hessian $\partial \Omega/\partial J$

```
>>> sdf.freqEigvalRatio(isotropic=True)
# 34.450028399901434
```

or the model's ratio of the largest and second-to-largest eigenvalue of the model frequency variance matrix

```
>>> sdf.freqEigvalRatio()
# 29.625538344985291
```

The fact that this ratio is so large means that an approximately one dimensional stream will form.

Similarly, we can calculate the angle between the frequency vector of the progenitor and of the model mean frequency vector

```
>>> sdf.misalignment()
# 0.0086441947505973005
```

which returns this angle in radians. We can also calculate the angle between the frequency vector of the progenitor and the principal eigenvector of $\partial\Omega/\partial J$

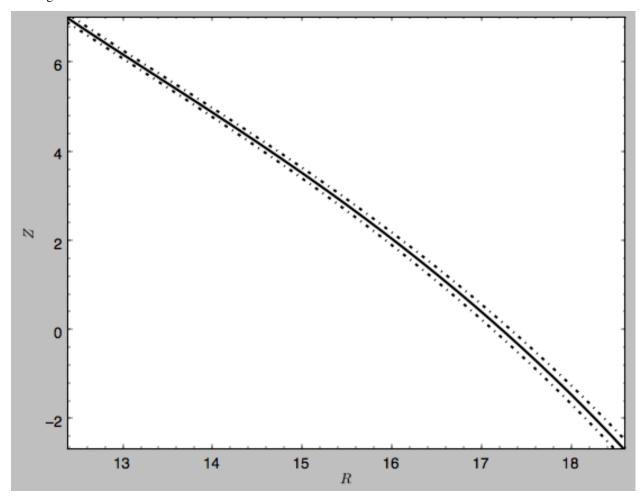
```
>>> sdf.misalignment(isotropic=True)
# 0.02238411611147997
```

(the reason these are obtained by specifying isotropic=True is that these would be the ratio of the eigenvalues or the angle if we assumed that the disrupted materials action distribution were isotropic).

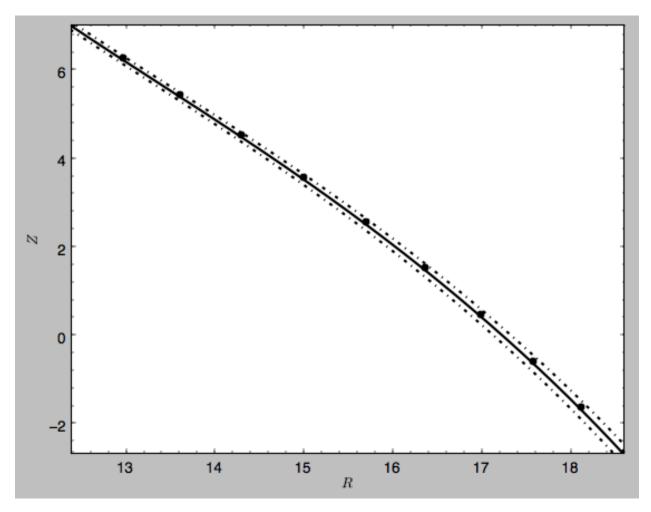
1.9.2 Calculating the average stream location (track)

We can display the stream track in various coordinate systems as follows

which gives

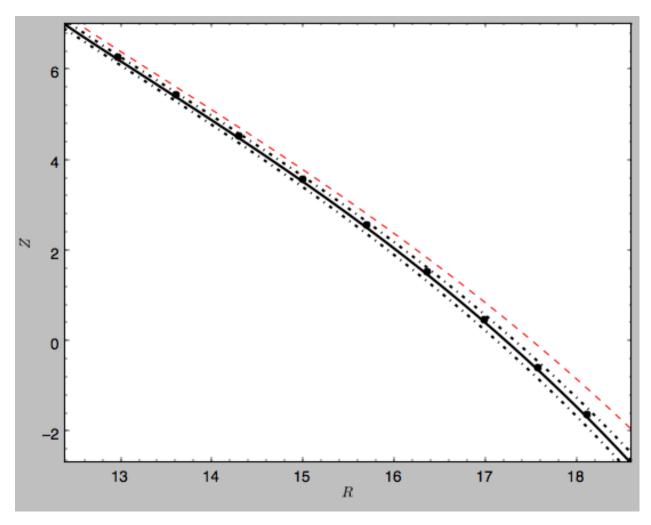


which shows the track in Galactocentric R and Z coordinates as well as an estimate of the spread around the track as the dash-dotted line. We can overplot the points along the track along which the $(\mathbf{x}, \mathbf{v}) \to (\Omega, \theta)$ transformation and the track position is explicitly calculated, by turning off the interpolation

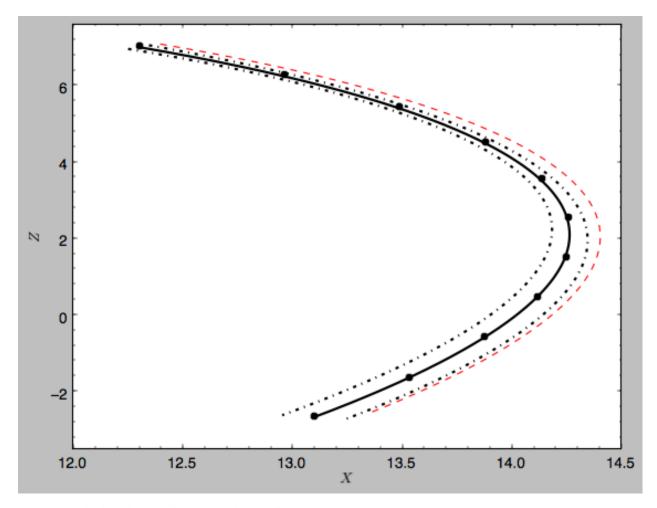


We can also overplot the orbit of the progenitor

to give

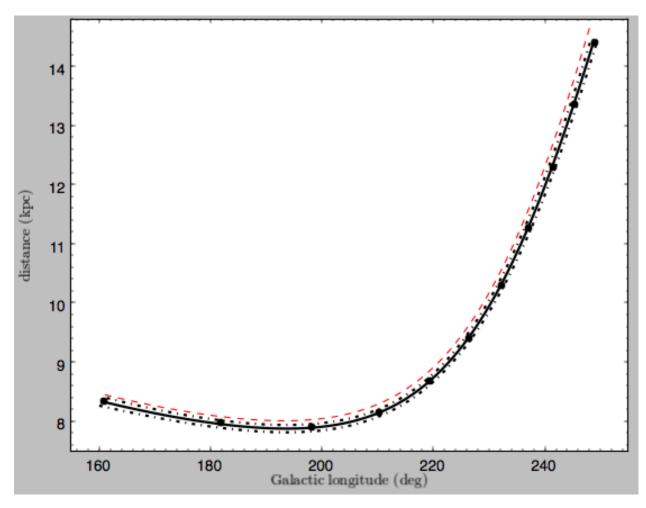


We can do the same in other coordinate systems, for example X and Z (as in Figure 1 of Bovy 2014)



or we can calculate the track in observable coordinates, e.g.,

which displays



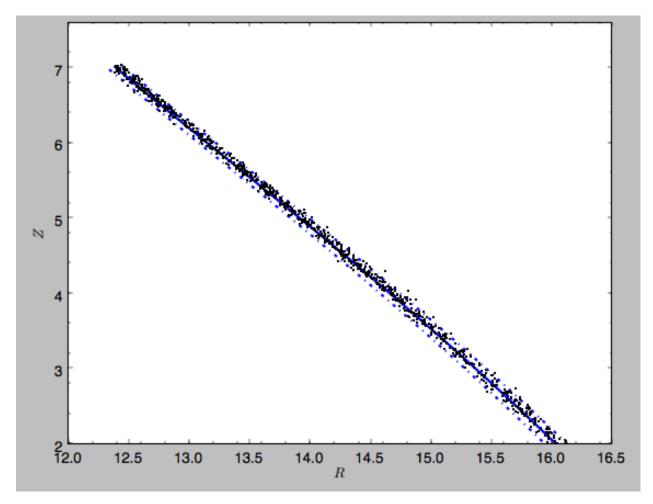
Coordinate transformations to physical coordinates are done using parameters set when initializing the sdf instance. See the help for ?streamdf for a complete list of initialization parameters.

1.9.3 Mock stream data generation

We can also easily generate mock data from the stream model. This uses streamdf.sample. For example,

```
>>> RvR= sdf.sample(n=1000)
```

which returns the sampled points as a set $(R, v_R, v_T, Z, v_Z, \phi)$ in natural galpy coordinates. We can plot these and compare them to the track location

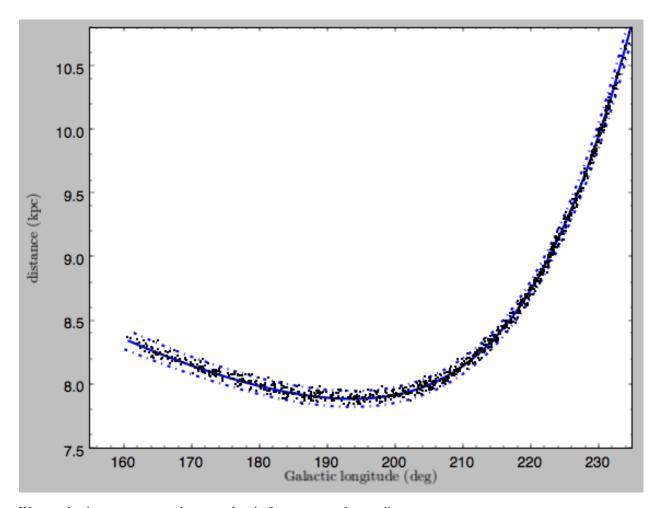


Similarly, we can generate mock data in observable coordinates

```
>>> lb= sdf.sample(n=1000,lb=True)
```

and plot it

which displays

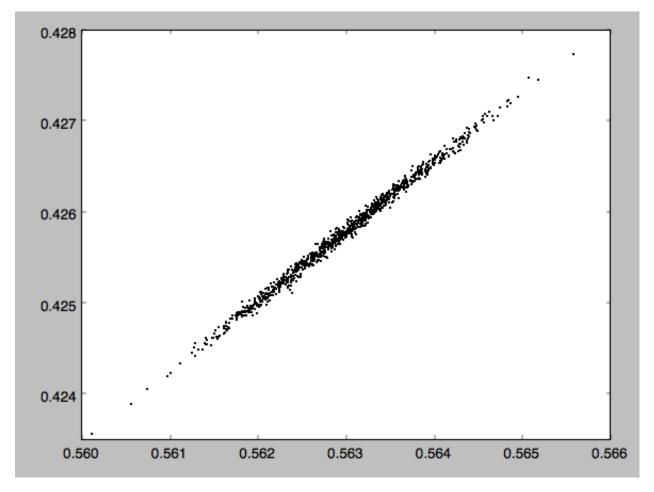


We can also just generate mock stream data in frequency-angle coordinates

```
>>> mockaA= sdf.sample(n=1000,returnaAdt=True)
```

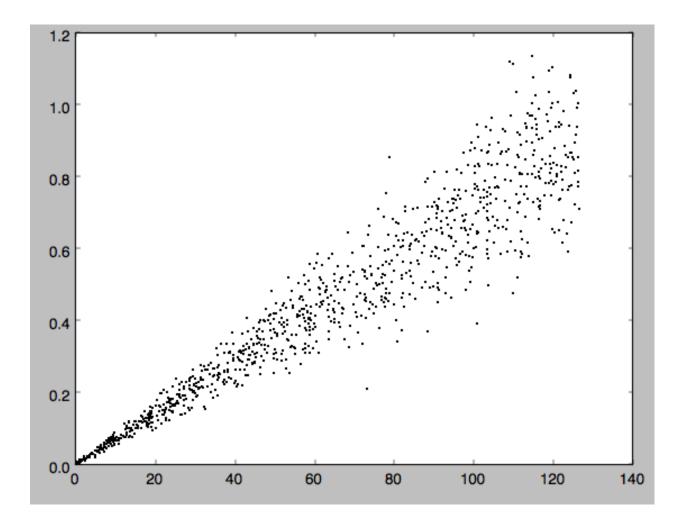
which returns a tuple with three components: an array with shape [3,N] of frequency vectors $(\Omega_R, \Omega_\phi, \Omega_Z)$, an array with shape [3,N] of angle vectors $(\theta_R, \theta_\phi, \theta_Z)$ and t_s , the stripping time. We can plot the vertical versus the radial frequency

```
>>> plot(mockaA[0][0],mockaA[0][2],'k.',ms=2.)
```



or we can plot the magnitude of the angle offset as a function of stripping time

```
>>> plot(mockaA[2], numpy.sqrt(numpy.sum((mockaA[1]-numpy.tile(sdf._progenitor_angle, \( (1000,1)).T) **2.,axis=0)),'k.',ms=2.)
```



1.9.4 Evaluating and marginalizing the full PDF

We can also evaluate the stream PDF, the probability of a (\mathbf{x}, \mathbf{v}) phase-space position in the stream. We can evaluate the PDF, for example, at the location of the progenitor

```
>>> sdf(obs.R(),obs.vR(),obs.vT(),obs.z(),obs.vz(),obs.phi())
# array([-33.16985861])
```

which returns the natural log of the PDF. If we go to slightly higher in Z and slightly smaller in R, the PDF becomes zero

```
>>> sdf(obs.R()-0.1,obs.vR(),obs.vT(),obs.z()+0.1,obs.vz(),obs.phi())
# array([-inf])
```

because this phase-space position cannot be reached by a leading stream star. We can also marginalize the PDF over unobserved directions. For example, similar to Figure 10 in Bovy (2014), we can evaluate the PDF p(X|Z) near a point on the track, say near Z=2 kpc (=0.25 in natural units. We first find the approximate Gaussian PDF near this point, calculated from the stream track and dispersion (see above)

```
>>> meanp, varp= sdf.gaussApprox([None,None,2./8.,None,None,None])
```

where the input is a array with entries [X,Y,Z,vX,vY,vZ] and we substitute None for directions that we want to establish the approximate PDF for. So the above expression returns an approximation to $p(X,Y,v_X,v_Y,v_Z|Z)$. This

approximation allows us to get a sense of where the PDF peaks and what its width is

```
>>> meanp[0]*8.

# 14.267559400127833

>>> numpy.sqrt(varp[0,0])*8.

# 0.04152968631186698
```

We can now evaluate the PDF p(X|Z) as a function of X near the peak

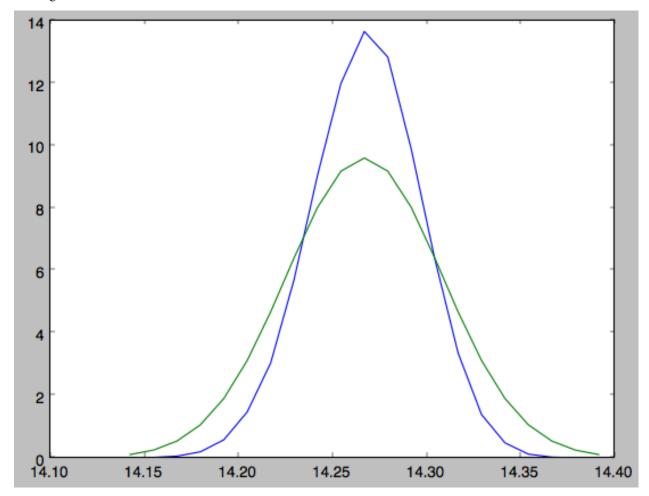
```
>>> xs= numpy.linspace(-3.*numpy.sqrt(varp[0,0]),3.*numpy.sqrt(varp[0,0]),21)+meanp[0]
>>> logps= numpy.array([sdf.callMarg([x,None,2./8.,None,None,None]) for x in xs])
>>> ps= numpy.exp(logps)
```

and we normalize the PDF

```
>>> ps/= numpy.sum(ps)*(xs[1]-xs[0])*8.
```

and plot it together with the Gaussian approximation

which gives



Sometimes it is hard to automatically determine the closest point on the calculated track if only one phase-space

coordinate is given. For example, this happens when evaluating p(Z|X) for X > 13 kpc here, where there are two branches of the track in Z (see the figure of the track above). In that case, we can determine the closest track point on one of the branches by hand and then provide this closest point as the basis of PDF calculations. The following example shows how this is done for the upper Z branch at X = 13.5 kpc, which is near Z = 5 kpc (Figure 10 in Bovy 2014).

```
>>> cindx= sdf.find_closest_trackpoint(13.5/8.,None,5.32/8.,None,None,xy=True)
```

gives the index of the closest point on the calculated track. This index can then be given as an argument for the PDF functions:

computes the approximate $p(Y, Z, v_X, v_Y, v_Z | X)$ near the upper Z branch. In Z, this PDF has mean and dispersion

```
>>> meanp[1]*8.

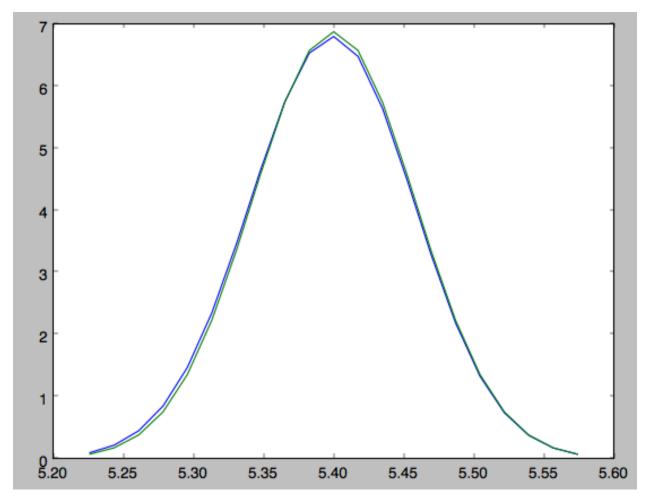
# 5.4005530328542077

>>> numpy.sqrt(varp[1,1])*8.

# 0.05796023309510244
```

We can then evaluate p(Z|X) for the upper branch as

and we can again plot this and the approximation



The approximate PDF in this case is very close to the correct PDF. When supplying the closest track point, care needs to be taken that this really is the closest track point. Otherwise the approximate PDF will not be quite correct.

1.9.5 Modeling gaps in streams

galpy also contains tools to model the effect of impacts due to dark-matter subhalos on streams (see Sanders, Bovy, & Erkal 2015). This is implemented as a subclass streamgapdf of streamdf, because they share many of the same methods. Setting up a streamgapdf object requires the same arguments and keywords as setting up a streamdf instance (to specify the smooth underlying stream model and the Galactic potential) as well as parameters that specify the impact (impact parameter and velocity, location and time of closest approach, mass and structure of the subhalo, and helper keywords that specify how the impact should be calculated). An example used in the paper (but not that with the modifications in Sec. 6.1) is as follows. Imports:

```
>>> from galpy.df import streamdf, streamgapdf
>>> from galpy.orbit import Orbit
>>> from galpy.potential import LogarithmicHaloPotential
>>> from galpy.actionAngle import actionAngleIsochroneApprox
>>> from galpy.util import bovy_conversion
```

Parameters for the smooth stream and the potential:

```
>>> lp= LogarithmicHaloPotential(normalize=1.,q=0.9)
>>> aAI= actionAngleIsochroneApprox(pot=1p,b=0.8)

(continues on next page)
```

(continued from previous page)

and the parameters of the impact

```
>>> GM= 10.**-2./bovy_conversion.mass_in_1010msol(V0,R0)
>>> rs= 0.625/R0
>>> impactb= 0.
>>> subhalovel= numpy.array([6.82200571,132.7700529,149.4174464])/V0
>>> timpact= 0.88/bovy_conversion.time_in_Gyr(V0,R0)
>>> impact_angle= -2.34
```

The setup is then

The streamgapdf implementation is currently not entirely complete (for example, one cannot yet evaluate the full phase-space PDF), but the model can be sampled as in the paper above. To compare the perturbed model to the unperturbed model, we also set up an unperturbed model of the same stream

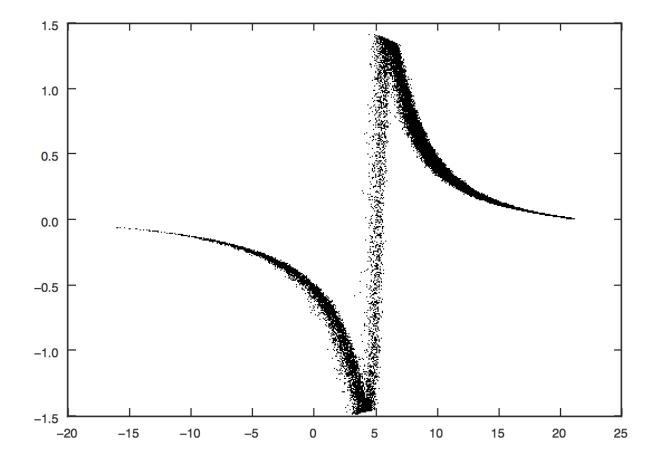
We can then sample positions and velocities for the perturbed and unperturbed preduction for the *same* particle by using the same random seed:

```
>>> numpy.random.seed(1)
>>> xv_mock_per= sdf_sanders15.sample(n=100000,xy=True).T
>>> numpy.random.seed(1) # should give same points
>>> xv_mock_unp= sdf_sanders15_unp.sample(n=100000,xy=True).T
```

and we can plot the offset due to the perturbation, for example,

```
>>> plot(xv_mock_unp[:,0]*R0,(xv_mock_per[:,0]-xv_mock_unp[:,0])*R0,'k,')
```

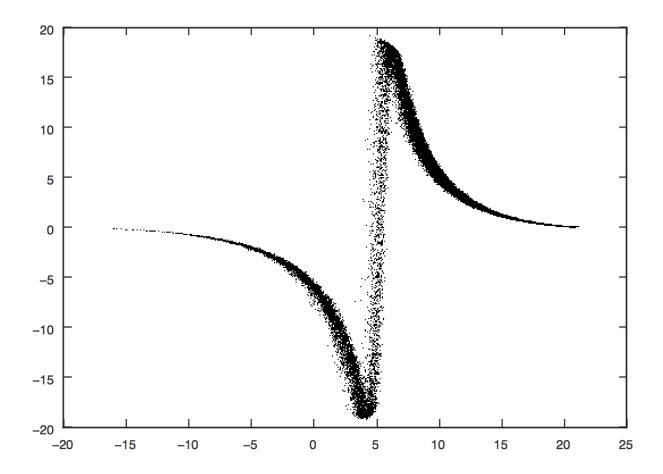
for the difference in X as a function of unperturbed X:



or

```
>>> plot(xv_mock_unp[:,0]*R0,(xv_mock_per[:,4]-xv_mock_unp[:,4])*V0,'k,')
```

for the difference in v_Y as a function of unperturbed X:



CHAPTER 2

Library reference

2.1 Orbit (galpy.orbit)

See Orbit initialization for a detailed explanation on how to set up Orbit instances.

2.1.1 Initialization

galpy.orbit.Orbit

```
Orbit.__init__ (vxvv=None, ro=None, vo=None, zo=None, solarmotion=None, radec=False, uvw=False, lb=False)

NAME:
__init__
PURPOSE:
Initialize an Orbit instance
```

INPUT:

vxvv - initial conditions (must all have the same phase-space dimension); can be either

- a) astropy (>v3.0) SkyCoord with arbitrary shape, including velocities (note that this turns *on* physical output even if ro and vo are not given)
- b) array of arbitrary shape (shape,phasedim) (shape of the orbits, followed by the phase-space dimension of the orbit); shape information is retained and used in outputs; elements can be either
 - 1) in Galactocentric cylindrical coordinates with phase-space coordinates arranged as [R,vR,vT(,z,vz,phi)]; needs to be in internal units (for Quantity input; see 'list' option below)
 - 2) [ra,dec,d,mu_ra, mu_dec,vlos] in [deg,deg,kpc,mas/yr,mas/yr,km/s] (ICRS; mu_ra = mu_ra * cos dec); (for Quantity input, see 'list' option below);

- 4) [ra,dec,d,U,V,W] in [deg,deg,kpc,km/s,km/s,kms]; (for Quantity input; see 'list' option below); ICRS frame
- 5) (l,b,d,mu_l, mu_b, vlos) in [deg,deg,kpc,mas/yr,mas/yr,km/s) (mu_l = mu_l * cos b); (for Quantity input; see 'list' option below)
- 6) [l,b,d,U,V,W] in [deg,deg,kpc,km/s,kms]; (for Quantity input; see 'list' option below)
- 5) and 6) also work when leaving out b and mu b/W
- c) lists of initial conditions, entries can be
 - 1) individual Orbit instances (of single objects)
 - 2) Quantity arrays arranged as in section 2) above (so things like [R,vR,vT,z,vz,phi], where R, vR, ... can be arbitrary shape Quantity arrays)
 - 3) list of Quantities (so things like [R1,vR1,...], where R1, vR1, ... are scalar Quantities
 - 4) None: assumed to be the Sun; if None occurs in a list it is assumed to be the Sun and all other items in the list are assumed to be [ra,dec,...]; cannot be combined with Quantity lists (2 and 3 above)
 - 5) lists of scalar phase-space coordinates arranged as in b) (so things like [R,vR,...] where R,vR are scalars in internal units

OPTIONAL INPUTS:

ro - distance from vantage point to GC (kpc; can be Quantity)

vo - circular velocity at ro (km/s; can be Quantity)

zo - offset toward the NGP of the Sun wrt the plane (kpc; can be Quantity; default = 20.8 pc from Bennett & Bovy 2019)

solarmotion - 'hogg' or 'dehnen', or 'schoenrich', or value in [-U,V,W]; can be Quantity

OUTPUT:

instance

HISTORY:

2018-10-13 - Written - Mathew Bub (UofT)

2019-01-01 - Better handling of unit/coordinate-conversion parameters and consistency checks - Bovy (UofT)

2019-02-01 - Handle array of SkyCoords in a faster way by making use of the fact that array of SkyCoords is processed correctly by Orbit

2019-02-18 - Don't support radec, lb, or uvw keywords to avoid slow coordinate transformations that would require ugly code to fix - Bovy (UofT)

2019-03-19 - Allow array vxvv and arbitrary shapes - Bovy (UofT)

galpy.orbit.Orbit.from_fit

NAME:

from fit

PURPOSE:

Initialize an Orbit using a fit to data

INPUT:

init_vxvv - initial guess for the fit (same representation [e.g.,radec=True] as vxvv data, except when customsky, then init_vxvv is assumed to be ra,dec)

vxvv - [:,6] array of positions and velocities along the orbit (if not lb=True or radec=True, these need to be in natural units [/ro,/vo], cannot be Quantities)

vxvv_err= [:,6] array of errors on positions and velocities along the orbit (if None, these are set to 0.01) (if not lb=True or radec=True, these need to be in natural units [/ro,/vo], cannot be Quantities)

pot= Potential to fit the orbit in

Keywords related to the input data:

radec= if True, input vxvv and vxvv are [ra,dec,d,mu_ra, mu_dec,vlos] in [deg,deg,kpc,mas/yr,mas/yr,km/s] (all J2000.0; mu_ra = mu_ra * cos dec); the attributes of the current Orbit are used to convert between these coordinates and Galactocentric coordinates

lb= if True, input vxvv and vxvv are [long,lat,d,mu_ll, mu_bb,vlos] in [deg,deg,kpc,mas/yr,mas/yr,km/s] (mu_ll = mu_ll * cos lat); the attributes of the current Orbit are used to convert between these coordinates and Galactocentric coordinates

customsky= if True, input vxvv and vxvv_err are [custom long,custom lat,d,mu_customll, mu_custombb,vlos] in [deg,deg,kpc,mas/yr,mas/yr,km/s] (mu_ll = mu_ll * cos lat) where custom longitude and custom latitude are a custom set of sky coordinates (e.g., ecliptic) and the proper motions are also expressed in these coordinats; you need to provide the functions lb_to_customsky and pmllpmbb_to_customsky to convert to the custom sky coordinates (these should have the same inputs and outputs as lb_to_radec and pmllpmbb_to_pmrapmdec); the attributes of the current Orbit are used to convert between these coordinates and Galactocentric coordinates

obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in kpc and km/s; entries can be Quantity) (default=Object-wide default) Cannot be an Orbit instance with the orbit of the reference point, as w/ the ra etc. functions Y is ignored and always assumed to be zero

lb_to_customsky= function that converts l,b,degree=False to the custom sky coordinates (like lb_to_radec); needs to be given when customsky=True

pmllpmbb_to_customsky= function that converts pmll,pmbb,l,b,degree=False to proper motions in the custom sky coordinates (like pmllpmbb_to_pmrapmdec); needs to be given when customsky=True

Keywords related to the orbit integrations:

```
tintJ= (default: 10) time to integrate orbits for fitting the orbit (can be Quantity)
               ntintJ= (default: 1000) number of time-integration points
               integrate_method= (default: 'dopr54_c') integration method to use
           Keywords related to the coordinate transformation:
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
               zo= offset toward the NGP of the Sun wrt the plane (kpc; can be Quantity; default = 20.8
               pc from Bennett & Bovy 2019)
               solarmotion= 'hogg' or 'dehnen', or 'schoenrich', or value in [-U,V,W]; can be Quantity
           disp= (False) display the optimizer's convergence message
      OUTPUT:
           Orbit instance
      HISTORY:
           2014-06-17 - Written - Bovy (IAS)
           2019-05-22 - Incorporated into new Orbit class as from_fit - Bovy (UofT)
galpy.orbit.Orbit.from name
classmethod Orbit.from_name(*args, **kwargs)
      NAME:
           from name
      PURPOSE:
           given the name of an object or a list of names, retrieve coordinate information for that object from
           SIMBAD and return a corresponding orbit
      INPUT:
           name - the name of the object or list of names; when loading a collection of objects (like 'mwglobu-
           larclusters'), lists are not allowed
           +standard Orbit initialization keywords:
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
               zo= offset toward the NGP of the Sun wrt the plane (kpc; can be Quantity; default = 20.8
               pc from Bennett & Bovy 2019)
               solarmotion= 'hogg' or 'dehnen', or 'schoenrich', or value in [-U,V,W]; can be Quantity
      OUTPUT:
           orbit containing the phase space coordinates of the named object
      HISTORY:
           2018-07-15 - Written - Mathew Bub (UofT)
           2019-05-21 - Generalized to multiple objects and incorporated into Orbits - Bovy (UofT)
```

2.1.2 Plotting

galpy.orbit.Orbit.animate

```
Orbit.animate(*args, **kwargs)
NAME:
animate
PURPOSE:
```

animate a previously calculated orbit (with reasonable defaults)

INPUT:

d1= first dimension to plot ('x', 'y', 'R', 'vR', 'vT', 'z', 'vz', ...); can be list with up to three entries for three subplots; each entry can also be a user-defined function of time (e.g., lambda t: o.R(t) for R)

d2= second dimension to plot; can be list with up to three entries for three subplots; each entry can also be a user-defined function of time (e.g., lambda t: o.R(t) for R)

width= (600) width of output div in px

height= (400) height of output div in px

xlabel= (pre-defined labels) label for the first dimension (or list of labels if d1 is a list); should only have to be specified when using a function as d1 and can then specify as, e.g., [None, YOUR LABEL', None] if d1 is a list of three xs and the first and last are standard entries)

ylabel= (pre-defined labels) label for the second dimension (or list of labels if d2 is a list); should only have to be specified when using a function as d2 and can then specify as, e.g., [None, YOUR LABEL', None] if d1 is a list of three xs and the first and last are standard entries)

json_filename= (None) if set, save the data necessary for the figure in this filename (e.g., json_filename= 'orbit_data/orbit.json'); this path is also used in the output HTML, so needs to be accessible

staticPlot= (False) if True, create a static plot that doesn't allow zooming, panning, etc.

ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)

vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)

use_physical= use to override Object-wide default for using a physical scale for output

OUTPUT:

IPython.display.HTML object with code to animate the orbit; can be directly shown in jupyter notebook or embedded in HTML pages; get a text version of the HTML using the _repr_html_() function

HISTORY:

```
2017-09-17-24 - Written - Bovy (UofT)
2019-03-11 - Adapted for multiple orbits - Bovy (UofT)
```

galpy.orbit.Orbit.plot

```
Orbit.plot(*args, **kwargs)
NAME:
plot
```

```
PURPOSE:
           plot a previously calculated orbit (with reasonable defaults)
      INPUT:
           d1= first dimension to plot ('x', 'y', 'R', 'vR', 'vT', 'z', 'vz', ...); can also be an expression, like
           'R*vR', or a user-defined function of time (e.g., lambda t: o.R(t) for R)
           d2= second dimension to plot; can also be an expression, like 'R*vR', or a user-defined function of
           time (e.g., lambda t: o.R(t) for R)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
           matplotlib.plot inputs+bovy_plot.plot inputs
      OUTPUT:
           sends plot to output device
      HISTORY:
           2010-07-26 - Written - Bovy (NYU)
           2010-09-22 - Adapted to more general framework - Bovy (NYU)
           2013-11-29 - added ra, dec kwargs and other derived quantities - Bovy (IAS)
           2014-06-11 - Support for plotting in physical coordinates - Bovy (IAS)
           2017-11-28 - Allow arbitrary functions of time to be plotted - Bovy (UofT)
           2019-04-13 - Edited for multiple Orbits - Bovy (UofT)
galpy.orbit.Orbit.plot3d
Orbit.plot3d(*args, **kwargs)
      NAME:
           plot3d
      PURPOSE:
           plot 3D aspects of an Orbit
      INPUT:
           d1= first dimension to plot ('x', 'y', 'R', 'vR', 'vT', 'z', 'vz', ...); can also be an expression, like
           'R*vR', or a user-defined function of time (e.g., lambda t: o.R(t) for R)
           d2= second dimension to plot
           d3= third dimension to plot
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
           bovy_plot3d args and kwargs
      OUTPUT:
```

```
plot
HISTORY:

2010-07-26 - Written - Bovy (NYU)

2010-09-22 - Adapted to more general framework - Bovy (NYU)

2010-01-08 - Adapted to 3D - Bovy (NYU)

2013-11-29 - added ra,dec kwargs and other derived quantities - Bovy (IAS)

2014-06-11 - Support for plotting in physical coordinates - Bovy (IAS)

2017-11-28 - Allow arbitrary functions of time to be plotted - Bovy (UofT)

2019-04-13 - Adapated for multiple orbits - Bovy (UofT)
```

In addition to these methods, any calculable attribute listed below can be plotted versus other attributes using plotATTR, where ATTR is an attribute like R, ll, etc. In this case, the y axis will have ATTR and the overrideable x axis default is time. For example, o.plotR() will plot the orbit's R vs time.

2.1.3 Attributes

- Orbit.name Name(s) of objects initialized using ``Orbit.from_name``
- Orbit.shape Tuple of Orbit dimensions
- Orbit.size Total number of elements in the Orbit instance

2.1.4 Methods

```
galpy.orbit.Orbit.__call__
Orbit.__call__ (*args, **kwargs)
NAME:
    __call__
PURPOSE:
    return the orbits at time t
INPUT:
    t - desired time (can be Quantity)
OUTPUT:
    an Orbit instance with initial conditions set to the phase-space at time t; shape of new Orbit is (shape_old,nt)
HISTORY:
    2019-03-05 - Written - Bovy (UofT)
    2019-03-20 - Implemented multiple times -> Orbits - Bovy (UofT)
```

```
galpy.orbit.Orbit. getitem
Orbit.__getitem__(key)
     NAME:
          __getitem__
     PURPOSE:
          get a subset of this instance's orbits
     INPUT:
          key - slice
     OUTPUT:
          For single item: Orbit instance, for multiple items: another Orbit instance
     HISTORY:
          2018-12-31 - Written - Bovy (UofT)
galpy.orbit.Orbit.bb
Orbit.bb(*args, **kwargs)
     NAME:
          bb
     PURPOSE:
          return Galactic latitude
     INPUT:
          t - (optional) time at which to get bb
          obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
          ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
          b(t) [*input_shape,nt]
     HISTORY:
          2019-02-21 - Written - Bovy (UofT
galpy.orbit.Orbit.dec
Orbit.dec(*args, **kwargs)
     NAME:
          dec
     PURPOSE:
          return the declination
     INPUT:
```

```
t - (optional) time at which to get dec
           obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
          ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
           dec(t) [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.dim
Orbit.dim()
     NAME:
           dim
     PURPOSE:
          return the dimension of the Orbit
     INPUT:
           (none)
     OUTPUT:
           dimension
     HISTORY:
           2011-02-03 - Written - Bovy (NYU)
galpy.orbit.Orbit.dist
Orbit.dist(*args, **kwargs)
     NAME:
           dist
     PURPOSE:
          return distance from the observer in kpc
     INPUT:
          t - (optional) time at which to get dist
           obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
               zero
          ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
           dist(t) in kpc [*input_shape,nt]
     HISTORY:
```

```
2019-02-21 - Written - Bovy (UofT)
```

```
galpy.orbit.Orbit.E
```

```
Orbit. E (*args, **kwargs)

NAME:

E

PURPOSE:

calculate the energy

INPUT:

t - (optional) time at which to get the energy (can be Quantity)

pot= Potential instance or list of such instances

vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)

use_physical= use to override Object-wide default for using a physical scale for output

OUTPUT:

energy [*input_shape,nt]

HISTORY:

2019-03-01 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.e

```
Orbit.e (analytic=False, pot=None, **kwargs)
NAME:
```

PURPOSE:

calculate the eccentricity, either numerically from the numerical orbit integration or using analytical means

INPUT:

analytic(= False) compute this analytically

pot - potential to use for analytical calculation

For 3D orbits different approximations for analytic=True are available (see the EccZmaxRperiRap method of actionAngle modules):

type= ('staeckel') type of actionAngle module to use

- 1) 'adiabatic': assuming motion splits into R and z
- 2) 'staeckel': assuming motion splits into u and v of prolate spheroidal coordinate system, exact for Staeckel potentials (incl. all spherical potentials)
- 3) 'spherical': for spherical potentials, exact

+actionAngle module setup kwargs for the corresponding actionAngle modules (actionAngleAdiabatic, actionAngleStaeckel, and actionAngleSpherical)

OUTPUT:

```
eccentricity [*input_shape]
      HISTORY:
           2019-02-25 - Written - Bovy (UofT)
galpy.orbit.Orbit.ER
Orbit.ER(*args, **kwargs)
      NAME:
           ER
      PURPOSE:
           calculate the radial energy
      INPUT:
           t - (optional) time at which to get the radial energy (can be Quantity)
           pot= Potential instance or list of such instances
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output (can be Quan-
           tity)
      OUTPUT:
           radial energy [*input_shape,nt]
      HISTORY:
           2019-03-01 - Written - Bovy (UofT)
galpy.orbit.Orbit.Ez
Orbit.Ez(*args, **kwargs)
      NAME:
           Ez
      PURPOSE:
           calculate the vertical energy
      INPUT:
           t - (optional) time at which to get the vertical energy (can be Quantity)
           pot= Potential instance or list of such instances
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output (can be Quan-
           tity)
      OUTPUT:
           vertical energy [*input_shape,nt]
      HISTORY:
           2019-03-01 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.flip

```
Orbit.flip(inplace=False)
NAME:
flip
```

PURPOSE:

'flip' an orbit's initial conditions such that the velocities are minus the original velocities; useful for quick backward integration; returns a new Orbit instance

INPUT:

inplace= (False) if True, flip the orbit in-place, that is, without returning a new instance and also flip the velocities of the integrated orbit (if it exists)

OUTPUT:

Orbit instance that has the velocities of the current orbit flipped (inplace=False) or just flips all velocities of current instance (inplace=True)

HISTORY:

2019-03-02 - Written - Bovy (UofT)

galpy.orbit.Orbit.integrate

```
Orbit.integrate (t, pot, method='symplec4_c', dt=None, numcores=2, force_map=False)
NAME:
```

integrate

PURPOSE:

integrate this Orbit instance with multiprocessing

INPUT:

t - list of times at which to output (0 has to be in this!) (can be Quantity)

pot - potential instance or list of instances

method = 'odeint' for scipy's odeint 'leapfrog' for a simple leapfrog implementation 'leapfrog_c' for a simple leapfrog implementation in C 'symplec4_c' for a 4th order symplectic integrator in C 'symplec6_c' for a 6th order symplectic integrator in C 'rk4_c' for a 4th-order Runge-Kutta integrator in C 'rk6_c' for a 6-th order Runge-Kutta integrator in C 'dopr54_c' for a 5-4 Dormand-Prince integrator in C 'dopr853 c' for a 8-5-3 Dormand-Prince integrator in C

dt - if set, force the integrator to use this basic stepsize; must be an integer divisor of output stepsize (only works for the C integrators that use a fixed stepsize) (can be Quantity)

numcores - number of cores to use for Python-based multiprocessing (pure Python or using force_map=True); default = OMP_NUM_THREADS

force_map= (False) if True, force use of Python-based multiprocessing (not recommended)

OUTPUT:

None (get the actual orbit using getOrbit())

HISTORY:

```
2018-10-13 - Written as parallel_map applied to regular Orbit integration - Mathew Bub (UofT)
           2018-12-26 - Written to use OpenMP C implementation - Bovy (UofT)
galpy.orbit.Orbit.integrate dxdv
Currently only supported for planarOrbit instances.
Orbit.integrate_dxdv (dxdv, t, pot, method='dopr54_c', dt=None, numcores=2, force_map=False,
                              rectIn=False, rectOut=False)
     NAME:
           integrate_dxdv
     PURPOSE:
           integrate the orbit and a small area of phase space
     INPUT:
           dxdv - [dR,dvR,dvT,dphi], shape=(*input shape,4)
           t - list of times at which to output (0 has to be in this!) (can be Quantity)
           pot - potential instance or list of instances
           dt - if set, force the integrator to use this basic stepsize; must be an integer divisor of output stepsize
           (only works for the C integrators that use a fixed stepsize) (can be Quantity)
           method = 'odeint' for scipy's odeint 'rk4_c' for a 4th-order Runge-Kutta integrator in C 'rk6_c'
               for a 6-th order Runge-Kutta integrator in C 'dopr54_c' for a 5-4 Dormand-Prince integrator in
               C 'dopr853_c' for a 8-5-3 Dormand-Prince integrator in C
           rectIn= (False) if True, input dxdv is in rectangular coordinates
           rectOut= (False) if True, output dxdv (that in orbit dxdv) is in rectangular coordinates
           numcores - number of cores to use for Python-based multiprocessing (pure Python or using
           force_map=True); default = OMP_NUM_THREADS
           force_map= (False) if True, force use of Python-based multiprocessing (not recommended)
     OUTPUT:
           (none) (get the actual orbit using getOrbit_dxdv(), the orbit that is integrated alongside with dxdv is
           stored as usual, any previous regular orbit integration will be erased!)
     HISTORY:
           2011-10-17 - Written - Bovy (IAS)
           2014-06-29 - Added rectIn and rectOut - Bovy (IAS)
           2019-05-21 - Parallelized and incorporated into new Orbits class - Bovy (UofT)
galpy.orbit.Orbit.getOrbit
Orbit.getOrbit()
     NAME:
```

getOrbit

return previously calculated orbits

PURPOSE:

```
INPUT:
          (none)
     OUTPUT:
          array orbit[*input_shape,nt,nphasedim]
     HISTORY:
          2019-03-02 - Written - Bovy (UofT)
galpy.orbit.Orbit.getOrbit dxdv
integrate_dxdv is currently only supported for planarOrbit instances. getOrbit_dxdv is therefore also
only supported for those types of Orbit.
Orbit.getOrbit_dxdv()
     NAME:
          getOrbit_dxdv
     PURPOSE:
          return a previously calculated integration of a small phase-space volume (with integrate_dxdv)
     INPUT:
          (none)
     OUTPUT:
          array orbit[*input_shape,nt,nphasedim]
     HISTORY:
          2019-05-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.helioX
Orbit.helioX(*args, **kwargs)
     NAME:
          helioX
     PURPOSE:
          return Heliocentric Galactic rectangular x-coordinate (aka "X")
     INPUT:
          t - (optional) time at which to get X
          obs=[X,Y,Z] - (optional) position and velocity of observer (in kpc and km/s) (default=Object-
               wide default) OR Orbit object that corresponds to the orbit of the observer Y is ignored and
               always assumed to be zero
          ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
          helioX(t) in kpc [*input_shape,nt]
     HISTORY:
          2019-02-21 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.helioY

```
Orbit.helioY(*args, **kwargs)
     NAME:
           helioY
     PURPOSE:
           return Heliocentric Galactic rectangular y-coordinate (aka "Y")
     INPUT:
          t - (optional) time at which to get Y
           obs=[X,Y,Z] - (optional) position and velocity of observer (in kpc and km/s) (default=Object-
               wide default) OR Orbit object that corresponds to the orbit of the observer Y is ignored and
               always assumed to be zero
          ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
          helioY(t) in kpc [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.helioZ
Orbit.helioZ(*args, **kwargs)
     NAME:
           helioZ
     PURPOSE:
           return Heliocentric Galactic rectangular z-coordinate (aka "Z")
     INPUT:
           t - (optional) time at which to get Z
           obs=[X,Y,Z] - (optional) position and velocity of observer (in kpc and km/s) (default=Object-
               wide default) OR Orbit object that corresponds to the orbit of the observer Y is ignored and
               always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
     OUTPUT:
          helioZ(t) in kpc [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.Jacobi
Orbit. Jacobi (*args, **kwargs)
     NAME:
           Jacobi
```

```
PURPOSE:
           calculate the Jacobi integral E - Omega L
      INPUT:
           t - (optional) time at which to get the Jacobi integral (can be Quantity)
           OmegaP= pattern speed (can be Quantity)
           pot= potential instance or list of such instances
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           Jacobi integral [*input_shape,nt]
      HISTORY:
           2019-03-01 - Written - Bovy (UofT)
galpy.orbit.Orbit.jp
Orbit.jp(pot=None, **kwargs)
      NAME:
           jp
      PURPOSE:
           calculate the azimuthal action
      INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           jp [*input_shape]
      HISTORY:
           2019-02-26 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.jr

```
Orbit.jr(pot=None, **kwargs)
     NAME:
           jr
     PURPOSE:
           calculate the radial action
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           jr [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.jz
Orbit.jz(pot=None, **kwargs)
     NAME:
           jΖ
     PURPOSE:
           calculate the vertical action
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
```

```
+actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           jz [*input_shape]
      HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.II
Orbit.11 (*args, **kwargs)
      NAME:
      PURPOSE:
           return Galactic longitude
      INPUT:
           t - (optional) time at which to get ll
           obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
               zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
      OUTPUT:
           l(t) [*input_shape,nt]
      HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.L
Orbit.L(*args, **kwargs)
      NAME:
           L
      PURPOSE:
           calculate the angular momentum at time t
      INPUT:
           t - (optional) time at which to get the angular momentum (can be Quantity)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
```

```
OUTPUT:
           angular momentum [*input_shape,nt,3]
     HISTORY:
           2019-03-01 - Written - Bovy (UofT)
galpy.orbit.Orbit.Lz
Orbit.Lz(*args, **kwargs)
     NAME:
          Lz
     PURPOSE:
           calculate the z-component of the angular momentum at time t
     INPUT:
           t - (optional) time at which to get the angular momentum (can be Quantity)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           z-component of the angular momentum [*input_shape,nt]
     HISTORY:
           2019-03-01 - Written - Bovy (UofT)
galpy.orbit.Orbit.Op
Orbit.Op (pot=None, **kwargs)
     NAME:
           Op
     PURPOSE:
           calculate the azimuthal frequency
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
```

```
+actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           Op [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.Or
Orbit.Or (pot=None, **kwargs)
     NAME:
           Or
     PURPOSE:
           calculate the radial frequency
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           Or [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.Oz
Orbit.Oz (pot=None, **kwargs)
     NAME:
           Oz
     PURPOSE:
```

```
calculate the vertical frequency
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           Oz [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.phasedim
Orbit.phasedim()
     NAME:
           phasedim
     PURPOSE:
           return the phase-space dimension of the problem (2 for 1D, 3 for 2D-axi, 4 for 2D, 5 for 3D-axi, 6
           for 3D)
     INPUT:
           (none)
     OUTPUT:
           phase-space dimension
     HISTORY:
           2018-12-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.phi
Orbit.phi(*args, **kwargs)
     NAME:
           phi
     PURPOSE:
           return azimuth
```

```
INPUT:
           t - (optional) time at which to get the azimuth
     OUTPUT:
           phi(t) [*input_shape,nt] in [-pi,pi]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.pmbb
Orbit.pmbb(*args, **kwargs)
     NAME:
           pmbb
     PURPOSE:
           return proper motion in Galactic latitude (in mas/yr)
     INPUT:
           t - (optional) time at which to get pmbb
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
                                                                                      kpc
                                                                                             and
                                                                                                     km/s)
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
     OUTPUT:
           pm_b(t) in mas/yr [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.pmdec
Orbit.pmdec(*args, **kwargs)
     NAME:
           pmdec
     PURPOSE:
           return proper motion in declination (in mas/yr)
     INPUT:
           t - (optional) time at which to get pmdec
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
```

```
OUTPUT:
           pm_dec(t) in mas/yr [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.pmll
Orbit.pmll(*args, **kwargs)
     NAME:
           pmll
     PURPOSE:
           return proper motion in Galactic longitude (in mas/yr)
     INPUT:
           t - (optional) time at which to get pmll
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
                                                                                      kpc
                                                                                             and
                                                                                                     km/s)
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
     OUTPUT:
           pm_l(t) in mas/yr [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.pmra
Orbit.pmra(*args, **kwargs)
     NAME:
           pmra
     PURPOSE:
           return proper motion in right ascension (in mas/yr)
     INPUT:
           t - (optional) time at which to get pmra
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
                                                                                      kpc
                                                                                             and
                                                                                                     km/s)
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
     OUTPUT:
           pm_ra(t) in mas / yr [*input_shape,nt]
```

```
HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.r
Orbit.r(*args, **kwargs)
     NAME:
           r
     PURPOSE:
           return spherical radius at time t
     INPUT:
           t - (optional) time at which to get the radius
           ro= (Object-wide default) physical scale for distances to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           r(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.R
Orbit.R(*args, **kwargs)
     NAME:
           R
     PURPOSE:
           return cylindrical radius at time t
     INPUT:
           t - (optional) time at which to get the radius (can be Quantity)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           R(t) [*input_shape,nt]
     HISTORY:
           2019-02-01 - Written - Bovy (UofT)
galpy.orbit.Orbit.ra
Orbit.ra(*args, **kwargs)
     NAME:
           ra
```

```
PURPOSE:
           return the right ascension
      INPUT:
           t - (optional) time at which to get ra
           obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
               zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
      OUTPUT:
           ra(t) [*input_shape,nt]
      HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.rap
Orbit.rap (analytic=False, pot=None, **kwargs)
      NAME:
           rap
      PURPOSE:
           calculate the apocenter radius, either numerically from the numerical orbit integration or using ana-
           lytical means
      INPUT:
           analytic(= False) compute this analytically
           pot - potential to use for analytical calculation
           For 3D orbits different approximations for analytic=True are available (see the EccZmaxRperiRap
           method of actionAngle modules):
               type= ('staeckel') type of actionAngle module to use
                 1) 'adiabatic': assuming motion splits into R and z
                 2) 'staeckel': assuming motion splits into u and v of prolate spheroidal coordinate system,
                    exact for Staeckel potentials (incl. all spherical potentials)
                 3) 'spherical': for spherical potentials, exact
               +actionAngle module setup kwargs for the corresponding actionAngle modules (actionAn-
               gleAdiabatic, actionAngleStaeckel, and actionAngleSpherical)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           R_ap [*input_shape]
      HISTORY:
           2019-02-25 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.reshape

```
Orbit.reshape (newshape)
     NAME:
           reshape
     PURPOSE:
           Change the shape of the Orbit instance
     INPUT:
           newshape - new shape (int or tuple of ints; see numpy.reshape)
     OUTPUT:
           (none; re-shaping is done in-place)
     HISTORY:
           2019-03-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.rguiding
Orbit.rguiding(*args, **kwargs)
     NAME:
           rguiding
     PURPOSE:
           calculate the guiding-center radius (the radius of a circular orbit with the same angular momentum)
     INPUT:
           pot= potential instance or list of such instances
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           R_guiding [*input_shape,nt]
     HISTORY:
           2019-03-02 - Written as thin wrapper around Potential.rl - Bovy (UofT)
galpy.orbit.Orbit.rperi
Orbit.rperi(analytic=False, pot=None, **kwargs)
     NAME:
           rperi
     PURPOSE:
           calculate the pericenter radius, either numerically from the numerical orbit integration or using ana-
           lytical means
     INPUT:
```

```
pot - potential to use for analytical calculation
           For 3D orbits different approximations for analytic=True are available (see the EccZmaxRperiRap
           method of actionAngle modules):
               type= ('staeckel') type of actionAngle module to use
                1) 'adiabatic': assuming motion splits into R and z
                2) 'staeckel': assuming motion splits into u and v of prolate spheroidal coordinate system,
                   exact for Staeckel potentials (incl. all spherical potentials)
                3) 'spherical': for spherical potentials, exact
               +actionAngle module setup kwargs for the corresponding actionAngle modules (actionAn-
               gleAdiabatic, actionAngleStaeckel, and actionAngleSpherical)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           R_peri [*input_shape]
      HISTORY:
           2019-02-25 - Written - Bovy (UofT)
galpy.orbit.Orbit.SkyCoord
Orbit.SkyCoord(*args, **kwargs)
      NAME:
           SkyCoord
      PURPOSE:
           return the positions and velocities as an astropy SkyCoord
      INPUT:
           t - (optional) time at which to get the position
           obs=[X,Y,Z] - (optional) position of observer (in kpc) (default=Object-wide default) OR Orbit
               object that corresponds to the orbit of the observer Y is ignored and always assumed to be
               zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
      OUTPUT:
           SkyCoord(t) [*input_shape,nt]
      HISTORY:
           2019-02-21 - Written - Bovy (UofT)
```

analytic(= False) compute this analytically

galpy.orbit.Orbit.time

```
Orbit.time(*args, **kwargs)
     NAME:
           time
     PURPOSE:
           return the times at which the orbit is sampled
     INPUT:
           t - (default: integration times) time at which to get the time (for consistency reasons); default is to
           return the list of times at which the orbit is sampled
           ro= (Object-wide default) physical scale for distances to use to convert
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           t(t)
     HISTORY:
           2019-02-28 - Written - Bovy (UofT)
galpy.orbit.Orbit.toLinear
Orbit.toLinear()
     NAME:
           toLinear
     PURPOSE:
           convert 3D orbits into 1D orbits (z)
     INPUT:
           (none)
     OUTPUT:
           linear Orbit instance
     HISTORY:
           2019-03-02 - Written - Bovy (UofT)
galpy.orbit.Orbit.toPlanar
Orbit.toPlanar()
     NAME:
           toPlanar
     PURPOSE:
           convert 3D orbits into 2D orbits
     INPUT:
```

```
(none)
      OUTPUT:
           planar Orbit instance
      HISTORY:
           2019-03-02 - Written - Bovy (UofT)
galpy.orbit.Orbit.Tp
Orbit.Tp (pot=None, **kwargs)
     NAME:
           Tp
      PURPOSE:
           calculate the azimuthal period
      INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           Tp [*input_shape]
      HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.Tr
Orbit.Tr (pot=None, **kwargs)
     NAME:
           Tr
      PURPOSE:
           calculate the radial period
      INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
```

```
1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
          use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
          Tr [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.TrTp
Orbit.TrTp (pot=None, **kwargs)
     NAME:
          TrTp
     PURPOSE:
          the 'ratio' between the radial and azimuthal period Tr/Tphi*pi
     INPUT:
          pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
          +actionAngle module setup kwargs
     OUTPUT:
          Tr/Tp*pi [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.turn_physical_off
Orbit.turn_physical_off()
     NAME:
           turn_physical_off
```

```
PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2019-02-28 - Written - Bovy (UofT)
galpy.orbit.Orbit.turn_physical_on
Orbit.turn_physical_on(ro=None, vo=None)
     NAME:
          turn_physical_on
     PURPOSE:
           turn on automatic returning of outputs in physical units
     INPUT:
          ro= reference distance (kpc; can be Quantity)
          vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
          (none)
     HISTORY:
          2019-02-28 - Written - Bovy (UofT)
           2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)
galpy.orbit.Orbit.Tz
Orbit.Tz (pot=None, **kwargs)
     NAME:
          Tz
     PURPOSE:
          calculate the vertical period
     INPUT:
          pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
```

```
4) 'spherical'
           +actionAngle module setup kwargs
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           Tz [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.U
Orbit.U(*args, **kwargs)
     NAME:
           U
     PURPOSE:
           return Heliocentric Galactic rectangular x-velocity (aka "U")
     INPUT:
           t - (optional) time at which to get U
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
     OUTPUT:
           U(t) in km/s [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.V
Orbit. V (*args, **kwargs)
     NAME:
     PURPOSE:
           return Heliocentric Galactic rectangular y-velocity (aka "V")
     INPUT:
           t - (optional) time at which to get U
```

```
obs=[X,Y,Z,vx,vv,vz] - (optional) position and velocity of observer (in
                                                                                                       km/s)
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
      OUTPUT:
           V(t) in km/s [*input_shape,nt]
      HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.vbb
Orbit.vbb(*args, **kwargs)
      NAME:
           vbb
      PURPOSE:
           return velocity in Galactic latitude (km/s)
      INPUT:
           t - (optional) time at which to get vbb (can be Quantity)
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer in the Galactocentric frame
               (in kpc and km/s) (default=[8.0,0.,0.,0.,220.,0.]; entries can be Quantity) OR Orbit object that
               corresponds to the orbit of the observer Y is ignored and always assumed to be zero
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
      OUTPUT:
           v_b(t) in km/s [*input_shape]
      HISTORY:
           2019-02-28 - Written - Bovy (UofT)
galpy.orbit.Orbit.vdec
Orbit.vdec(*args, **kwargs)
      NAME:
           vdec
      PURPOSE:
           return velocity in declination (km/s)
      INPUT:
           t - (optional) time at which to get vdec (can be Quantity)
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer in the Galactocentric frame
               (in kpc and km/s) (default=[8.0,0.,0.,0.,220.,0.]; entries can be Quantity) OR Orbit object that
               corresponds to the orbit of the observer Y is ignored and always assumed to be zero
```

```
ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
      OUTPUT:
           v_dec(t) in km/s [*input_shape]
      HISTORY:
           2019-02-28 - Written - Bovy (UofT)
galpy.orbit.Orbit.vll
Orbit.vll(*args, **kwargs)
      NAME:
           vll
      PURPOSE:
           return the velocity in Galactic longitude (km/s)
      INPUT:
           t - (optional) time at which to get vll (can be Quantity)
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer in the Galactocentric frame
               (in kpc and km/s) (default=[8.0,0.,0.,0.,220.,0.]; entries can be Quantity) OR Orbit object that
               corresponds to the orbit of the observer Y is ignored and always assumed to be zero
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)
      OUTPUT:
           v_l(t) in km/s [*input_shape]
      HISTORY:
           2019-02-28 - Written - Bovy (UofT)
galpy.orbit.Orbit.vlos
Orbit.vlos(*args, **kwargs)
      NAME:
           vlos
      PURPOSE:
           return the line-of-sight velocity (in km/s)
      INPUT:
           t - (optional) time at which to get vlos
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
```

```
OUTPUT:
           vlos(t) in km/s [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.vphi
Orbit.vphi(*args, **kwargs)
     NAME:
           vphi
     PURPOSE:
           return angular velocity
     INPUT:
           t - (optional) time at which to get the angular velocity
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           vphi(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.vR
Orbit.vR(*args, **kwargs)
     NAME:
           vR
     PURPOSE:
           return radial velocity at time t
     INPUT:
           t - (optional) time at which to get the radial velocity
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           vR(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
```

```
galpy.orbit.Orbit.vra

Orbit.vra (*args, **kwargs)

NAME:

vra

PURPOSE:

return velocity in right ascension (km/s)

INPUT:
```

t - (optional) time at which to get vra (can be Quantity)

obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer in the Galactocentric frame (in kpc and km/s) (default=[8.0,0.,0.,0.,220.,0.]; entries can be Quantity) OR Orbit object that corresponds to the orbit of the observer Y is ignored and always assumed to be zero

ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)

vo= (Object-wide default) physical scale for velocities to use to convert (can be Quantity)

OUTPUT:

```
v_ra(t) in km/s [*input_shape]
HISTORY:
```

2019-02-28 - Written - Bovy (UofT)

galpy.orbit.Orbit.vT

```
Orbit .vT (*args, **kwargs)

NAME:

vT

PURPOSE:

return tangential velocity at time t

INPUT:

t - (optional) time at which to get the tangential velocity

vo= (Object-wide default) physical scale for velocities to use to convert

use_physical= use to override Object-wide default for using a physical scale for output

OUTPUT:

vT(t) [*input_shape,nt]

HISTORY:

2019-02-20 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.vx

```
Orbit.vx(*args, **kwargs)
NAME:
vx
```

```
PURPOSE:
           return x velocity at time t
      INPUT:
           t - (optional) time at which to get the velocity
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           vx(t) [*input_shape,nt]
      HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.vy
Orbit.vy(*args, **kwargs)
      NAME:
           vy
      PURPOSE:
           return y velocity at time t
      INPUT:
           t - (optional) time at which to get the velocity
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           vy(t) [*input_shape,nt]
      HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.vz
Orbit.vz (*args, **kwargs)
     NAME:
           VΖ
      PURPOSE:
           return vertical velocity
      INPUT:
           t - (optional) time at which to get the vertical velocity
           vo= (Object-wide default) physical scale for velocities to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
```

```
OUTPUT:
           vz(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.W
Orbit.W(*args, **kwargs)
     NAME:
           W
     PURPOSE:
           return Heliocentric Galactic rectangular z-velocity (aka "W")
     INPUT:
           t - (optional) time at which to get W
           obs=[X,Y,Z,vx,vy,vz] - (optional) position and velocity of observer (in
                                                                                     kpc
                                                                                            and
                                                                                                    km/s)
               (default=Object-wide default) OR Orbit object that corresponds to the orbit of the observer Y is
               ignored and always assumed to be zero
           ro= distance in kpc corresponding to R=1. (default=Object-wide default)
           vo= velocity in km/s corresponding to v=1. (default=Object-wide default)
     OUTPUT:
           W(t) in km/s [*input_shape,nt]
     HISTORY:
           2019-02-21 - Written - Bovy (UofT)
galpy.orbit.Orbit.wp
Orbit.wp (pot=None, **kwargs)
     NAME:
           wp
     PURPOSE:
           calculate the azimuthal angle
     INPUT:
           pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
            4) 'spherical'
           +actionAngle module setup kwargs
```

```
OUTPUT:
          wp [*input_shape]
     HISTORY:
          2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.wr
Orbit.wr (pot=None, **kwargs)
     NAME:
           wr
     PURPOSE:
          calculate the radial angle
     INPUT:
          pot - potential
           type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
           3) 'isochroneApprox'
           4) 'spherical'
          +actionAngle module setup kwargs
     OUTPUT:
          wr [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.wz
Orbit.wz (pot=None, **kwargs)
     NAME:
          wz
     PURPOSE:
          calculate the vertical angle
     INPUT:
          pot - potential
          type= ('staeckel') type of actionAngle module to use
            1) 'adiabatic'
            2) 'staeckel'
            3) 'isochroneApprox'
```

```
4) 'spherical'
           +actionAngle module setup kwargs
     OUTPUT:
           wz [*input_shape]
     HISTORY:
           2019-02-27 - Written - Bovy (UofT)
galpy.orbit.Orbit.x
Orbit.x(*args, **kwargs)
     NAME:
           \mathbf{X}
     PURPOSE:
           return x
     INPUT:
           t - (optional) time at which to get x
           ro= (Object-wide default) physical scale for distances to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           x(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.y
Orbit.y(*args, **kwargs)
     NAME:
     PURPOSE:
           return y
     INPUT:
           t - (optional) time at which to get y
           ro= (Object-wide default) physical scale for distances to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
     OUTPUT:
           y(t) [*input_shape,nt]
     HISTORY:
           2019-02-20 - Written - Bovy (UofT)
```

galpy.orbit.Orbit.z

```
Orbit.z(*args, **kwargs)
      NAME:
           Z
      PURPOSE:
           return vertical height
      INPUT:
           t - (optional) time at which to get the vertical height
           ro= (Object-wide default) physical scale for distances to use to convert
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           z(t) [*input_shape,nt]
      HISTORY:
           2019-02-20 - Written - Bovy (UofT)
galpy.orbit.Orbit.zmax
Orbit.zmax(analytic=False, pot=None, **kwargs)
      NAME:
           zmax
      PURPOSE:
           calculate the maximum vertical height, either numerically from the numerical orbit integration or
           using analytical means
      INPUT:
           analytic(= False) compute this analytically
           pot - potential to use for analytical calculation
           For 3D orbits different approximations for analytic=True are available (see the EccZmaxRperiRap
           method of actionAngle modules):
               type= ('staeckel') type of actionAngle module to use
                 1) 'adiabatic': assuming motion splits into R and z
                 2) 'staeckel': assuming motion splits into u and v of prolate spheroidal coordinate system,
                    exact for Staeckel potentials (incl. all spherical potentials)
                3) 'spherical': for spherical potentials, exact
               +actionAngle module setup kwargs for the corresponding actionAngle modules (actionAn-
               gleAdiabatic, actionAngleStaeckel, and actionAngleSpherical)
           ro= (Object-wide default) physical scale for distances to use to convert (can be Quantity)
           use_physical= use to override Object-wide default for using a physical scale for output
      OUTPUT:
           Z_max [*input_shape]
```

```
HISTORY:
     2019-02-25 - Written - Bovy (UofT)
```

2.2 Potential (galpy.potential)

```
2.2.1 3D potentials
General instance routines
Use as Potential-instance.method(...)
galpy.potential.Potential. add
Potential._{add}_{(b)}
     NAME:
          __add__
     PURPOSE:
          Add Force or Potential instances together to create a multi-component potential (e.g., pot=
          pot1+pot2+pot3)
     INPUT:
          b - Force or Potential instance or a list thereof
     OUTPUT:
          List of Force or Potential instances that represents the combined potential
     HISTORY:
          2019-01-27 - Written - Bovy (UofT)
          2020-04-22 - Added check that unit systems of combined potentials are compatible - Bovy (UofT)
galpy.potential.Potential.__mul__
Potential. mul (b)
     NAME:
          __mul__
     PURPOSE:
          Multiply a Force or Potential's amplitude by a number
     INPUT:
          b - number
     OUTPUT:
          New instance with amplitude = (old amplitude) x b
```

HISTORY:

2019-01-27 - Written - Bovy (UofT)

galpy.potential.Potential.__call__

Warning: galpy potentials do *not* necessarily approach zero at infinity. To compute, for example, the escape velocity or whether or not an orbit is unbound, you need to take into account the value of the potential at infinity. E.g., $v_{\rm esc}(r) = \sqrt{2[\Phi(\infty) - \Phi(r)]}$.

```
Potential.___call___(R, z, phi=0.0, t=0.0, dR=0, dphi=0)
     NAME:
           __call__
     PURPOSE:
           evaluate the potential at (R,z,phi,t)
     INPUT:
           R - Cylindrical Galactocentric radius (can be Quantity)
           z - vertical height (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           Phi(R,z,t)
     HISTORY:
           2010-04-16 - Written - Bovy (NYU)
galpy.potential.Potential.dens
Potential.dens (R, z, phi=0.0, t=0.0, forcepoisson=False)
     NAME:
           dens
     PURPOSE:
           evaluate the density rho(R,z,t)
     INPUT:
           R - Cylindrical Galactocentric radius (can be Quantity)
           z - vertical height (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     KEYWORDS:
           forcepoisson= if True, calculate the density through the Poisson equation, even if an explicit expres-
           sion for the density exists
     OUTPUT:
           rho (R,z,phi,t)
```

```
HISTORY:
           2010-08-08 - Written - Bovy (NYU)
galpy.potential.Potential.dvcircdR
Potential.dvcircdR(R, phi=None, t=0.0)
     NAME:
           dvcircdR
     PURPOSE:
           calculate the derivative of the circular velocity at R wrt R in this potential
     INPUT:
           R - Galactocentric radius (can be Quantity)
           phi= (None) azimuth to use for non-axisymmetric potentials
           t - time (optional; can be Quantity)
     OUTPUT:
           derivative of the circular rotation velocity wrt R
     HISTORY:
           2013-01-08 - Written - Bovy (IAS)
           2016-06-28 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
galpy.potential.Potential.epifreq
Potential.epifreq(R, t=0.0)
     NAME:
           epifreq
     PURPOSE:
           calculate the epicycle frequency at R in this potential
     INPUT:
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           epicycle frequency
     HISTORY:
           2011-10-09 - Written - Bovy (IAS)
```

galpy.potential.Potential.flattening

```
Potential.flattening (R, z, t=0.0)
     NAME:
          flattening
     PURPOSE:
          calculate the potential flattening, defined as sqrt(fabs(z/R F_R/F_z))
     INPUT:
          R - Galactocentric radius (can be Quantity)
          z - height (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          flattening
     HISTORY:
          2012-09-13 - Written - Bovy (IAS)
galpy.potential.Potential.lindbladR
Potential.lindbladR(OmegaP, m=2, t=0.0, **kwargs)
     NAME:
          lindbladR
     PURPOSE:
          calculate the radius of a Lindblad resonance
     INPUT:
          OmegaP - pattern speed (can be Quantity)
          m= order of the resonance (as in m(O-Op)=kappa (negative m for outer) use
                                                                                        m='corotation'
               for corotation +scipy.optimize.brentq xtol,rtol,maxiter kwargs
          t - time (optional; can be Quantity)
     OUTPUT:
          radius of Linblad resonance, None if there is no resonance
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.mass
Potential.mass (R, z=None, t=0.0, forceint=False)
     NAME:
          mass
     PURPOSE:
```

```
evaluate the mass enclosed
```

INPUT:

```
R - Cylindrical Galactocentric radius (can be Quantity)
```

z= (None) vertical height (can be Quantity)

t - time (optional; can be Quantity)

KEYWORDS:

forceint= if True, calculate the mass through integration of the density, even if an explicit expression for the mass exists

OUTPUT:

- 1) for spherical potentials: M(<R) [or if z is None], when the mass is implemented explicitly, the mass enclosed within $r = sqrt(R^2+z^2)$ is returned when not z is None; forceint will integrate between -z and z, so the two are inconsistent (If you care to have this changed, raise an issue on github)
- 2) for axisymmetric potentials: M(<R,<fabs(Z))

HISTORY:

```
2014-01-29 - Written - Bovy (IAS)
2019-08-15 - Added spherical warning - Bovy (UofT)
```

galpy.potential.Potential.nemo_accname

```
Potential.nemo_accname() NAME:
```

nemo accname

PURPOSE:

return the accname potential name for use of this potential with NEMO

INPUT:

(none)

OUTPUT:

Acceleration name

HISTORY:

2014-12-18 - Written - Bovy (IAS)

galpy.potential.Potential.nemo accpars

```
Potential.nemo_accpars(vo, ro) NAME:
```

nemo_accpars

PURPOSE:

return the accpars potential parameters for use of this potential with NEMO

INPUT:

```
vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          accpars string
     HISTORY:
           2014-12-18 - Written - Bovy (IAS)
galpy.potential.Potential.omegac
Potential.omegac (R, t=0.0)
     NAME:
          omegac
     PURPOSE:
          calculate the circular angular speed at R in this potential
     INPUT:
          R - Galactocentric radius (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          circular angular speed
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.phiforce
Potential.phiforce (R, z, phi=0.0, t=0.0)
     NAME:
          phiforce
     PURPOSE:
          evaluate the azimuthal force F_phi (R,z,phi,t)
     INPUT:
          R - Cylindrical Galactocentric radius (can be Quantity)
           z - vertical height (can be Quantity)
           phi - azimuth (rad; can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          F_{phi}(R,z,phi,t)
     HISTORY:
           2010-07-10 - Written - Bovy (NYU)
```

galpy.potential.Potential.phi2deriv

```
Potential.phi2deriv (R, Z, phi=0.0, t=0.0)
     NAME:
           phi2deriv
     PURPOSE:
           evaluate the second azimuthal derivative
     INPUT:
           R - Galactocentric radius (can be Quantity)
           Z - vertical height (can be Quantity)
           phi - Galactocentric azimuth (can be Quantity)
           t - time (can be Quantity)
     OUTPUT:
           d2Phi/dphi2
     HISTORY:
           2013-09-24 - Written - Bovy (IAS)
galpy.potential.Potential.plot
Potential.plot (t=0.0, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, nzs=21, effective=False,
                     Lz=None, phi=None, xy=False, xrange=None, yrange=None, justcontours=False, lev-
                     els=None, cntrcolors=None, ncontours=21, savefilename=None)
     NAME:
           plot
     PURPOSE:
           plot the potential
     INPUT:
           t= time to plot potential at
           rmin= minimum R (can be Quantity) [xmin if xy]
           rmax= maximum R (can be Quantity) [ymax if xy]
           nrs= grid in R
           zmin= minimum z (can be Quantity) [ymin if xy]
           zmax= maximum z (can be Quantity) [ymax if xy]
           nzs= grid in z
           phi= (None) azimuth to use for non-axisymmetric potentials
           xy= (False) if True, plot the potential in X-Y
           effective= (False) if True, plot the effective potential Phi + Lz^2/2/R^2
           Lz= (None) angular momentum to use for the effective potential when effective=True
           justcontours= (False) if True, just plot contours
```

```
savefilename - save to or restore from this savefile (pickle)
           xrange, yrange= can be specified independently from rmin, zmin, etc.
           levels= (None) contours to plot
           ncontours - number of contours when levels is None
           cntrcolors= (None) colors of the contours (single color or array with length ncontours)
     OUTPUT:
           plot to output device
     HISTORY:
           2010-07-09 - Written - Bovy (NYU)
           2014-04-08 - Added effective = - Bovy (IAS)
galpy.potential.Potential.plotDensity
Potential.plotDensity(t=0.0, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, nzs=21,
                               phi=None, xy=False, ncontours=21, savefilename=None, aspect=None,
                               log=False, justcontours=False)
     NAME:
           plotDensity
     PURPOSE:
           plot the density of this potential
     INPUT:
           t= time to plot potential at
           rmin= minimum R (can be Quantity) [xmin if xy]
           rmax= maximum R (can be Quantity) [ymax if xy]
           nrs= grid in R
           zmin= minimum z (can be Quantity) [ymin if xy]
           zmax= maximum z (can be Quantity) [ymax if xy]
           nzs= grid in z
           phi= (None) azimuth to use for non-axisymmetric potentials
           xy= (False) if True, plot the density in X-Y
           ncontours= number of contours
           justcontours= (False) if True, just plot contours
           savefilename= save to or restore from this savefile (pickle)
           log= if True, plot the log density
     OUTPUT:
           plot to output device
     HISTORY:
           2014-01-05 - Written - Bovy (IAS)
```

galpy.potential.Potential.plotEscapecurve

```
Potential.plotEscapecurve(*args, **kwargs)
          plotEscapecurve
     PURPOSE:
          plot the escape velocity curve for this potential (in the z=0 plane for non-spherical potentials)
     INPUT:
          Rrange - range (can be Quantity)
          grid= number of points to plot
          savefilename= save to or restore from this savefile (pickle)
          +bovy_plot(*args,**kwargs)
     OUTPUT:
          plot to output device
     HISTORY:
          2010-08-08 - Written - Bovy (NYU)
galpy.potential.Potential.plotRotcurve
Potential.plotRotcurve(*args, **kwargs)
     NAME:
          plotRotcurve
     PURPOSE:
          plot the rotation curve for this potential (in the z=0 plane for non-spherical potentials)
     INPUT:
          Rrange - range (can be Quantity)
          grid= number of points to plot
          savefilename=- save to or restore from this savefile (pickle)
          +bovy_plot(*args,**kwargs)
     OUTPUT:
          plot to output device
     HISTORY:
          2010-07-10 - Written - Bovy (NYU)
galpy.potential.Potential.Rphideriv
Potential. Rphideriv (R, Z, phi=0.0, t=0.0)
     NAME:
          Rphideriv
```

```
PURPOSE:
          evaluate the mixed radial, azimuthal derivative
     INPUT:
          R - Galactocentric radius (can be Quantity)
          Z - vertical height (can be Quantity)
           phi - Galactocentric azimuth (can be Quantity)
          t - time (can be Quantity)
     OUTPUT:
          d2Phi/dphidR
     HISTORY:
           2014-06-30 - Written - Bovy (IAS)
galpy.potential.Potential.R2deriv
Potential.R2deriv (R, Z, phi=0.0, t=0.0)
     NAME:
          R2deriv
     PURPOSE:
          evaluate the second radial derivative
     INPUT:
          R - Galactocentric radius (can be Quantity)
          Z - vertical height (can be Quantity)
          phi - Galactocentric azimuth (can be Quantity)
          t - time (can be Quantity)
     OUTPUT:
          d2phi/dR2
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.r2deriv
Potential.r2deriv(R, z, phi=0.0, t=0.0)
     NAME:
          r2deriv
     PURPOSE:
          evaluate the second spherical radial derivative
     INPUT:
```

```
R - Cylindrical Galactocentric radius (can be Quantity)
           z - vertical height (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           d2phi/dr2
     HISTORY:
           2018-03-21 - Written - Webb (UofT)
galpy.potential.Potential.Rzderiv
Potential.Rzderiv (R, Z, phi=0.0, t=0.0)
     NAME:
           Rzderiv
     PURPOSE:
           evaluate the mixed R,z derivative
     INPUT:
           R - Galactocentric radius (can be Quantity)
           Z - vertical height (can be Quantity)
           phi - Galactocentric azimuth (can be Quantity)
           t - time (can be Quantity)
     OUTPUT:
           d2phi/dz/dR
     HISTORY:
           2013-08-26 - Written - Bovy (IAS)
galpy.potential.Potential.Rforce
Potential. Rforce (R, z, phi=0.0, t=0.0)
     NAME:
           Rforce
     PURPOSE:
           evaluate cylindrical radial force F_R (R,z)
     INPUT:
           R - Cylindrical Galactocentric radius (can be Quantity)
           z - vertical height (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
```

```
OUTPUT:
          F_R(R,z,phi,t)
     HISTORY:
           2010-04-16 - Written - Bovy (NYU)
galpy.potential.Potential.rforce
Potential.rforce(*args, **kwargs)
     NAME:
          rforce
     PURPOSE:
          evaluate spherical radial force F_r (R,z)
     INPUT:
          R - Cylindrical Galactocentric radius (can be Quantity)
          z - vertical height (can be Quantity)
          phi - azimuth (optional; can be Quantity)
          t - time (optional; can be Quantity)
          v - current velocity in cylindrical coordinates (optional, but required when including dissipative
          forces; can be a Quantity)
     OUTPUT:
          F_r(R,z,phi,t)
     HISTORY:
           2016-06-20 - Written - Bovy (UofT)
galpy.potential.Potential.rl
Potential.rl(lz, t=0.0)
     NAME:
          rl
     PURPOSE:
          calculate the radius of a circular orbit of Lz
     INPUT:
          lz - Angular momentum (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          radius
     HISTORY:
           2012-07-30 - Written - Bovy (IAS@MPIA)
```

NOTE:

seems to take about ~0.5 ms for a Miyamoto-Nagai potential; ~0.75 ms for a MWPotential

galpy.potential.Potential.rtide

Potential.rtide (R, z, phi=0.0, t=0.0, M=None)

NAME:

rtide

PURPOSE:

Calculate the tidal radius for object of mass M assuming a circular orbit as

$$r_t^3 = \frac{GM_s}{\Omega^2 - \mathrm{d}^2\Phi/\mathrm{d}r^2}$$

where M_s is the cluster mass, Ω is the circular frequency, and Φ is the gravitational potential. For non-spherical potentials, we evaluate $\Omega^2=(1/r)(\mathrm{d}\Phi/\mathrm{d}r)$ and evaluate the derivatives at the given position of the cluster.

INPUT:

R - Galactocentric radius (can be Quantity)

z - height (can be Quantity)

phi - azimuth (optional; can be Quantity)

t - time (optional; can be Quantity)

M - (default = None) Mass of object (can be Quantity)

OUTPUT:

Tidal Radius

HISTORY:

2018-03-21 - Written - Webb (UofT)

galpy.potential.Potential.surfdens

Potential.surfdens (R, z, phi=0.0, t=0.0, forcepoisson=False)

NAME:

surfdens

PURPOSE:

evaluate the surface density $\Sigma(R, z, \phi, t) = \int_{-z}^{+z} dz' \rho(R, z', \phi, t)$

INPUT:

R - Cylindrical Galactocentric radius (can be Quantity)

z - vertical height (can be Quantity)

phi - azimuth (optional; can be Quantity)

t - time (optional; can be Quantity)

KEYWORDS:

```
forcepoisson= if True, calculate the surface density through the Poisson equation, even if an explicit
           expression for the surface density exists
     OUTPUT:
           Sigma (R,z,phi,t)
     HISTORY:
           2018-08-19 - Written - Bovy (UofT)
galpy.planar.Potential.toPlanar
Potential.toPlanar()
     NAME:
           toPlanar
     PURPOSE:
           convert a 3D potential into a planar potential in the mid-plane
     INPUT:
           (none)
     OUTPUT:
           planarPotential
     HISTORY:
           unknown
galpy.potential.Potential.toVertical
Potential.toVertical(R, phi=None, t0=0.0)
     NAME:
           toVertical
     PURPOSE:
           convert a 3D potential into a linear (vertical) potential at R
     INPUT:
           R - Galactocentric radius at which to create the vertical potential (can be Quantity)
           phi= (None) Galactocentric azimuth at which to create the vertical potential (can be Quantity); re-
           quired for non-axisymmetric potential
           t0=(0.) time at which to create the vertical potential (can be Quantity)
     OUTPUT:
           linear (vertical) potential: Phi(z,phi,t) = Phi(R,z,phi,t)-Phi(R,0,phi0,t0) where phi0 and t0 are the phi
           and t inputs
     HISTORY
           unknown
```

galpy.potential.Potential.ttensor

```
Potential.ttensor(R, z, phi=0.0, t=0.0, eigenval=False)
     NAME:
          ttensor
     PURPOSE:
          Calculate the tidal tensor Tij=-d(Psi)(dxidxj)
     INPUT:
          R - Galactocentric radius (can be Quantity)
          z - height (can be Quantity)
          phi - azimuth (optional; can be Quantity)
          t - time (optional; can be Quantity)
          eigenval - return eigenvalues if true (optional; boolean)
     OUTPUT:
          Tidal Tensor
     HISTORY:
          2018-03-21 - Written - Webb (UofT)
galpy.potential.Potential.turn_physical_off
Potential.turn_physical_off()
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
galpy.potential.Potential.turn physical on
Potential.turn_physical_on(ro=None, vo=None)
     NAME:
          turn_physical_on
     PURPOSE:
          turn on automatic returning of outputs in physical units
```

```
INPUT:
           ro= reference distance (kpc; can be Quantity)
           vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
           (none)
     HISTORY:
           2016-01-30 - Written - Bovy (UofT)
           2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)
galpy.potential.Potential.vcirc
Potential.vcirc(R, phi=None, t=0.0)
           NAME:
               vcirc
           PURPOSE:
               calculate the circular velocity at R in this potential
           INPUT:
               R - Galactocentric radius (can be Quantity)
               phi= (None) azimuth to use for non-axisymmetric potentials
               t - time (optional; can be Quantity)
           OUTPUT:
               circular rotation velocity
           HISTORY:
               2011-10-09 - Written - Bovy (IAS)
     2016-06-15 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
galpy.potential.Potential.verticalfreq
Potential.verticalfreq(R, t=0.0)
     NAME:
           verticalfreq
     PURPOSE:
           calculate the vertical frequency at R in this potential
     INPUT:
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           vertical frequency
```

```
HISTORY:
          2012-07-25 - Written - Bovy (IAS@MPIA)
galpy.potential.Potential.vesc
Potential.vesc(R, t=0.0)
     NAME:
          vesc
     PURPOSE:
          calculate the escape velocity at R for this potential
     INPUT:
          R - Galactocentric radius (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          escape velocity
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.Potential.vterm
Potential.vterm(l, t=0.0, deg=True)
     NAME:
          vterm
     PURPOSE:
          calculate the terminal velocity at 1 in this potential
     INPUT:
          1 - Galactic longitude [deg/rad; can be Quantity)
          t - time (optional; can be Quantity)
          deg= if True (default), 1 in deg
     OUTPUT:
          terminal velocity
     HISTORY:
           2013-05-31 - Written - Bovy (IAS)
galpy.potential.Potential.z2deriv
Potential.z2deriv(R, Z, phi=0.0, t=0.0)
     NAME:
          z2deriv
```

```
PURPOSE:
          evaluate the second vertical derivative
     INPUT:
          R - Galactocentric radius (can be Quantity)
          Z - vertical height (can be Quantity)
           phi - Galactocentric azimuth (can be Quantity)
          t - time (can be Quantity)
     OUTPUT:
          d2phi/dz2
     HISTORY:
           2012-07-25 - Written - Bovy (IAS@MPIA)
galpy.potential.Potential.zforce
Potential.zforce (R, z, phi=0.0, t=0.0)
     NAME:
          zforce
     PURPOSE:
          evaluate the vertical force F_z (R,z,t)
     INPUT:
          R - Cylindrical Galactocentric radius (can be Quantity)
          z - vertical height (can be Quantity)
          phi - azimuth (optional; can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          F_z(R,z,phi,t)
     HISTORY:
          2010-04-16 - Written - Bovy (NYU)
In addition to these, the NFWPotential also has methods to calculate virial quantities
galpy.potential.Potential.conc
Potential.conc (H=70.0, Om=0.3, t=0.0, overdens=200.0, wrtcrit=False, ro=None, vo=None)
     NAME:
          conc
     PURPOSE:
          return the concentration
     INPUT:
```

```
H= (default: 70) Hubble constant in km/s/Mpc
           Om= (default: 0.3) Omega matter
           t - time (optional; can be Quantity)
           overdens= (200) overdensity which defines the virial radius
           wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density
           ro= distance scale in kpc or as Quantity (default: object-wide, which if not set is 8 kpc))
           vo= velocity scale in km/s or as Quantity (default: object-wide, which if not set is 220 km/s))
      OUTPUT:
           concentration (scale/rvir)
      HISTORY:
           2014-04-03 - Written - Bovy (IAS)
galpy.potential.Potential.mvir
Potential.mvir (H=70.0, Om=0.3, t=0.0, overdens=200.0, wrtcrit=False, forceint=False, ro=None,
                      vo=None, use_physical=False)
      NAME:
           mvir
      PURPOSE:
           calculate the virial mass
      INPUT:
           H= (default: 70) Hubble constant in km/s/Mpc
           Om= (default: 0.3) Omega matter
           overdens= (200) overdensity which defines the virial radius
           wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density
           ro= distance scale in kpc or as Quantity (default: object-wide, which if not set is 8 kpc))
           vo= velocity scale in km/s or as Quantity (default: object-wide, which if not set is 220 km/s))
      KEYWORDS:
           forceint= if True, calculate the mass through integration of the density, even if an explicit expression
           for the mass exists
      OUTPUT:
           M(<rvir)
      HISTORY:
           2014-09-12 - Written - Bovy (IAS)
```

galpy.potential.NFWPotential.rvir

```
NFWPotential.rvir (H=70.0, Om=0.3, t=0.0, overdens=200.0, wrtcrit=False, ro=None, vo=None, 
                                                                    use_physical=False)
               NAME:
                             rvir
               PURPOSE:
                              calculate the virial radius for this density distribution
               INPUT:
                              H= (default: 70) Hubble constant in km/s/Mpc
                              Om= (default: 0.3) Omega matter
                              overdens= (200) overdensity which defines the virial radius
                              wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density
                              ro= distance scale in kpc or as Quantity (default: object-wide, which if not set is 8 kpc))
                              vo= velocity scale in km/s or as Quantity (default: object-wide, which if not set is 220 km/s))
               OUTPUT:
                              virial radius
               HISTORY:
                              2014-01-29 - Written - Bovy (IAS)
General 3D potential routines
Use as method (...)
galpy.potential.dvcircdR
galpy.potential.dvcircdR(Pot, R, phi=None, t=0.0)
               NAME:
                              dvcircdR
               PURPOSE:
                              calculate the derivative of the circular velocity wrt R at R in potential Pot
               INPUT:
                              Pot - Potential instance or list of such instances
                              R - Galactocentric radius (can be Quantity)
                              phi= (None) azimuth to use for non-axisymmetric potentials
                             t= time (optional; can be Quantity)
               OUTPUT:
                              derivative of the circular rotation velocity wrt R
               HISTORY:
```

2013-01-08 - Written - Bovy (IAS)

```
2016-06-28 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
galpy.potential.epifreq
galpy.potential.epifreq(Pot, R, t=0.0)
     NAME:
           epifreq
     PURPOSE:
           calculate the epicycle frequency at R in the potential Pot
     INPUT:
           Pot - Potential instance or list thereof
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           epicycle frequency
     HISTORY:
           2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluateDensities
galpy.potential.evaluateDensities (Pot, R, z, phi=None, t=0.0, forcepoisson=False)
     NAME:
           evaluateDensities
     PURPOSE:
           convenience function to evaluate a possible sum of densities
     INPUT:
           Pot - potential or list of potentials (dissipative forces in such a list are ignored)
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (can be Quantity)
           t - time (can be Quantity)
           forcepoisson= if True, calculate the density through the Poisson equation, even if an explicit expres-
           sion for the density exists
     OUTPUT:
           rho(R,z)
     HISTORY:
```

```
2010-08-08 - Written - Bovy (NYU)
2013-12-28 - Added forcepoisson - Bovy (IAS)
```

galpy.potential.evaluatephiforces

```
galpy.potential.evaluatephiforces (Pot, R, z, phi=None, t=0.0, v=None)
      NAME:
           evaluatephiforces
      PURPOSE:
           convenience function to evaluate a possible sum of potentials
      INPUT: Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
           v - current velocity in cylindrical coordinates (optional, but required when including dissipative forces;
           can be a Quantity)
      OUTPUT:
           F_{phi}(R,z,phi,t)
      HISTORY:
           2010-04-16 - Written - Bovy (NYU)
           2018-03-16 - Added velocity input for dissipative forces - Bovy (UofT)
```

galpy.potential.evaluatePotentials

Warning: galpy potentials do *not* necessarily approach zero at infinity. To compute, for example, the escape velocity or whether or not an orbit is unbound, you need to take into account the value of the potential at infinity. E.g., $v_{\rm esc}(r) = \sqrt{2[\Phi(\infty) - \Phi(r)]}$.

```
phi - azimuth (can be Quantity)
           t - time (can be Quantity)
           dR= dphi=, if set to non-zero integers, return the dR, dphi't derivative instead
     OUTPUT:
           Phi(R,z)
     HISTORY:
           2010-04-16 - Written - Bovy (NYU)
galpy.potential.evaluatephi2derivs
galpy.potential.evaluatephi2derivs (Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluatephi2derivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           d2Phi/d2phi(R,z,phi,t)
     HISTORY:
           2018-03-28 - Written - Bovy (UofT)
galpy.potential.evaluateRphiderivs
galpy.potential.evaluateRphiderivs(Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluateRphiderivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
```

```
t - time (optional; can be Quantity)
     OUTPUT:
           d2Phi/d2R(R,z,phi,t)
     HISTORY:
           2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluateR2derivs
galpy.potential.evaluateR2derivs (Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluateR2derivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials (dissipative forces in such a list are ignored)
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           d2Phi/d2R(R,z,phi,t)
     HISTORY:
           2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluater2derivs
galpy.potential.evaluater2derivs(Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluater2derivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
```

```
d2phi/dr2(R,z,phi,t)
     HISTORY:
           2018-03-28 - Written - Bovy (UofT)
galpy.potential.evaluateRzderivs
galpy.potential.evaluateRzderivs(Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluateRzderivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials (dissipative forces in such a list are ignored)
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           d2Phi/dz/dR(R,z,phi,t)
     HISTORY:
           2013-08-28 - Written - Bovy (IAS)
galpy.potential.evaluateRforces
galpy.potential.evaluateRforces(Pot, R, z, phi=None, t=0.0, v=None)
     NAME:
           evaluateRforce
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity))
           t - time (optional; can be Quantity)
           v - current velocity in cylindrical coordinates (optional, but required when including dissipative
           forces; can be a Quantity)
     OUTPUT:
```

```
F_R(R,z,phi,t)
     HISTORY:
           2010-04-16 - Written - Bovy (NYU)
           2018-03-16 - Added velocity input for dissipative forces - Bovy (UofT)
galpy.potential.evaluaterforces
galpy.potential.evaluaterforces (Pot, R, z, phi=None, t=0.0, v=None)
     NAME:
           evaluaterforces
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
           v - current velocity in cylindrical coordinates (optional, but required when including dissipative
           forces; can be a Quantity)
     OUTPUT:
           F_r(R,z,phi,t)
     HISTORY:
           2016-06-10 - Written - Bovy (UofT)
galpy.potential.evaluateSurfaceDensities
galpy.potential.evaluateSurfaceDensities (Pot, R, z, phi=None, t=0.0, forcepoisson=False)
     NAME:
           evaluateSurfaceDensities
     PURPOSE:
           convenience function to evaluate a possible sum of surface densities
     INPUT:
           Pot - potential or list of potentials (dissipative forces in such a list are ignored)
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (can be Quantity)
           t - time (can be Quantity)
```

```
forcepoisson= if True, calculate the surface density through the Poisson equation, even if an explicit
           expression for the surface density exists
     OUTPUT:
           Sigma(R,z)
     HISTORY:
           2018-08-20 - Written - Bovy (UofT)
galpy.potential.evaluatez2derivs
galpy.potential.evaluatez2derivs(Pot, R, z, phi=None, t=0.0)
     NAME:
           evaluatez2derivs
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials (dissipative forces in such a list are ignored)
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           d2Phi/d2z(R,z,phi,t)
     HISTORY:
           2012-07-25 - Written - Bovy (IAS)
galpy.potential.evaluatezforces
galpy.potential.evaluatezforces(Pot, R, z, phi=None, t=0.0, v=None)
     NAME:
           evaluatezforces
     PURPOSE:
           convenience function to evaluate a possible sum of potentials
     INPUT:
           Pot - a potential or list of potentials
           R - cylindrical Galactocentric distance (can be Quantity)
           z - distance above the plane (can be Quantity)
           phi - azimuth (optional; can be Quantity)
           t - time (optional; can be Quantity)
```

```
v - current velocity in cylindrical coordinates (optional, but required when including dissipative
           forces; can be a Quantity)
     OUTPUT:
           F_z(R,z,phi,t)
     HISTORY:
           2010-04-16 - Written - Bovy (NYU)
           2018-03-16 - Added velocity input for dissipative forces - Bovy (UofT)
galpy.potential.flatten
galpy.potential.flatten(Pot)
     NAME:
           flatten
     PURPOSE:
           flatten a possibly nested list of Potential instances into a flat list
     INPUT:
           Pot - list (possibly nested) of Potential instances
     OUTPUT:
           Flattened list of Potential instances
     HISTORY:
           2018-03-14 - Written - Bovy (UofT)
galpy.potential.flattening
galpy.potential.flattening(Pot, R, z, t=0.0)
     NAME:
           flattening
     PURPOSE:
           calculate the potential flattening, defined as sqrt(fabs(z/R F_R/F_z))
     INPUT:
           Pot - Potential instance or list thereof
           R - Galactocentric radius (can be Quantity)
           z - height (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           flattening
     HISTORY:
           2012-09-13 - Written - Bovy (IAS)
```

galpy.potential.lindbladR

```
galpy.potential.lindbladR(Pot, OmegaP, m=2, t=0.0, **kwargs)
          lindbladR
     PURPOSE:
          calculate the radius of a Lindblad resonance
     INPUT:
          Pot - Potential instance or list of such instances
          OmegaP - pattern speed (can be Quantity)
          m= order of the resonance (as in m(O-Op)=kappa (negative m for outer) use
                                                                                       m='corotation'
              for corotation
          +scipy.optimize.brentq xtol,rtol,maxiter kwargs
          t - time (optional; can be Quantity)
     OUTPUT:
          radius of Linblad resonance, None if there is no resonance
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.nemo_accname
galpy.potential.nemo_accname(Pot)
     NAME:
          nemo_accname
     PURPOSE:
          return the accname potential name for use of this potential or list of potentials with NEMO
     INPUT:
          Pot - Potential instance or list of such instances
     OUTPUT:
          Acceleration name in the correct format to give to accname=
     HISTORY:
          2014-12-18 - Written - Bovy (IAS)
galpy.potential.nemo accpars
galpy.potential.nemo_accpars(Pot, vo, ro)
     NAME:
          nemo_accpars
     PURPOSE:
```

```
return the accpars potential parameters for use of this potential or list of potentials with NEMO
     INPUT:
           Pot - Potential instance or list of such instances
           vo - velocity unit in km/s
           ro - length unit in kpc
     OUTPUT:
           accpars string in the corrct format to give to accpars
     HISTORY:
           2014-12-18 - Written - Bovy (IAS)
galpy.potential.omegac
galpy.potential.omegac (Pot, R, t=0.0)
     NAME:
           omegac
     PURPOSE:
           calculate the circular angular speed velocity at R in potential Pot
     INPUT:
           Pot - Potential instance or list of such instances
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           circular angular speed
     HISTORY:
           2011-10-09 - Written - Bovy (IAS)
galpy.potential.plotDensities
galpy.potential.plotDensities(Pot, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5, nzs=21,
                                          phi=None, xy=False, t=0.0, ncontours=21, savefilename=None,
                                          aspect=None, log=False, justcontours=False)
     NAME:
           plotDensities
     PURPOSE:
           plot the density a set of potentials
     INPUT:
           Pot - Potential or list of Potential instances
           rmin= minimum R (can be Quantity) [xmin if xy]
           rmax= maximum R (can be Quantity) [ymax if xy]
```

```
nrs= grid in R
           zmin= minimum z (can be Quantity) [ymin if xy]
           zmax= maximum z (can be Quantity) [ymax if xy]
           nzs= grid in z
           phi= (None) azimuth to use for non-axisymmetric potentials
           t=(0.) time to use to evaluate potential
           xy= (False) if True, plot the density in X-Y
           ncontours= number of contours
           justcontours= (False) if True, just plot contours
           savefilename= save to or restore from this savefile (pickle)
           log= if True, plot the log density
     OUTPUT:
           plot to output device
     HISTORY:
           2013-07-05 - Written - Bovy (IAS)
galpy.potential.plotEscapecurve
galpy.potential.plotEscapecurve(Pot, *args, **kwargs)
     NAME:
           plotEscapecurve
     PURPOSE:
           plot the escape velocity curve for this potential (in the z=0 plane for non-spherical potentials)
     INPUT:
           Pot - Potential or list of Potential instances
           Rrange= Range in R to consider (can be Quantity)
           grid= grid in R
           savefilename= save to or restore from this savefile (pickle)
           +bovy_plot.bovy_plot args and kwargs
     OUTPUT:
           plot to output device
     HISTORY:
           2010-08-08 - Written - Bovy (NYU)
```

galpy.potential.plotPotentials

```
galpy.potential.plotPotentials(Pot, rmin=0.0, rmax=1.5, nrs=21, zmin=-0.5, zmax=0.5,
                                           nzs=21, phi=None, xy=False, t=0.0, effective=False, Lz=None,
                                           ncontours=21, savefilename=None, aspect=None, justcon-
                                           tours=False, levels=None, cntrcolors=None)
     NAME:
           plotPotentials
     PURPOSE:
           plot a set of potentials
     INPUT:
           Pot - Potential or list of Potential instances
           rmin= minimum R (can be Quantity) [xmin if xy]
           rmax= maximum R (can be Quantity) [ymax if xy]
           nrs= grid in R
           zmin= minimum z (can be Quantity) [ymin if xy]
           zmax= maximum z (can be Quantity) [ymax if xy]
           nzs= grid in z
           phi= (None) azimuth to use for non-axisymmetric potentials
           t=(0.) time to use to evaluate potential
           xy= (False) if True, plot the potential in X-Y
           effective= (False) if True, plot the effective potential Phi + Lz^2/2/R^2
           Lz= (None) angular momentum to use for the effective potential when effective=True
          justcontours= (False) if True, just plot contours
           levels= (None) contours to plot
           ncontours - number of contours when levels is None
           cntrcolors= (None) colors of the contours (single color or array with length ncontours)
           savefilename= save to or restore from this savefile (pickle)
     OUTPUT:
           plot to output device
     HISTORY:
           2010-07-09 - Written - Bovy (NYU)
galpy.potential.plotRotcurve
galpy.potential.plotRotcurve(Pot, *args, **kwargs)
     NAME:
          plotRotcurve
     PURPOSE:
```

```
plot the rotation curve for this potential (in the z=0 plane for non-spherical potentials)
     INPUT:
           Pot - Potential or list of Potential instances
           Rrange - Range in R to consider (needs to be in the units that you are plotting; can be Quantity)
           grid= grid in R
           phi= (None) azimuth to use for non-axisymmetric potentials
           savefilename= save to or restore from this savefile (pickle)
           +bovy_plot.bovy_plot args and kwargs
     OUTPUT:
           plot to output device
     HISTORY:
           2010-07-10 - Written - Bovy (NYU)
           2016-06-15 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
galpy.potential.rl
galpy.potential.rl (Pot, lz, t=0.0)
     NAME:
           rl
     PURPOSE:
           calculate the radius of a circular orbit of Lz
     INPUT:
           Pot - Potential instance or list thereof
           lz - Angular momentum (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           radius
     HISTORY:
           2012-07-30 - Written - Bovy (IAS@MPIA)
     NOTE:
           seems to take about ~0.5 ms for a Miyamoto-Nagai potential; ~0.75 ms for a MWPotential
galpy.potential.rtide
galpy.potential.rtide(Pot, R, z, phi=0.0, t=0.0, M=None)
     NAME:
           rtide
     PURPOSE:
```

Calculate the tidal radius for object of mass M assuming a circular orbit as

$$r_t^3 = \frac{GM_s}{\Omega^2 - \mathrm{d}^2\Phi/\mathrm{d}r^2}$$

where M_s is the cluster mass, Ω is the circular frequency, and Φ is the gravitational potential. For non-spherical potentials, we evaluate $\Omega^2 = (1/r)(\mathrm{d}\Phi/\mathrm{d}r)$ and evaluate the derivatives at the given position of the cluster.

INPUT:

Pot - Potential instance or list of such instances

R - Galactocentric radius (can be Quantity)

z - height (can be Quantity)

phi - azimuth (optional; can be Quantity)

t - time (optional; can be Quantity)

M - (default = None) Mass of object (can be Quantity)

OUTPUT:

Tidal Radius

HISTORY:

2018-03-21 - Written - Webb (UofT)

galpy.potential.to amuse

galpy.potential.to_amuse (Pot, t=0.0, tgalpy=0.0, reverse=False, ro=None, vo=None) NAME:

to amuse

PURPOSE:

Return an AMUSE representation of a galpy Potential or list of Potentials

INPUT:

Pot - Potential instance or list of such instances

t= (0.) Initial time in AMUSE (can be in internal galpy units or AMUSE units)

tgalpy= (0.) Initial time in galpy (can be in internal galpy units or AMUSE units); because AMUSE initial times have to be positive, this is useful to set if the initial time in galpy is negative

reverse= (False) set whether the galpy potential evolves forwards or backwards in time (default: False); because AMUSE can only integrate forward in time, this is useful to integrate backward in time in AMUSE

ro= (default taken from Pot) length unit in kpc

vo= (default taken from Pot) velocity unit in km/s

OUTPUT:

AMUSE representation of Pot

HISTORY:

2019-08-04 - Written - Bovy (UofT)

```
2019-08-12 - Implemented actual function - Webb (UofT)
galpy.potential.ttensor
galpy.potential.ttensor(Pot, R, z, phi=0.0, t=0.0, eigenval=False)
     NAME:
          ttensor
     PURPOSE:
          Calculate the tidal tensor Tij=-d(Psi)(dxidxj)
     INPUT:
          Pot - Potential instance or list of such instances
          R - Galactocentric radius (can be Quantity)
          z - height (can be Quantity)
          phi - azimuth (optional; can be Quantity)
          t - time (optional; can be Quantity)
           eigenval - return eigenvalues if true (optional; boolean)
     OUTPUT:
          Tidal Tensor
     HISTORY:
           2018-03-21 - Written - Webb (UofT)
galpy.potential.turn_physical_off
galpy.potential.turn_physical_off(Pot)
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
```

galpy.potential.turn_physical_on

```
galpy.potential.turn_physical_on(Pot, ro=None, vo=None)
          turn_physical_on
     PURPOSE:
          turn on automatic returning of outputs in physical units
     INPUT:
          ro= reference distance (kpc; can be Quantity)
          vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
galpy.potential.vcirc
galpy.potential.vcirc(Pot, R, phi=None, t=0.0)
     NAME:
          vcirc
     PURPOSE:
          calculate the circular velocity at R in potential Pot
     INPUT:
          Pot - Potential instance or list of such instances
          R - Galactocentric radius (can be Quantity)
          phi= (None) azimuth to use for non-axisymmetric potentials
          t= time (optional; can be Quantity)
     OUTPUT:
          circular rotation velocity
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
          2016-06-15 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
galpy.potential.verticalfreq
galpy.potential.verticalfreq(Pot, R, t=0.0)
     NAME:
          verticalfreq
     PURPOSE:
```

```
calculate the vertical frequency at R in the potential Pot
     INPUT:
           Pot - Potential instance or list thereof
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           vertical frequency
     HISTORY:
           2012-07-25 - Written - Bovy (IAS@MPIA)
galpy.potential.vesc
galpy.potential.vesc(Pot, R, t=0.0)
     NAME:
           vesc
     PURPOSE:
           calculate the escape velocity at R for potential Pot
     INPUT:
           Pot - Potential instances or list thereof
           R - Galactocentric radius (can be Quantity)
           t - time (optional; can be Quantity)
     OUTPUT:
           escape velocity
     HISTORY:
           2011-10-09 - Written - Bovy (IAS)
galpy.potential.vterm
galpy.potential.vterm(Pot, l, t=0.0, deg=True)
     NAME:
           vterm
     PURPOSE:
           calculate the terminal velocity at l in this potential
     INPUT:
           Pot - Potential instance
           1 - Galactic longitude [deg/rad; can be Quantity)
           t - time (optional; can be Quantity)
           deg= if True (default), 1 in deg
```

OUTPUT:

terminal velocity

HISTORY:

In addition to these, the following methods are available to compute expansion coefficients for the SCFPotential class for a given density

galpy.potential.scf_compute_coeffs

Note: This function computes Acos and Asin as defined in Hernquist & Ostriker (1992), except that we multiply Acos and Asin by 2 such that the density from *Galpy's Hernquist Potential* corresponds to $Acos = \delta_{0n}\delta_{0l}\delta_{0m}$ and Asin = 0.

For a given $\rho(R, z, \phi)$ we can compute $A\cos$ and $A\sin$ through the following equation

$$\begin{bmatrix} A\cos \\ A\sin \end{bmatrix}_{nlm} = \frac{4a^3}{I_{nl}} \int_{\xi=0}^{\infty} \int_{\cos(\theta)=-1}^{1} \int_{\phi=0}^{2\pi} (1+\xi)^2 (1-\xi)^{-4} \rho(R,z,\phi) \Phi_{nlm}(\xi,\cos(\theta),\phi) d\phi d\cos(\theta) d\xi$$

Where

$$\Phi_{nlm}(\xi,\cos(\theta),\phi) = -\frac{\sqrt{2l+1}}{a2^{2l+1}} \sqrt{\frac{(l-m)!}{(l+m)!}} (1+\xi)^l (1-\xi)^{l+1} C_n^{2l+3/2}(\xi) P_{lm}(\cos(\theta)) \begin{bmatrix} \cos(m\phi) \\ \sin(m\phi) \end{bmatrix}$$

$$I_{nl} = -K_{nl} \frac{4\pi}{a2^{8l+6}} \frac{\Gamma(n+4l+3)}{n!(n+2l+3/2)[\Gamma(2l+3/2)]^2} \qquad K_{nl} = \frac{1}{2}n(n+4l+3) + (l+1)(2l+1)$$

 P_{lm} is the Associated Legendre Polynomials whereas C_n^{α} is the Gegenbauer polynomial.

Also note $\xi = \frac{r-a}{r+a}$, and n, l and m are integers bounded by 0 <= n < N , 0 <= l < L, and 0 <= m <= l

galpy.potential.scf_compute_coeffs(dens, N, L, a=1.0, radial_order=None, costheta order=None, phi order=None)

NAME:

scf_compute_coeffs

PURPOSE:

Numerically compute the expansion coefficients for a given triaxial density

INPUT:

dens - A density function that takes a parameter R, z and phi

N - size of the Nth dimension of the expansion coefficients

L - size of the Lth and Mth dimension of the expansion coefficients

a - parameter used to shift the basis functions

radial_order - Number of sample points of the radial integral. If None, radial_order=max(20, N + 3/2L + 1)

costheta_order - Number of sample points of the costheta integral. If None, If costheta_order=max(20, L + 1)

phi_order - Number of sample points of the phi integral. If None, If costheta_order=max(20, L + 1)

OUTPUT:

(Acos,Asin) - Expansion coefficients for density dens that can be given to SCFPotential.__init__ HISTORY:

2016-05-27 - Written - Aladdin

galpy.potential.scf_compute_coeffs_axi

Note: This function computes Acos and Asin as defined in Hernquist & Ostriker (1992), except that we multiply Acos by 2 such that the density from *Galpy's Hernquist Potential* corresponds to $Acos = \delta_{0n}\delta_{0l}\delta_{0m}$.

Further note that this function is a specification of $scf_compute_coeffs$ where $Acos_{nlm} = 0$ at $m \neq 0$ and $Asin_{nlm} = None$

For a given $\rho(R, z)$ we can compute $A\cos$ and $A\sin$ through the following equation

$$Acos_{nlm} = \frac{8\pi a^3}{I_{nl}} \int_{\xi=0}^{\infty} \int_{\cos(\theta)=-1}^{1} (1+\xi)^2 (1-\xi)^{-4} \rho(R,z) \Phi_{nlm}(\xi,\cos(\theta)) d\cos(\theta) d\xi \qquad Asin_{nlm} = None$$

Where

$$\Phi_{nlm}(\xi,\cos(\theta)) = -\frac{\sqrt{2l+1}}{a2^{2l+1}}(1+\xi)^l(1-\xi)^{l+1}C_n^{2l+3/2}(\xi)P_{l0}(\cos(\theta))\delta_{m0}$$

$$I_{nl} = -K_{nl}\frac{4\pi}{a2^{8l+6}}\frac{\Gamma(n+4l+3)}{n!(n+2l+3/2)[\Gamma(2l+3/2)]^2} \qquad K_{nl} = \frac{1}{2}n(n+4l+3) + (l+1)(2l+1)$$

 P_{lm} is the Associated Legendre Polynomials whereas C_n^{α} is the Gegenbauer polynomial.

Also note $\xi = \frac{r-a}{r+a}$, and n, l and m are integers bounded by 0 <= n < N , 0 <= l < L, and m = 0

galpy.potential.scf_compute_coeffs_axi(dens, N, L, a=1.0, radial_order=None, costheta order=None)

NAME:

scf_compute_coeffs_axi

PURPOSE:

Numerically compute the expansion coefficients for a given axi-symmetric density

INPUT:

dens - A density function that takes a parameter R and z

N - size of the Nth dimension of the expansion coefficients

L - size of the Lth dimension of the expansion coefficients

a - parameter used to shift the basis functions

radial_order - Number of sample points of the radial integral. If None, radial_order=max(20, N + 3/2L + 1)

costheta_order - Number of sample points of the costheta integral. If None, If costheta_order= $\max(20, L+1)$

OUTPUT:

(Acos,Asin) - Expansion coefficients for density dens that can be given to SCFPotential.__init__

HISTORY:

2016-05-20 - Written - Aladdin

galpy.potential.scf_compute_coeffs_spherical

Note: This function computes Acos and Asin as defined in Hernquist & Ostriker (1992), except that we multiply Acos by 2 such that the density from *Galpy's Hernquist Potential* corresponds to $Acos = \delta_{0n}\delta_{0l}\delta_{0m}$.

Futher note that this function is a specification of scf_compute_coeffs_axi where $Acos_{nlm} = 0$ at $l \neq 0$

For a given $\rho(r)$ we can compute $A\cos$ and $A\sin$ through the following equation

$$Acos_{nlm} = \frac{16\pi a^3}{I_{nl}} \int_{\xi=0}^{\infty} (1+\xi)^2 (1-\xi)^{-4} \rho(r) \Phi_{nlm}(\xi) d\xi \qquad Asin_{nlm} = None$$

Where

$$\Phi_{nlm}(\xi, \cos(\theta)) = -\frac{1}{2a} (1 - \xi) C_n^{3/2}(\xi) \delta_{l0} \delta_{m0}$$

$$I_{n0} = -K_{n0} \frac{1}{4a} \frac{(n+2)(n+1)}{(n+3/2)}$$
 $K_{nl} = \frac{1}{2}n(n+3) + 1$

 C_n^{α} is the Gegenbauer polynomial.

Also note $\xi = \frac{r-a}{r+a}$, and n, l and m are integers bounded by 0 <= n < N , l = m = 0

scf_compute_coeffs_spherical

PURPOSE:

Numerically compute the expansion coefficients for a given spherical density

INPUT:

dens - A density function that takes a parameter R

N - size of expansion coefficients

a - parameter used to shift the basis functions

radial_order - Number of sample points of the radial integral. If None, radial_order=max(20, N + 1)

OUTPUT:

(Acos, Asin) - Expansion coefficients for density dens that can be given to SCFPotential.__init__

HISTORY:

2016-05-18 - Written - Aladdin

Specific potentials

All of the following potentials can also be modified by the specific WrapperPotentials listed below.

Spherical potentials

Burkert potential

class galpy.potential.BurkertPotential(amp=1.0, a=2.0, normalize=False, ro=None, vo=None)

BurkertPotential.py: Potential with a Burkert density

$$\rho(r) = \frac{\text{amp}}{(1 + r/a)(1 + [r/a]^2)}$$

__init__ (amp=1.0, a=2.0, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a Burkert-density potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

a = scale radius (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2013-04-10 - Written - Bovy (IAS)

Double power-law density spherical potential

class galpy.potential.TwoPowerSphericalPotential (amp=1.0, a=5.0, alpha=1.5, beta=3.5, normalize=False, ro=None, vo=None)

Class that implements spherical potentials that are derived from two-power density models

$$\rho(r) = \frac{\mathrm{amp}}{4 \, \pi \, a^3} \, \frac{1}{(r/a)^\alpha \, (1 + r/a)^{\beta - \alpha}} \label{eq:rho}$$

__init__ (amp=1.0, a=5.0, alpha=1.5, beta=3.5, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a two-power-density potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

alpha - inner power

beta - outer power

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Started - Bovy (NYU)

Spherical Cored Dehnen potential

class galpy.potential.DehnenCoreSphericalPotential (amp=1.0, a=1.0, normal-ize=False, ro=None, vo=None)

Class that implements the Dehnen Spherical Potential from Dehnen (1993) with alpha=0 (corresponding to an inner core)

$$\rho(r) = \frac{\text{amp}}{12 \pi a^3} \frac{1}{(1 + r/a)^4}$$

__init__ (amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a cored Dehnen Spherical Potential; note that the amplitude definition used here does NOT match that of Dehnen (1993)

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

alpha - inner power, restricted to [0, 3)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2019-10-07 - Started - Starkman (UofT)

Spherical Dehnen potential

class galpy.potential.DehnenSphericalPotential(amp=1.0, a=1.0, alpha=1.5, normalize=False, ro=None, vo=None)

Class that implements the Dehnen Spherical Potential from Dehnen (1993)

$$\rho(r) = \frac{\text{amp}(3-\alpha)}{4\pi a^3} \frac{1}{(r/a)^{\alpha} (1+r/a)^{4-\alpha}}$$

__init__ (amp=1.0, a=1.0, alpha=1.5, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a Dehnen Spherical Potential; note that the amplitude definitio used here does NOT match that of Dehnen (1993)

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

alpha - inner power, restricted to [0, 3)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2019-10-07 - Started - Starkman (UofT)

Hernquist potential

class galpy.potential.HernquistPotential(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

Class that implements the Hernquist potential

$$\rho(r) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(r/a) (1 + r/a)^3}$$

__init__(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

Initialize a Hernquist potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass (note that amp is $2 \times [total]$ for the chosen definition of the Hernquist potential)

a - scale radius (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Written - Bovy (NYU)

Homogeneous sphere potential

class galpy.potential.HomogeneousSpherePotential(amp=1.0, R=1.1, normalize=False, ro=None, vo=None)

Class that implements the homogeneous sphere potential for $\rho(r) = \rho_0 = \text{constant}$ for all r < R and zero otherwise. The potential is given by

$$\Phi(r) = \operatorname{amp} \times \left\{ \begin{array}{ll} (r^2 - 3R^2), & \text{for } r < R \\ -\frac{2R^3}{r} & \text{for } r \ge R \end{array} \right.$$

We have that $\rho_0 = 3 \operatorname{amp}/[2\pi G]$.

__init__(amp=1.0, R=1.1, normalize=False, ro=None, vo=None)
NAME:

init

PURPOSE:

initialize a homogeneous sphere potential

INPUT:

amp= amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

R= size of the sphere (can be Quantity)

normalize= if True, normalize such that vc(1..0.)=1.., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1..0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2019-12-20 - Written - Bovy (UofT)

Isochrone potential

class galpy.potential.IsochronePotential(amp=1.0, b=1.0, normalize=False, ro=None, vo=None)

Class that implements the Isochrone potential

$$\Phi(r) = -\frac{\text{amp}}{b + \sqrt{b^2 + r^2}}$$

with amp = GM the total mass.

__init__ (amp=1.0, b=1.0, normalize=False, ro=None, vo=None)
NAME:

.

__init__

PURPOSE:

initialize an isochrone potential

INPUT:

amp= amplitude to be applied to the potential, the total mass (default: 1); can be a Quantity with units of mass or Gxmass

b= scale radius of the isochrone potential (can be Quantity)

normalize= if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2013-09-08 - Written - Bovy (IAS)

Jaffe potential

class galpy.potential.JaffePotential(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

Class that implements the Jaffe potential

$$\rho(r) = \frac{\mathrm{amp}}{4 \, \pi \, a^3} \, \frac{1}{(r/a)^2 \, (1 + r/a)^2}$$

__init__ (amp=1.0, a=1.0, normalize=False, ro=None, vo=None)
NAME:

. .

__init__

PURPOSE:

Initialize a Jaffe potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Written - Bovy (NYU)

Kepler potential

class galpy.potential.**KeplerPotential** (*amp=1.0*, *normalize=False*, *ro=None*, *vo=None*) Class that implements the Kepler (point mass) potential

$$\Phi(r) = -\frac{\mathrm{amp}}{r}$$

with amp = GM the mass.

__init__ (amp=1.0, normalize=False, ro=None, vo=None)
NAME:

vi iivii.

__init__

PURPOSE:

initialize a Kepler, point-mass potential

INPUT:

amp - amplitude to be applied to the potential, the mass of the point mass (default: 1); can be a Quantity with units of mass density or Gxmass density

alpha - inner power

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-10 - Written - Bovy (NYU)

NFW potential

class galpy.potential.NFWPotential (amp=1.0, a=1.0, normalize=False, conc=None, mvir=None, vo=None, ro=None, H=70.0, Om=0.3, overdens=200.0, wrtcrit=False)

Class that implements the NFW potential

$$\rho(r) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(r/a) (1 + r/a)^2}$$

__init__ (amp=1.0, a=1.0, normalize=False, conc=None, mvir=None, vo=None, ro=None, H=70.0, Om=0.3, overdens=200.0, wrtcrit=False)

NAME:

__init__

PURPOSE:

Initialize a NFW potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

normalize - if True, normalize such that vc(1,0)=1, or, if given as a number, such that the force is this fraction of the force necessary to make vc(1,0)=1.

Alternatively, NFW potentials can be initialized using

conc= concentration

mvir= virial mass in 10^12 Msolar

in which case you also need to supply the following keywords

H= (default: 70) Hubble constant in km/s/Mpc

Om= (default: 0.3) Omega matter

overdens= (200) overdensity which defines the virial radius

wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Written - Bovy (NYU)

2014-04-03 - Initialization w/ concentration and mass - Bovy (IAS)

Plummer potential

class galpy.potential.PlummerPotential(amp=1.0, b=0.8, normalize=False, ro=None, vo=None)

Class that implements the Plummer potential

$$\Phi(R,z) = -\frac{\text{amp}}{\sqrt{R^2 + z^2 + b^2}}$$

with amp = GM the total mass.

__init__(amp=1.0, b=0.8, normalize=False, ro=None, vo=None)
NAME:

NAIVIE.

__init__

PURPOSE:

initialize a Plummer potential

INPUT:

amp - amplitude to be applied to the potential, the total mass (default: 1); can be a Quantity with units of mass or Gxmass

b - scale parameter (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2015-06-15 - Written - Bovy (IAS)

Power-law density spherical potential

class galpy.potential.PowerSphericalPotential(amp=1.0, alpha=1.0, normalize=False, r1=1.0, ro=None, vo=None)

Class that implements spherical potentials that are derived from power-law density models

$$\rho(r) = \frac{\text{amp}}{r_1^3} \left(\frac{r_1}{r}\right)^{\alpha}$$

__init__ (amp=1.0, alpha=1.0, normalize=False, r1=1.0, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a power-law-density potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

alpha - inner power

r1=(1.) reference radius for amplitude (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-10 - Written - Bovy (NYU)

Power-law density spherical potential with an exponential cut-off

class galpy.potential.PowerSphericalPotentialwCutoff (amp=1.0, alpha=1.0, rc=1.0, normalize=False, r1=1.0, ro=None, vo=None)

Class that implements spherical potentials that are derived from power-law density models

$$\rho(r) = \operatorname{amp}\left(\frac{r_1}{r}\right)^{\alpha} \exp\left(-(r/rc)^2\right)$$

__init__ (amp=1.0, alpha=1.0, rc=1.0, normalize=False, r1=1.0, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a power-law-density potential

INPUT:

amp= amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

alpha= inner power

rc= cut-off radius (can be Quantity)

r1=(1.) reference radius for amplitude (can be Quantity)

normalize= if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2013-06-28 - Written - Bovy (IAS)

Pseudo-isothermal potential

class galpy.potential.PseudoIsothermalPotential(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

Class that implements the pseudo-isothermal potential

$$\rho(r) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{1 + (r/a)^2}$$

 $_$ init $_$ (amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a pseudo-isothermal potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - core radius (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2015-12-04 - Started - Bovy (UofT)

Spherical Shell Potential

class galpy.potential.SphericalShellPotential (amp=1.0, a=0.75, normalize=False, ro=None, vo=None)

Class that implements the potential of an infinitesimally-thin, spherical shell

$$\rho(r) = \frac{\mathrm{amp}}{4\pi \, a^2} \, \delta(r - a)$$

with amp = GM the mass of the shell.

__init__(amp=1.0, a=0.75, normalize=False, ro=None, vo=None)
NAME:

init

PURPOSE:

initialize a spherical shell potential

INPUT:

```
amp - mass of the shell (default: 1); can be a Quantity with units of mass or Gxmass a=(0.75) radius of the shell (can be Quantity) normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.; note that because the force is always zero at r < a, this does not work if a > 1 ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)
```

OUTPUT:

(none)

HISTORY:

2018-08-04 - Written - Bovy (UofT)

Axisymmetric potentials

Double exponential disk potential

Class that implements the double exponential disk potential

$$\rho(R, z) = \operatorname{amp} \exp\left(-R/h_R - |z|/h_z\right)$$

initialize a double-exponential disk potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

hr - disk scale-length (can be Quantity)

hz - scale-height (can be Quantity)

tol - relative accuracy of potential-evaluations

maxiter - scipy.integrate keyword

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

DoubleExponentialDiskPotential object

HISTORY:

2010-04-16 - Written - Bovy (NYU)

2013-01-01 - Re-implemented using faster integration techniques - Bovy (IAS)

Flattened Power-law potential

Flattening is in the potential as in Evans (1994) rather than in the density

class galpy.potential.FlattenedPowerPotential (amp=1.0, alpha=0.5, q=0.9, core=1e-08, normalize=False, r1=1.0, ro=None, vo=None)

Class that implements a power-law potential that is flattened in the potential (NOT the density)

$$\Phi(R, z) = -\frac{\text{amp } r_1^{\alpha}}{\alpha \left(R^2 + (z/q)^2 + \text{core}^2\right)^{\alpha/2}}$$

and the same as LogarithmicHaloPotential for $\alpha = 0$

See Figure 1 in Evans (1994) for combinations of alpha and q that correspond to positive densities

__init__ (amp=1.0, alpha=0.5, q=0.9, core=1e-08, normalize=False, r1=1.0, ro=None, vo=None)
NAME:

init

PURPOSE:

initialize a flattened power-law potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of velocity-squared

alpha - power

q - flattening

core - core radius (can be Quantity)

r1=(1.) reference radius for amplitude (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2013-01-09 - Written - Bovy (IAS)

Interpolated axisymmetric potential

The interpRZPotential class provides a general interface to generate interpolated instances of general three-dimensional, axisymmetric potentials or lists of such potentials. This interpolated potential can be used in any function where other three-dimensional galpy potentials can be used. This includes functions that use C to speed up calculations, if the interpRZPotential instance was set up with enable_c=True. Initialize as

```
>>> from galpy import potential
>>> ip= potential.interpRZPotential(potential.MWPotential,interpPot=True)
```

which sets up an interpolation of the potential itself only. The potential and all different forces and functions (dens, "vcirc", epifreq, verticalfreq, dvcircdR) are interpolated separately and one needs to specify that these need to be interpolated separately (so, for example, one needs to set interpRforce=True to interpolate the radial force, or interpvcirc=True to interpolate the circular velocity).

When points outside the grid are requested within the python code, the instance will fall back on the original (non-interpolated) potential. However, when the potential is used purely in C, like during orbit integration in C or during action—angle evaluations in C, there is no way for the potential to fall back onto the original potential and nonsense or NaNs will be returned. Therefore, when using interpRZPotential in C, one must make sure that the whole relevant part of the (R, z) plane is covered. One more time:

Warning: When an interpolated potential is used purely in C, like during orbit integration in C or during action—angle evaluations in C, there is no way for the potential to fall back onto the original potential and nonsense or NaNs will be returned. Therefore, when using interpRZPotential in C, one must make sure that the whole relevant part of the (R, z) plane is covered.

```
class galpy.potential.interpRZPotential(RZPot=None,
                                                                           rgrid = (-4.605170185988091,
                                                       2.995732273553991,
                                                                             101), zgrid=(0.0,
                                                       101),
                                                               logR=True,
                                                                              interpPot=False,
                                                                                                 inter-
                                                       pRforce=False,
                                                                         interpzforce=False,
                                                                                                interp-
                                                       Dens=False.
                                                                      interpvcirc=False,
                                                                                           interpdvcir-
                                                       cdr=False, interpepifreq=False, interpvertical-
                                                       freq=False, ro=None, vo=None, use_c=False,
                                                       enable c=False, zsym=True, numcores=None)
     Class that interpolates a given potential on a grid for fast orbit integration
        _init__(RZPot=None, rgrid=(-4.605170185988091, 2.995732273553991, 101), zgrid=(0.0, 1.0,
                  101), logR=True, interpPot=False, interpRforce=False, interpzforce=False, interp-
                  Dens=False, interpvcirc=False, interpdvcircdr=False, interpepifreq=False, interpverti-
                  calfreq=False, ro=None, vo=None, use_c=False, enable_c=False, zsym=True, num-
                  cores=None)
          NAME:
               init
          PURPOSE:
               Initialize an interpRZPotential instance
          INPUT:
               RZPot - RZPotential to be interpolated
               rgrid - R grid to be given to linspace as in rs= linspace(*rgrid)
               zgrid - z grid to be given to linspace as in zs= linspace(*zgrid)
               logR - if True, rgrid is in the log of R so logrs= linspace(*rgrid)
```

```
interpPot, interpRforce, interpzforce, interpDens,interpvcirc, interpepifreq, interpverticalfreq, interpdvcircdr= if True, interpolate these functions

use_c= use C to speed up the calculation of the grid

enable_c= enable use of C for interpolations

zsym= if True (default), the potential is assumed to be symmetric around z=0 (so you can use, e.g., zgrid=(0.,1.,101)).

numcores= if set to an integer, use this many cores (only used for vcirc, dvcircdR, epifreq, and verticalfreq; NOT NECESSARILY FASTER, TIME TO MAKE SURE)

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:
   instance

HISTORY:

2010-07-21 - Written - Bovy (NYU)
```

Interpolated SnapshotRZ potential

This class is built on the interpRZPotential class; see the documentation of that class *here* for additional information on how to setup objects of the InterpSnapshotRZPotential class.

2013-01-24 - Started with new implementation - Bovy (IAS)

```
class galpy.potential.InterpSnapshotRZPotential(s,
                                                                           ro=None,
                                                                                             vo=None,
                                                                  rgrid=(-4.605170185988091,
                                                                  2.995732273553991, 101), zgrid=(0.0,
                                                                         101),
                                                                                  interpepifreq=False,
                                                                  interpverticalfreq=False,
                                                                                               interp-
                                                                  Pot=True, enable\_c=True, logR=True,
                                                                  zsym=True, numcores=None, naz-
                                                                  imuths=4, use_pkdgrav=False)
     Interpolated axisymmetrized potential extracted from a simulation output (see interpRZPotential and
     SnapshotRZPotential)
        init (s, ro=None, vo=None, rgrid=(-4.605170185988091, 2.995732273553991, 101), zgrid=(0.0,
                  1.0, 101), interpepifreq=False, interpvertical freq=False, interpPot=True, enable c=True,
                  logR=True, zsym=True, numcores=None, nazimuths=4, use_pkdgrav=False)
          NAME:
               __init__
          PURPOSE:
               Initialize an InterpSnapshotRZPotential instance
          INPUT:
               s - a simulation snapshot loaded with pynbody
               rgrid - R grid to be given to linspace as in rs= linspace(*rgrid)
               zgrid - z grid to be given to linspace as in zs= linspace(*zgrid)
               logR - if True, rgrid is in the log of R so logrs= linspace(*rgrid)
```

interpPot, interpepifreq, interpverticalfreq= if True, interpolate these functions (interpPot=True also interpolates the R and zforce)

enable_c= enable use of C for interpolations

zsym= if True (default), the potential is assumed to be symmetric around z=0 (so you can use, e.g., zgrid=(0.1.101)).

numcores= if set to an integer, use this many cores

nazimuths= (4) number of azimuths to average over

use_pkdgrav= (False) use PKDGRAV to calculate the snapshot's potential and forces (CUR-RENTLY NOT IMPLEMENTED)

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

instance

HISTORY:

2013 - Written - Rok Roskar (ETH)

2014-11-24 - Edited for merging into main galpy - Bovy (IAS)

Kuzmin disk potential

class galpy.potential.KuzminDiskPotential(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)

Class that implements the Kuzmin Disk potential

$$\Phi(R,z) = -\frac{\mathrm{amp}}{\sqrt{R^2 + (a+|z|)^2}}$$

with amp = GM the total mass.

__init__(amp=1.0, a=1.0, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a Kuzmin disk Potential

INPUT:

amp - amplitude to be applied to the potential, the total mass (default: 1); can be a Quantity with units of mass density or Gxmass density

a - scale length (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

KuzminDiskPotential object

HISTORY:

2016-05-09 - Written - Aladdin

Kuzmin-Kutuzov Staeckel potential

class galpy.potential.KuzminKutuzovStaeckelPotential (amp=1.0, ac=5.0, Delta=1.0, normalize=False, ro=None, vo=None)

Class that implements the Kuzmin-Kutuzov Staeckel potential

$$\Phi(R,z) = -\frac{\mathrm{amp}}{\sqrt{\lambda} + \sqrt{\nu}}$$

(see, e.g., Batsleer & Dejonghe 1994)

__init__(amp=1.0, ac=5.0, Delta=1.0, normalize=False, ro=None, vo=None)

__init__

PURPOSE:

initialize a Kuzmin-Kutuzov Staeckel potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

ac - axis ratio of the coordinate surfaces; (a/c) = sqrt(-alpha) / sqrt(-gamma) (default: 5.)

Delta - focal distance that defines the spheroidal coordinate system (default: 1.); Delta=sqrt(gamma-alpha) (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2015-02-15 - Written - Trick (MPIA)

Logarithmic halo potential

class galpy.potential.LogarithmicHaloPotential (amp=1.0, core=1e-08, q=1.0, b=None, normalize=False, ro=None, vo=None)

Class that implements the logarithmic potential

$$\Phi(R, z) = \frac{\text{amp}}{2} \ln \left[R^2 + \left(\frac{z}{q}\right)^2 + \text{core}^2 \right]$$

Alternatively, the potential can be made triaxial by adding a parameter b

$$\Phi(x,y,z) = \frac{\text{amp}}{2} \ln \left[x^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{q}\right)^2 + \text{core}^2 \right]$$

```
With these definitions, \sqrt{\text{amp}} is the circular velocity at r \gg \text{core} at (y, z) = (0, 0).
  \texttt{_init}_{\texttt{\_\_}}(amp=1.0, core=1e-08, q=1.0, b=None, normalize=False, ro=None, vo=None)
     NAME:
           __init__
     PURPOSE:
          initialize a logarithmic potential
     INPUT:
          amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of
          velocity-squared
          core - core radius at which the logarithm is cut (can be Quantity)
          q - potential flattening (z/q)^{**}2.
          b= (None) if set, shape parameter in y-direction (y \rightarrow y/b); see definition)
          normalize - if True, normalize such that vc(1..0.)=1, or, if given as a number, such that the force
          is this fraction of the force necessary to make vc(1.,0.)=1.
          ro=, vo= distance and velocity scales for translation into internal units (default from configuration
          file)
     OUTPUT:
          (none)
     HISTORY:
          2010-04-02 - Started - Bovy (NYU)
```

Miyamoto-Nagai potential

Class that implements the Miyamoto-Nagai potential

$$\Phi(R,z) = -\frac{\mathrm{amp}}{\sqrt{R^2 + (a+\sqrt{z^2+b^2})^2}}$$

with amp = GM the total mass.

__init__(amp=1.0, a=1.0, b=0.1, normalize=False, ro=None, vo=None)
NAME:

init

PURPOSE:

initialize a Miyamoto-Nagai potential

INPUT:

amp - amplitude to be applied to the potential, the total mass (default: 1); can be a Quantity with units of mass or Gxmass

a - scale length (can be Quantity)

b - scale height (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Started - Bovy (NYU)

Three Miyamoto-Nagai disk approximation to an exponential disk

class galpy.potential.MN3ExponentialDiskPotential(amp=1.0,

hz=0.0625, sech=False, posdens=False, normalize=False,

ro=None, vo=None

class that implements the three Miyamoto-Nagai approximation to a radially-exponential disk potential of Smith et al. 2015

$$\rho(R, z) = \operatorname{amp} \exp(-R/h_R - |z|/h_z)$$

or

$$\rho(R, z) = \operatorname{amp} \exp(-R/h_R) \operatorname{sech}^2(-|z|/h_z)$$

depending on whether sech=True or not. This density is approximated using three Miyamoto-Nagai disks

NAME:

___init___

PURPOSE:

initialize a 3MN approximation to an exponential disk potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass density or Gxmass density

hr - disk scale-length (can be Quantity)

hz - scale-height (can be Quantity)

sech= (False) if True, hz is the scale height of a sech vertical profile (default is exponential vertical profile)

posdens= (False) if True, allow only positive density solutions (Table 2 in Smith et al. rather than Table 1)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

```
OUTPUT:
```

MN3ExponentialDiskPotential object

HISTORY:

2015-02-07 - Written - Bovy (IAS)

Razor-thin exponential disk potential

class galpy.potential.RazorThinExponentialDiskPotential(amp=1.0,

hr=0.33333333333333333, maxiter=20, tol=0.001, normalize=False, ro=None, vo=None, new=True, glorder=100)

Class that implements the razor-thin exponential disk potential

$$\rho(R, z) = \text{amp exp}(-R/h_R) \delta(z)$$

NAME:

___init__

PURPOSE:

initialize a razor-thin-exponential disk potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of surface-mass or Gxsurface-mass

hr - disk scale-length (can be Quantity)

tol - relative accuracy of potential-evaluations

maxiter - scipy.integrate keyword

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

RazorThinExponentialDiskPotential object

HISTORY:

2012-12-27 - Written - Bovy (IAS)

Ring potential

```
a=0.75.
class galpy.potential.RingPotential (amp=1.0,
                                                                             normalize=False,
                                                                                                  ro=None.
      Class that implements the potential of an infinitesimally-thin, circular ring
                                          \rho(R,z) = \frac{\text{amp}}{2\pi R_0} \, \delta(R - R_0) \, \delta(z)
      with amp = GM the mass of the ring.
      init (amp=1.0, a=0.75, normalize=False, ro=None, vo=None)
           NAME:
                __init__
           PURPOSE:
               initialize a circular ring potential
           INPUT:
               amp - mass of the ring (default: 1); can be a Quantity with units of mass or Gxmass
               a = (0.75) radius of the ring (can be Quantity)
               normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force
               is this fraction of the force necessary to make vc(1.,0.)=1; note that because the force is always
               positive at r < a, this does not work if a > 1
               ro=, vo= distance and velocity scales for translation into internal units (default from configuration
               file)
           OUTPUT:
               (none)
           HISTORY:
               2018-08-04 - Written - Bovy (UofT)
Axisymmetrized N-body snapshot potential
class galpy.potential.SnapshotRZPotential(s, num_threads=None, nazimuths=4, ro=None,
      Class that implements an axisymmetrized version of the potential of an N-body snapshot (requires pynbody)
      evaluate, Rforce, and zforce calculate a hash for the array of points that is passed in by the user. The hash
      and corresponding potential/force arrays are stored - if a subsequent request matches a previously computed
      hash, the previous results are returned and not recalculated.
        _init___(s, num_threads=None, nazimuths=4, ro=None, vo=None)
```

NAME:

__init__

Initialize a SnapshotRZ potential object

PURPOSE:

INPUT:

```
s - a simulation snapshot loaded with pynbody
```

num threads= (4) number of threads to use for calculation

nazimuths= (4) number of azimuths to average over

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

instance

HISTORY:

2013 - Written - Rok Roskar (ETH)

2014-11-24 - Edited for merging into main galpy - Bovy (IAS)

Ellipsoidal triaxial potentials

galpy has very general support for implementing triaxial (or the oblate and prolate special cases) of ellipsoidal potentials through the general EllipsoidalPotential class. These potentials have densities that are uniform on ellipsoids, thus only functions of $m^2 = x^2 + \frac{y^2}{b^2} + \frac{z^2}{c^2}$. New potentials of this type can be implemented by inheriting from this class and implementing the _mdens (self,m), _psi(self,m), and _mdens_deriv functions for the density, its integral with respect to m^2 , and its derivative with respect to m, respectively. For adding a C implementation, follow similar steps (use PerfectEllipsoidPotential as an example to follow).

Perfect Ellipsoid potential

class galpy.potential.PerfectEllipsoidPotential (amp=1.0, a=5.0, b=1.0, c=1.0, zvec=None, pa=None, glo-rder=50, normalize=False, ro=None, ve=None)

Potential of the perfect ellipsoid (de Zeeuw 1985):

$$\rho(x, y, z) = \frac{\text{amp a}}{\pi^2 bc} \frac{1}{(m^2 + a^2)^2}$$

where amp = GM is the total mass and $m^2 = x^2 + y^2/b^2 + z^2/c^2$.

 $_$ init $_$ (amp=1.0, a=5.0, b=1.0, c=1.0, zvec=None, pa=None, glorder=50, normalize=False, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a perfect ellipsoid potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units that depend on the specific spheroidal potential

a - scale radius (can be Quantity)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

zvec= (None) If set, a unit vector that corresponds to the z axis

pa= (None) If set, the position angle of the x axis (rad or Quantity)

glorder= (50) if set, compute the relevant force and potential integrals with Gaussian quadrature of this order

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2018-08-06 - Started - Bovy (UofT)

Double power-law density triaxial potential

class galpy.potential.TwoPowerTriaxialPotential (amp=1.0, a=5.0, alpha=1.5, beta=3.5, b=1.0, c=1.0, zvec=None, pa=None, glorder=50, normalize=False, ro=None, vo=None)

Class that implements triaxial potentials that are derived from two-power density models

$$\rho(x, y, z) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(m/a)^{\alpha} (1 + m/a)^{\beta - \alpha}}$$

with

$$m^2 = x'^2 + \frac{y'^2}{h^2} + \frac{z'^2}{c^2}$$

and (x', y', z') is a rotated frame wrt (x, y, z) specified by parameters zvec and pa which specify (a) zvec: the location of the z' axis in the (x, y, z) frame and (b) pa: the position angle of the x' axis wrt the \tilde{x} axis, that is, the x axis after rotating to zvec.

Note that this general class of potentials does *not* automatically revert to the special TriaxialNFWPotential, TriaxialHernquistPotential, or TriaxialJaffePotential when using their (alpha,beta) values (like TwoPowerSphericalPotential).

__init__ (amp=1.0, a=5.0, alpha=1.5, beta=3.5, b=1.0, c=1.0, zvec=None, pa=None, glorder=50, normalize=False, ro=None, vo=None)

NAME:

___init___

PURPOSE:

initialize a triaxial two-power-density potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

alpha - inner power $(0 \le alpha \le 3)$

beta - outer power (beta > 2)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

zvec= (None) If set, a unit vector that corresponds to the z axis

pa= (None) If set, the position angle of the x axis (rad or Quantity)

glorder= (50) if set, compute the relevant force and potential integrals with Gaussian quadrature of this order

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2016-05-30 - Started - Bovy (UofT)

2018-08-07 - Re-written using the general EllipsoidalPotential class - Bovy (UofT)

Triaxial Jaffe potential

class galpy.potential.TriaxialJaffePotential (amp=1.0, a=2.0, b=1.0, c=1.0, zvec=None, pa=None, normalize=False, glorder=50, ro=None, vo=None)

Class that implements the Jaffe potential

$$\rho(x, y, z) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(m/a)^2 (1 + m/a)^2}$$

with

$$m^2 = x'^2 + \frac{y'^2}{b^2} + \frac{z'^2}{c^2}$$

and (x',y',z') is a rotated frame wrt (x,y,z) specified by parameters zvec and pa which specify (a) zvec: the location of the z' axis in the (x,y,z) frame and (b) pa: the position angle of the x' axis wrt the \tilde{x} axis, that is, the x axis after rotating to zvec.

 $_$ init $_$ (amp=1.0, a=2.0, b=1.0, c=1.0, zvec=None, pa=None, normalize=False, glorder=50, ro=None, vo=None)

NAME:

__init__

PURPOSE:

Initialize a Jaffe potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

zvec= (None) If set, a unit vector that corresponds to the z axis

pa= (None) If set, the position angle of the x axis

glorder= (50) if set, compute the relevant force and potential integrals with Gaussian quadrature of this order

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Written - Bovy (UofT)

2018-08-07 - Re-written using the general EllipsoidalPotential class - Bovy (UofT)

Triaxial Hernquist potential

class galpy.potential.TriaxialHernquistPotential (amp=1.0, a=2.0, normalize=False, b=1.0, c=1.0, zvec=None, pa=None, glorder=50, ro=None, vo=None)

Class that implements the triaxial Hernquist potential

$$\rho(x, y, z) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(m/a) (1 + m/a)^3}$$

with

$$m^2 = x'^2 + \frac{y'^2}{b^2} + \frac{z'^2}{c^2}$$

and (x', y', z') is a rotated frame wrt (x, y, z) specified by parameters zvec and pa which specify (a) zvec: the location of the z' axis in the (x, y, z) frame and (b) pa: the position angle of the x' axis wrt the \tilde{x} axis, that is, the x axis after rotating to zvec.

 $_$ _init $_$ (amp=1.0, a=2.0, normalize=False, b=1.0, c=1.0, zvec=None, pa=None, glorder=50, ro=None, vo=None)

NAME:

__init__

PURPOSE:

Initialize a triaxial Hernquist potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

zvec= (None) If set, a unit vector that corresponds to the z axis

pa= (None) If set, the position angle of the x axis

glorder= (50) if set, compute the relevant force and potential integrals with Gaussian quadrature of this order

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2010-07-09 - Written - Bovy (UofT)

2018-08-07 - Re-written using the general EllipsoidalPotential class - Bovy (UofT)

Triaxial NFW potential

class galpy.potential.TriaxialNFWPotential (amp=1.0, a=2.0, b=1.0, c=1.0, zvec=None, pa=None, normalize=False, conc=None, mvir=None, glorder=50, vo=None, ro=None, H=70.0, Om=0.3, overdens=200.0, wrtcrit=False)

Class that implements the triaxial NFW potential

$$\rho(x, y, z) = \frac{\text{amp}}{4 \pi a^3} \frac{1}{(m/a) (1 + m/a)^2}$$

with

$$m^2 = x'^2 + \frac{y'^2}{b^2} + \frac{z'^2}{c^2}$$

and (x', y', z') is a rotated frame wrt (x, y, z) specified by parameters zvec and pa which specify (a) zvec: the location of the z' axis in the (x, y, z) frame and (b) pa: the position angle of the x' axis wrt the \tilde{x} axis, that is, the x axis after rotating to zvec.

__init__ (amp=1.0, a=2.0, b=1.0, c=1.0, zvec=None, pa=None, normalize=False, conc=None, mvir=None, glorder=50, vo=None, ro=None, H=70.0, Om=0.3, overdens=200.0, wrtcrit=False)

NAME:

__init__

PURPOSE:

Initialize a triaxial NFW potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

zvec= (None) If set, a unit vector that corresponds to the z axis

pa= (None) If set, the position angle of the x axis

glorder= (50) if set, compute the relevant force and potential integrals with Gaussian quadrature of this order

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

Alternatively, NFW potentials can be initialized using

conc= concentration

mvir= virial mass in 10^12 Msolar

in which case you also need to supply the following keywords

H= (default: 70) Hubble constant in km/s/Mpc

Om= (default: 0.3) Omega matter

overdens= (200) overdensity which defines the virial radius

wrtcrit= (False) if True, the overdensity is wrt the critical density rather than the mean matter density

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2016-05-30 - Written - Bovy (UofT)

2018-08-06 - Re-written using the general EllipsoidalPotential class - Bovy (UofT)

Note that the Ferrers potential listed below is a potential of this type, but it is currently not implemented using the EllipsoidalPotential class.

Spiral, bar, other triaxial, and miscellaneous potentials

Dehnen bar potential

class galpy.potential.DehnenBarPotential (amp=1.0, omegab=None, rb=None, chi=0.8, rolr=0.9, barphi=0.4363323129985824, tform=-4.0, tsteady=None, beta=0.0, alpha=0.01, Af=None, ro=None, vo=None)

Class that implements the Dehnen bar potential (Dehnen 2000), generalized to 3D following Monari et al. (2016)

$$\Phi(R, z, \phi) = A_b(t) \cos(2(\phi - \Omega_b t))) \left(\frac{R}{r}\right)^2 \times \begin{cases} -(R_b/r)^3, & \text{for } r \ge R_b \\ (r/R_b)^3 - 2, & \text{for } r \le R_b. \end{cases}$$

where $r^2 = R^2 + z^2$ is the spherical radius and

$$A_b(t) = A_f \left(\frac{3}{16} \xi^5 - \frac{5}{8} \xi^3 + \frac{15}{16} \xi + \frac{1}{2} \right), \xi = 2 \frac{t/T_b - t_{\text{form}}}{T_{\text{steady}}} - 1, \text{ if } t_{\text{form}} \le \frac{t}{T_b} \le t_{\text{form}} + T_{\text{steady}}$$

and

$$A_b(t) = \begin{cases} 0, & \frac{t}{T_b} < t_{\text{form}} \\ A_f, & \frac{t}{T_b} > t_{\text{form}} + T_{\text{steady}} \end{cases}$$

where

$$T_b = \frac{2\pi}{\Omega_b}$$

is the bar period and the strength can also be specified using α

$$\alpha = 3 \, \frac{A_f}{v_0^2} \, \left(\frac{R_b}{r_0}\right)^3 \, .$$

__init__ (amp=1.0, omegab=None, rb=None, chi=0.8, rolr=0.9, barphi=0.4363323129985824, tform=-4.0, tsteady=None, beta=0.0, alpha=0.01, Af=None, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a Dehnen bar potential

INPUT:

amp - amplitude to be applied to the potential (default: 1., see alpha or Ab below)

barphi - angle between sun-GC line and the bar's major axis (in rad; default=25 degree; or can be Quantity))

tform - start of bar growth / bar period (default: -4)

tsteady - time from tform at which the bar is fully grown / bar period (default: -tform/2, st the perturbation is fully grown at tform/2)

Either provide:

a) rolr - radius of the Outer Lindblad Resonance for a circular orbit (can be Quantity)

chi - fraction R_bar / R_CR (corotation radius of bar)

alpha - relative bar strength (default: 0.01)

beta - power law index of rotation curve (to calculate OLR, etc.)

b) omegab - rotation speed of the bar (can be Quantity)

rb - bar radius (can be Quantity)

Af - bar strength (can be Quantity)

OUTPUT:

(none)

HISTORY:

2010-11-24 - Started - Bovy (NYU)

2017-06-23 - Converted to 3D following Monari et al. (2016) - Bovy (UofT/CCA)

Ferrers potential

class galpy.potential.FerrersPotential (amp=1.0, a=1.0, n=2, b=0.35, c=0.2375, omegab=0.0, pa=0.0, normalize=False, ro=None, vo=None)

Class that implements triaxial Ferrers potential for the ellipsoidal density profile with the short axis along the z-direction

$$\rho(x, y, z) = \frac{\text{amp}}{\pi^{1.5} a^3 bc} \frac{\Gamma(n + \frac{5}{2})}{\Gamma(n + 1)} (1 - (m/a)^2)^n$$

with

$$m^2 = x'^2 + \frac{y'^2}{b^2} + \frac{z'^2}{c^2}$$

and (x', y', z') is a rotated frame wrt (x, y, z) so that the major axis is aligned with x'.

Note that this potential has not yet been optimized for speed and has no C implementation, so orbit integration is currently slow.

 $_$ init $_$ (amp=1.0, a=1.0, n=2, b=0.35, c=0.2375, omegab=0.0, pa=0.0, normalize=False, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a Ferrers potential

INPUT:

amp - total mass of the ellipsoid determines the amplitude of the potential; can be a Quantity with units of mass or Gxmass

a - scale radius (can be Quantity)

n - power of Ferrers density (n > 0)

b - y-to-x axis ratio of the density

c - z-to-x axis ratio of the density

omegab - rotation speed of the ellipsoid (can be Quantity)

pa= (None) If set, the position angle of the x axis (rad or Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

Moving object potential

class galpy.potential.MovingObjectPotential(orbit, pot=None, amp=1.0, ro=None,

Class that implements the potential coming from a moving object by combining any galpy potential with an integrated galpy orbit.

```
init (orbit, pot=None, amp=1.0, ro=None, vo=None)
           NAME:
               __init__
           PURPOSE:
               initialize a MovingObjectPotential
           INPUT:
               orbit - the Orbit of the object (Orbit object)
               pot - A potential object or list of potential objects representing the potential of the moving object;
               should be spherical, but this is not checked [default= PlummerPotential(amp=0.06,b=0.01)]
               amp (=1.) another amplitude to apply to the potential
               ro=, vo= distance and velocity scales for translation into internal units (default from configuration
               file)
           OUTPUT:
               (none)
           HISTORY:
               2011-04-10 - Started - Bovy (NYU)
               2018-10-18 - Re-implemented to represent general object potentials using galpy potential models
               - James Lane (UofT)
Softened-needle bar potential
class galpy.potential.SoftenedNeedleBarPotential(amp=1.0, a=4.0, b=0.0, c=1.0, nor-
                                                                     malize=False, pa=0.4, omegab=1.8,
                                                                     ro=None, vo=None)
     Class that implements the softened needle bar potential from Long & Murali (1992)
```

$$\Phi(x,y,z) = \frac{\mathrm{amp}}{2a} \, \ln \left(\frac{x-a+T_-}{x+a+T_+} \right)$$

where

$$T_{\pm} = \sqrt{(a \pm x)^2 + y^2 + (b + \sqrt{z^2 + c^2})^2}$$

For a prolate bar, set b to zero.

 $_$ init $_$ (amp=1.0, a=4.0, b=0.0, c=1.0, normalize=False, pa=0.4, omegab=1.8, ro=None, NAME:

__init__

PURPOSE:

initialize a softened-needle bar potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass

a= (4.) Bar half-length (can be Quantity)

b= (1.) Triaxial softening length (can be Quantity)

c=(1.) Prolate softening length (can be Quantity)

pa=(0.4) The position angle of the x axis (rad or Quantity)

omegab= (1.8) Pattern speed (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

(none)

HISTORY:

2016-11-02 - Started - Bovy (UofT)

Spiral arms potential

class galpy.potential.SpiralArmsPotential (amp=1, ro=None, vo=None, $amp_units='density', N=2, alpha=0.2, r_ref=1, phi_ref=0, Rs=0.3, H=0.125, omega=0, Cs=[1])$

Class that implements the spiral arms potential from (Cox and Gomez 2002). Should be used to modulate an existing potential (density is positive in the arms, negative outside).

$$\Phi(R, \phi, z) = -4\pi G H \,\rho_0 exp\left(-\frac{R - r_{ref}}{R_s}\right) \sum \frac{C_n}{K_n D_n} \,\cos(n\gamma) \,\mathrm{sech}^{B_n}\left(\frac{K_n z}{B_n}\right)$$

where

$$K_n = \frac{nN}{R\sin(\alpha)}$$

$$B_n = K_n H (1 + 0.4K_n H)$$

$$D_n = \frac{1 + K_n H + 0.3(K_n H)^2}{1 + 0.3K_n H}$$

and

$$\gamma = N \left[\phi - \phi_{ref} - \frac{\ln(R/r_{ref})}{\tan(\alpha)} \right]$$

The default of $C_n = [1]$ gives a sinusoidal profile for the potential. An alternative from Cox and Gomez (2002) creates a density that behaves approximately as a cosine squared in the arms but is separated by a flat interarm region by setting

$$C_n = \left[\frac{8}{3\pi}, \frac{1}{2}, \frac{8}{15\pi}\right]$$

__init__ (amp=1, ro=None, vo=None, amp_units='density', N=2, alpha=0.2, r_ref=1, phi_ref=0, Rs=0.3, H=0.125, omega=0, Cs=[1])

NAME: init

PURPOSE: initialize a spiral arms potential

INPUT:

amp amplitude to be applied to the potential (default: 1); can be a Quantity with units of density. $(amp = 4\pi G\rho_0)$

ro distance scales for translation into internal units (default from configuration file)

vo velocity scales for translation into internal units (default from configuration file)

N number of spiral arms

alpha pitch angle of the logarithmic spiral arms in radians (can be Quantity)

r_ref fiducial radius where $\rho = \rho_0$ (r_0 in the paper by Cox and Gomez) (can be Quantity)

phi_ref reference angle $(\phi_p(r_0))$ in the paper by Cox and Gomez) (can be Quantity)

Rs radial scale length of the drop-off in density amplitude of the arms (can be Quantity)

H scale height of the stellar arm perturbation (can be Quantity)

Cs list of constants multiplying the $\cos(n\gamma)$ terms

omega rotational pattern speed of the spiral arms (can be Quantity)

OUTPUT: (none)

HISTORY: Started - 2017-05-12 Jack Hong (UBC)

Completed - 2017-07-04 Jack Hong (UBC)

All galpy potentials can also be made to rotate using the SolidBodyRotationWrapperPotential listed in the section on wrapper potentials *below*.

General Poisson solvers for disks and halos

Disk potential using SCF basis-function-expansion

class galpy.potential.**DiskSCFPotential** (amp=1.0, normalize=False, dens=<function DiskSCFPotential.<lambda>>, Sigma={'amp': 'exp', 0.037037037037037035, $hz=\{'h':$ 'type': Sigma_amp=None, 'exp'}, dSigmadR=None, Hz=None, d2SigmadR2=None, dHzdz=None, N=10, L=10, a=1.0, $radial_order=None$, costheta_order=None, phi_order=None, ro=None, vo=None)

Class that implements a basis-function-expansion technique for solving the Poisson equation for disk (+halo) systems. We solve the Poisson equation for a given density $\rho(R, \phi, z)$ by introducing K helper function pairs $[\Sigma_i(R), h_i(z)]$, with $h_i(z) = \mathrm{d}^2 H(z)/\mathrm{d}z^2$ and search for solutions of the form

$$\Phi(R, \phi, z = \Phi_{\text{ME}}(R, \phi, z) + 4\pi G \sum_{i} \Sigma_{i}(r) H_{i}(z),$$

where r is the spherical radius $r^2 = R^2 + z^2$. We can solve for $\Phi_{\rm ME}(R,\phi,z)$ by solving

$$\frac{\Delta\Phi_{\mathrm{ME}}(R,\phi,z)}{4\pi G} = \rho(R,\phi,z) - \sum_{i} \left\{ \Sigma_{i}(r) \, h_{i}(z) + \frac{\mathrm{d}^{2}\Sigma_{i}(r)}{\mathrm{d}r^{2}} \, H_{i}(z) + \frac{2}{r} \, \frac{\mathrm{d}\Sigma_{i}(r)}{\mathrm{d}r} \left[H_{i}(z) + z \, \frac{\mathrm{d}H_{i}(z)}{\mathrm{d}z} \right] \right\} \, . \label{eq:phiME}$$

We solve this equation by using the *SCFPotential* class and methods ($scf_compute_coeffs_axi$ or $scf_compute_coeffs$ depending on whether $\rho(R,\phi,z)$ is axisymmetric or not). This technique works very well if the disk portion of the potential can be exactly written as $\rho_{disk} = \sum_i \Sigma_i(R) \, h_i(z)$, because the effective density on the right-hand side of this new Poisson equation is then not 'disky' and can be well represented using spherical harmonics. But the technique is general and can be used to compute the potential of any disk+halo potential; the closer the disk is to $\rho_{disk} \approx \sum_i \Sigma_i(R) \, h_i(z)$, the better the technique works.

This technique was introduced by Kuijken & Dubinski (1995) and was popularized by Dehnen & Binney (1998). The current implementation is a slight generalization of the technique in those papers and uses the SCF approach of Hernquist & Ostriker (1992) to solve the Poisson equation for $\Phi_{\rm ME}(R,\phi,z)$ rather than solving it on a grid using spherical harmonics and interpolating the solution (as done in Dehnen & Binney 1998).

```
normalize=False,
                                                                 DiskSCFPotential.<lambda>>,
___init___(amp=1.0,
                                               dens=<function
           Sigma={'amp':
                              1.0,
                                    'h':
                                            0.3333333333333333333333
                                                                              'exp'},
                                                                                      hz=\{'h':
                                                                   'type':
           0.037037037037037035,
                                     'type':
                                                exp',
                                                         Sigma\_amp=None,
                                                                              dSigmadR=None,
           d2SigmadR2=None, Hz=None, dHzdz=None, N=10, L=10, a=1.0, radial_order=None,
           costheta_order=None, phi_order=None, ro=None, vo=None)
    NAME:
```

__init__

PURPOSE:

initialize a DiskSCF Potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); cannot have units currently

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

dens= function of R,z[,phi optional] that gives the density [in natural units, cannot return a Quantity currently]

N=, L=, a=, radial_order=, costheta_order=, phi_order= keywords setting parameters for SCF solution for Phi_ME (see $scf_compute_coeffs_axi$ or $scf_compute_coeffs$ depending on whether $\rho(R, \phi, z)$ is axisymmetric or not)

Either:

- (a) Sigma= Dictionary of surface density (example: {'type':'exp','h':1./3.,'amp':1.,'Rhole':0.} for amp x exp(-Rhole/R-R/h))

 hz= Dictionary of vertical profile, either 'exp' or 'sech2' (example {'type':'exp','h':1./27.} for exp(-|z|/h)/[2h], sech2 is sech^2(z/[2h])/[4h])
- (b) Sigma= function of R that gives the surface density

dSigmadR= function that gives d Sigma / d R

d2SigmadR2= function that gives d^2 Sigma / d R^2

Sigma_amp= amplitude to apply to all Sigma functions

hz= function of z that gives the vertical profile

Hz= function of z such that $d^2 Hz(z) / dz^2 = hz$

dHzdz = function of z that gives d Hz(z) / d z

In both of these cases lists of arguments can be given for multiple disk components; can't mix (a) and (b) in these lists; if hz is a single item the same vertical profile is assumed for all Sigma

OUTPUT:

DiskSCFPotential object

HISTORY:

2016-12-26 - Written - Bovy (UofT)

Hernquist & Ostriker Self-Consistent-Field-type potential

class galpy.potential.SCFPotential(amp=1.0, Acos=array([[[1]]]), Asin=None, a=1.0, normalize=False, ro=None, vo=None)

Class that implements the Hernquist & Ostriker (1992) Self-Consistent-Field-type potential. Note that we divide the amplitude by 2 such that $Acos = \delta_{0n}\delta_{0l}\delta_{0m}$ and Asin = 0 corresponds to Galpy's Hernquist Potential.

$$\rho(r,\theta,\phi) = \frac{amp}{2} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{l} N_{lm} P_{lm}(\cos(\theta)) \tilde{\rho}_{nl}(r) \left(A_{cos,nlm} \cos(m\phi) + A_{sin,nlm} \sin(m\phi) \right)$$

where

$$\tilde{\rho}_{nl}(r) = \frac{K_{nl}}{\sqrt{\pi}} \frac{(ar)^l}{(r/a)(a+r)^{2l+3}} C_n^{2l+3/2}(\xi)$$

$$\Phi(r,\theta,\phi) = \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \sum_{m=0}^{\infty} N_{lm} P_{lm}(\cos(\theta)) \tilde{\Phi}_{nl}(r) \left(A_{cos,nlm} \cos(m\phi) + A_{sin,nlm} \sin(m\phi) \right)$$

where

$$\tilde{\Phi}_{nl}(r) = -\sqrt{4\pi}K_{nl}\frac{(ar)^l}{(a+r)^{2l+1}}C_n^{2l+3/2}(\xi)$$

where

$$\xi = \frac{r-a}{r+a} \qquad N_{lm} = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} (2-\delta_{m0}) \qquad K_{nl} = \frac{1}{2}n(n+4l+3) + (l+1)(2l+1)$$

and P_{lm} is the Associated Legendre Polynomials whereas C_n^{α} is the Gegenbauer polynomial.

__init__ (amp=1.0, Acos=array([[[1]]]), Asin=None, a=1.0, normalize=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a SCF Potential

INPUT:

amp - amplitude to be applied to the potential (default: 1); can be a Quantity with units of mass or Gxmass

Acos - The real part of the expansion coefficent (NxLxL matrix, or optionally NxLx1 if Asin=None)

Asin - The imaginary part of the expansion coefficient (NxLxL matrix or None)

a - scale length (can be Quantity)

normalize - if True, normalize such that vc(1.,0.)=1., or, if given as a number, such that the force is this fraction of the force necessary to make vc(1.,0.)=1.

ro=, vo= distance and velocity scales for translation into internal units (default from configuration file)

OUTPUT:

SCFPotential object

HISTORY:

2016-05-13 - Written - Aladdin

Dissipative forces

Chandrasekhar dynamical friction

class galpy.potential.ChandrasekharDynamicalFrictionForce (amp=1.0, GMs=0.1, gamma=1.0, rhm=0.0, dens=None, sigmar=None,

sigmar=None, const_lnLambda=False, minr=0.0001, maxr=25.0, nr=501, ro=None, vo=None)

Class that implements the Chandrasekhar dynamical friction force

$$\mathbf{F}(\mathbf{x}, \mathbf{v}) = -2\pi \left[G M \right] \left[G \rho(\mathbf{x}) \right] \ln[1 + \Lambda^2] \left[\operatorname{erf}(X) - \frac{2X}{\sqrt{\pi}} \exp\left(-X^2 \right) \right] \frac{\mathbf{v}}{|\mathbf{v}|^3}$$

on a mass (e.g., a satellite galaxy or a black hole) M at position ${\bf x}$ moving at velocity ${\bf v}$ through a background density ρ . The quantity X is the usual $X=|{\bf v}|/[\sqrt{2}\sigma_r(r)]$. The factor Λ that goes into the Coulomb logarithm is taken to be

$$\Lambda = \frac{r/\gamma}{\max\left(r_{\rm hm}, GM/|\mathbf{v}|^2\right)},\,$$

where γ is a constant. This γ should be the absolute value of the logarithmic slope of the density $\gamma = |\mathrm{d} \ln \rho / \mathrm{d} \ln r|$, although for $\gamma < 1$ it is advisable to set $\gamma = 1$. Implementation here roughly follows 2016MN-RAS.463..858P and earlier work.

__init__ (amp=1.0, GMs=0.1, gamma=1.0, rhm=0.0, dens=None, sigmar=None, const_lnLambda=False, minr=0.0001, maxr=25.0, nr=501, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a Chandrasekhar Dynamical Friction force

INPUT:

amp - amplitude to be applied to the potential (default: 1)

GMs - satellite mass; can be a Quantity with units of mass or Gxmass; can be adjusted after initialization by setting obj.GMs= where obj is your ChandrasekharDynamicalFrictionForce instance (note that the mass of the satellite can *not* be changed simply by multiplying the instance by a number, because he mass is not only used as an amplitude)

rhm - half-mass radius of the satellite (set to zero for a black hole; can be a Quantity); can be adjusted after initialization by setting obj.rhm= where obj is your ChandrasekharDynamicalFrictionForce instance

```
gamma - Free-parameter in \Lambda
```

dens - Potential instance or list thereof that represents the density [default: LogarithmicHaloPotential(normalize=1.,q=1.)]

sigmar= (None) function that gives the velocity dispersion as a function of r (has to be in natural units!); if None, computed from the dens potential using the spherical Jeans equation (in galpy.df.jeans) assuming zero anisotropy; if set to a lambda function, *the object cannot be pick-led* (so set it to a real function)

cont_lnLambda= (False) if set to a number, use a constant ln(Lambda) instead with this value

minr= (0.0001) minimum r at which to apply dynamical friction: at r < minr, friction is set to zero (can be a Quantity)

Interpolation:

maxr= (25) maximum r for which sigmar gets interpolated; for best performance set this to the maximum r you will consider (can be a Quantity)

nr= (501) number of radii to use in the interpolation of sigmar

You can check that sigmar is interpolated correctly by comparing the methods sigmar [the interpolated version] and sigmar_orig [the original or directly computed version]

OUTPUT:

(none)

HISTORY:

```
2011-12-26 - Started - Bovy (NYU)
```

2018-03-18 - Re-started: updated to r dependent Lambda form and integrated into galpy framework - Bovy (UofT)

2018-07-23 - Calculate sigmar from the Jeans equation and interpolate it; allow GMs and rhm to be set on the fly - Bovy (UofT)

Helper classes

Mixin to compute forces and second potential derivatives numerically

class galpy.potential.NumericalPotentialDerivativesMixin(kwargs)

Mixin to add numerical derivatives to a Potential class, use as, e.g.,

```
class PotWithNumericalDerivs (Potential, NumericalPotentialDerivativesMixin):
    def __init__ (self, *args, **kwargs):
        NumericalPotentialDerivativesMixin.__init__ (self, kwargs) # *not* **kwargs!
        # Remainder of initialization
        ...

def __evaluate(self,R,z,phi=0.,t=0.):
    # Evaluate the potential

# All forces and second derivatives then computed by___
        NumericalPotentialDerivativesMixin
```

to add numerical derivatives to a new potential class PotWithNumericalDerivs that only implements the potential itself, but not the forces. The class may implement any of the forces or second derivatives, all non-implemented forces/second-derivatives will be computed numerically by adding this Mixin

The step used to compute the first (force) and second derivatives can be controlled at object instantiation by the keyword arguments dR, dz, dphi (for the forces; 1e-8 default) and dR2, dz2, and dphi2 (for the second derivatives; 1e-4 default)

```
__init__(kwargs)
Initialize self. See help(type(self)) for accurate signature.
```

2.2.2 Milky-Way-like potentials

galpy contains various simple models for the Milky Way's gravitational potential. The recommended model, described in Bovy (2015), is included as galpy.potential.MWPotential2014. This potential was fit to a large variety of data on the Milky Way and thus serves as both a simple and accurate model for the Milky Way's potential (see Bovy 2015 for full information on how this potential was fit). Note that this potential assumes a circular velocity of 220 km/s at the solar radius at 8 kpc. This potential is defined as

```
>>> bp= PowerSphericalPotentialwCutoff(alpha=1.8,rc=1.9/8.,normalize=0.05)
>>> mp= MiyamotoNagaiPotential(a=3./8.,b=0.28/8.,normalize=.6)
>>> np= NFWPotential(a=16/8.,normalize=.35)
>>> MWPotential2014= bp+mp+np
```

and can thus be used like any list of Potentials. The mass of the dark-matter halo in MWPotential2014 is on the low side of estimates of the Milky Way's halo mass; if you want to adjust it, for example making it 50% larger, you can simply multiply the halo part of MWPotential2014 by 1.5 as (this type of multiplication works for *any* potential in galpy)

```
>>> MWPotential2014[2] *= 1.5
```

If one wants to add the supermassive black hole at the Galactic center, this can be done by

for a black hole with a mass of $4 \times 10^6 \, M_\odot$. If you want to take into account dynamical friction for, say, an object of mass $5 \times 10^{10} \, M_\odot$ and a half-mass radius of 5 kpc, do

where we have specified the parameters of the dynamical friction with units; alternatively, convert them directly to galpy natural units as

```
>>> cdf= ChandrasekharDynamicalFrictionForce(GMs=5.*10.**10./bovy_conversion.mass_in_

shipping msol(220.,8.),

rhm=5./8.,

dens=MWPotential2014)

>>> MWPotential2014wDF= MWPotential2014+cdf
```

As explained in *this section*, *without* this black hole or dynamical friction, MWPotential2014 can be used with Dehnen's gyrfalcON code using accname=PowSphwCut+MiyamotoNagai+NFW and accpars=0,1001. 79126907,1.8,1.9#0,306770.418682,3.0,0.28#0,16.0,162.958241887.

galpy also contains other models for the Milky Way's potential from the literature in the galpy.potential. mwpotentials module (which also contains MWPotential2014). Currently, these are:

- McMillan17: the potential model from McMillan (2017)
- Irrgang13I: model I from Irrgang et al. (2013), which is an updated version of the classic Allen & Santillan (1991)
- Irrgang13II and Irrgang13III: model II and III from Irrgang et al. (2013)
- DehnenBinney98I, DehnenBinney98II, DehnenBinney98III, and DehnenBinney98IV for models 1 through 4 from Dehnen & Binney (1998).

Unlike MWPotential2014, these potentials have physical units turned on, using as the unit scaling parameters ro and vo the distance to the Galactic center and the circular velocity at the Sun's radius of each potential. These can be obtained using the galpy.util.bovy_conversion.get_physical function, e.g.,

```
>>> from galpy.potential.mwpotentials import McMillan17
>>> from galpy.util.bovy_conversion import get_physical
>>> get_physical(McMillan17)
# {'ro': 8.21, 'vo': 233.1}
```

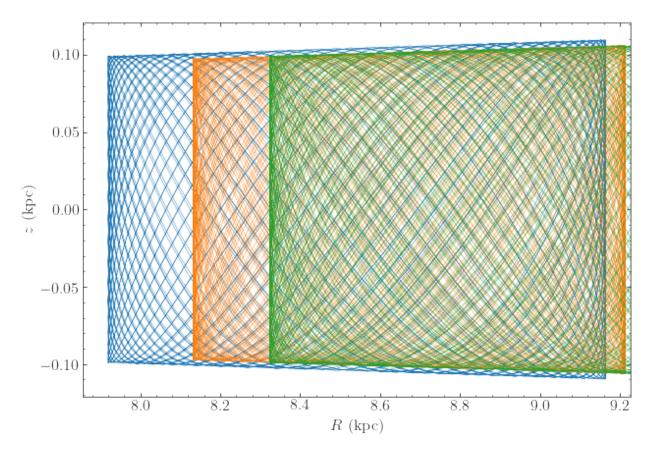
This function returns the unit-conversion parameters as a dictionary, so they can be easily passed to other functions. For example, when integrating an orbit in these potentials and either initializing the orbit using observed coordinates or converting the integrated orbit to observed coordinates, it is important to use the same unit-conversion parameters (otherwise an error will be raised). For example, to obtain the orbit of the Sun in the McMillan17 potential, we do

```
>>> from galpy.orbit import Orbit
>>> o= Orbit(**get_physical(McMillan17))
```

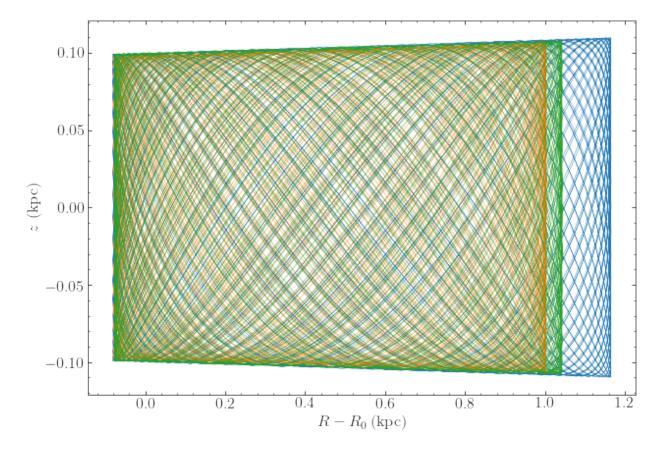
As an example, we integrate the Sun's orbit for 10 Gyr in MWPotential2014, McMillan17 and Irrgang13I

```
>>> from galpy.potential.mwpotentials import MWPotential2014, McMillan17, Irrgang13I
>>> from galpy.orbit import Orbit
>>> from galpy.util.bovy_conversion import get_physical
>>> from astropy import units
>>> times= numpy.linspace(0.,10.,3001)*units.Gyr
>>> o_mwp14= Orbit(ro=8.,vo=220.) # Need to set these by hand
>>> o_mcm17= Orbit(**get_physical(McMillan17))
>>> o_irrI= Orbit(**get_physical(Irrgang13I))
>>> o_mwp14.integrate(times,MWPotential2014)
>>> o_mcm17.integrate(times,McMillan17)
>>> o_irrI.integrate(times,Irrgang13I)
>>> o_mwp14.plot(lw=0.6)
>>> o_mcm17.plot(overplot=True,lw=0.6)
>>> o_irrI.plot(overplot=True,lw=0.6)
```

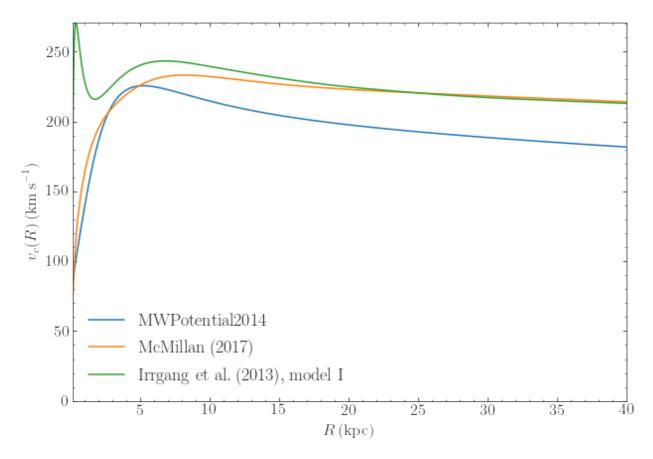
which gives



Much of the difference between these orbits is due to the different present Galactocentric radius of the Sun, if we simply plot the difference with respect to the present Galactocentric radius, they agree better



We can also compare the rotation curves of these different models



An older version galpy.potential.MWPotential of MWPotential2014 that was not fit to data on the Milky Way is defined as

```
>>> mp= MiyamotoNagaiPotential(a=0.5,b=0.0375,normalize=.6)
>>> np= NFWPotential(a=4.5,normalize=.35)
>>> hp= HernquistPotential(a=0.6/8,normalize=0.05)
>>> MWPotential= mp+np+hp
```

but galpy.potential.MWPotential2014 supersedes galpy.potential.MWPotential and its use is no longer recommended.

2.2.3 2D potentials

General instance routines

Use as Potential-instance.method(...)

galpy.potential.planarPotential.__add__

```
planarPotential.__add__(b)
NAME:
__add__
PURPOSE:
```

```
Add planarPotential instances together to create a multi-component potential (e.g., pot=
          pot1+pot2+pot3)
     INPUT:
          b - planarPotential instance or a list thereof
     OUTPUT:
          List of planarPotential instances that represents the combined potential
     HISTORY:
          2019-01-27 - Written - Bovy (UofT)
galpy.potential.planarPotential.__mul__
NAME:
          __mul__
     PURPOSE:
          Multiply a planarPotential's amplitude by a number
     INPUT:
          b - number
     OUTPUT:
          New instance with amplitude = (old amplitude) x b
     HISTORY:
          2019-01-27 - Written - Bovy (UofT)
galpy.potential.planarPotential.__call__
planarPotential. __call__(R, phi=0.0, t=0.0, dR=0, dphi=0)
     NAME:
          __call__
     PURPOSE:
          evaluate the potential
     INPUT:
          R - Cylindrica radius (can be Quantity)
          phi= azimuth (optional; can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          Phi(R(,phi,t)))
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
```

galpy.potential.planarPotential.phiforce

```
planarPotential.phiforce (R, phi=0.0, t=0.0)
          phiforce
     PURPOSE:
          evaluate the phi force
     INPUT:
          R - Cylindrical radius (can be Quantity)
          phi= azimuth (optional; can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F_{phi}(R,(phi,t))
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarPotential.Rforce
planarPotential.Rforce (R, phi=0.0, t=0.0)
     NAME:
          Rforce
     PURPOSE:
          evaluate the radial force
     INPUT:
          R - Cylindrical radius (can be Quantity)
          phi= azimuth (optional; can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F_R(R,(phi,t))
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarPotential.turn physical off
planarPotential.turn_physical_off()
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
```

```
INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
galpy.potential.planarPotential.turn physical on
planarPotential.turn_physical_on(ro=None, vo=None)
     NAME:
          turn_physical_on
     PURPOSE:
          turn on automatic returning of outputs in physical units
     INPUT:
          ro= reference distance (kpc; can be Quantity)
          vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
          2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)
General axisymmetric potential instance routines
Use as Potential-instance.method(...)
galpy.potential.planarAxiPotential.epifreq
Potential.epifreq(R, t=0.0)
     NAME:
          epifreq
     PURPOSE:
          calculate the epicycle frequency at R in this potential
     INPUT:
          R - Galactocentric radius (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          epicycle frequency
```

```
HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.lindbladR
Potential.lindbladR(OmegaP, m=2, t=0.0, **kwargs)
     NAME:
          lindbladR
     PURPOSE:
          calculate the radius of a Lindblad resonance
     INPUT:
          OmegaP - pattern speed (can be Quantity)
          m= order of the resonance (as in m(O-Op)=kappa (negative m for outer) use
                                                                                       m='corotation'
              for corotation +scipy.optimize.brentq xtol,rtol,maxiter kwargs
          t - time (optional; can be Quantity)
     OUTPUT:
          radius of Linblad resonance, None if there is no resonance
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.omegac
Potential.omegac (R, t=0.0)
     NAME:
          omegac
     PURPOSE:
          calculate the circular angular speed at R in this potential
     INPUT:
          R - Galactocentric radius (can be Quantity)
          t - time (optional; can be Quantity)
     OUTPUT:
          circular angular speed
     HISTORY:
          2011-10-09 - Written - Bovy (IAS)
galpy.potential.planarAxiPotential.plot
planarAxiPotential.plot(*args, **kwargs)
     NAME: plot
```

```
PURPOSE: plot the potential
     INPUT: Rrange - range (can be Quantity) grid - number of points to plot savefilename - save to or restore from
          this savefile (pickle) +bovy_plot(*args,**kwargs)
     OUTPUT: plot to output device
     HISTORY: 2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarAxiPotential.plotEscapecurve
planarAxiPotential.plotEscapecurve(*args, **kwargs)
     NAME:
          plotEscapecurve
     PURPOSE:
          plot the escape velocity curve for this potential
     INPUT:
          Rrange - range (can be Quantity)
          grid - number of points to plot
          savefilename - save to or restore from this savefile (pickle)
          +bovy_plot(*args,**kwargs)
     OUTPUT:
          plot to output device
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.planarAxiPotential.plotRotcurve
planarAxiPotential.plotRotcurve(*args, **kwargs)
     NAME:
          plotRotcurve
     PURPOSE:
          plot the rotation curve for this potential
     INPUT:
          Rrange - range (can be Quantity)
          grid - number of points to plot
          savefilename - save to or restore from this savefile (pickle)
          +bovy_plot(*args,**kwargs)
     OUTPUT:
          plot to output device
     HISTORY:
```

```
2010-07-13 - Written - Bovy (NYU)
```

galpy.potential.planarAxiPotential.vcirc

```
Potential.veire (R, phi=None, t=0.0)

NAME:
vcirc

PURPOSE:
calculate the circular velocity at R in this potential

INPUT:
R - Galactocentric radius (can be Quantity)
phi= (None) azimuth to use for non-axisymmetric potentials
t - time (optional; can be Quantity)

OUTPUT:
circular rotation velocity

HISTORY:
2011-10-09 - Written - Bovy (IAS)

2016-06-15 - Added phi= keyword for non-axisymmetric potential - Bovy (UofT)
```

galpy.potential.planarAxiPotential.vesc

```
Potential.vesc (R, t=0.0)

NAME:

vesc

PURPOSE:

calculate the escape velocity at R for this potential INPUT:

R - Galactocentric radius (can be Quantity)

t - time (optional; can be Quantity)

OUTPUT:

escape velocity

HISTORY:

2011-10-09 - Written - Bovy (IAS)
```

General 2D potential routines

```
Use as method (...)
```

galpy.potential.evaluateplanarphiforces

```
galpy.potential.evaluateplanarphiforces(Pot, R, phi=None, t=0.0)
     NAME:
           evaluateplanarphiforces
     PURPOSE:
          evaluate the phiforce of a (list of) planarPotential instance(s)
     INPUT:
          Pot - (list of) planarPotential instance(s)
          R - Cylindrical radius (can be Quantity)
          phi= azimuth (optional; can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F_{phi}(R(,phi,t))
     HISTORY:
           2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluateplanarPotentials
galpy.potential.evaluateplanarPotentials(Pot, R, phi=None, t=0.0, dR=0, dphi=0)
     NAME:
          evaluateplanarPotentials
     PURPOSE:
          evaluate a (list of) planarPotential instance(s)
     INPUT:
           Pot - (list of) planarPotential instance(s)
           R - Cylindrical radius (can be Quantity)
           phi= azimuth (optional; can be Quantity)
           t= time (optional; can be Quantity)
           dR=, dphi= if set to non-zero integers, return the dR,dphi't derivative instead
     OUTPUT:
          Phi(R(,phi,t))
     HISTORY:
           2010-07-13 - Written - Bovy (NYU)
```

galpy.potential.evaluateplanarRforces

```
galpy.potential.evaluateplanarRforces(Pot, R, phi=None, t=0.0)
          evaluateplanarRforces
     PURPOSE:
          evaluate the Rforce of a (list of) planarPotential instance(s)
     INPUT:
          Pot - (list of) planarPotential instance(s)
          R - Cylindrical radius (can be Quantity)
          phi= azimuth (optional can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F_R(R(,phi,t))
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluateplanarR2derivs
galpy.potential.evaluateplanarR2derivs(Pot, R, phi=None, t=0.0)
     NAME:
          evaluateplanarR2derivs
     PURPOSE:
          evaluate the second radial derivative of a (list of) planarPotential instance(s)
     INPUT:
          Pot - (list of) planarPotential instance(s)
          R - Cylindrical radius (can be Quantity)
          phi= azimuth (optional; can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F_R(R(,phi,t))
     HISTORY:
          2010-10-09 - Written - Bovy (IAS)
galpy.potential.LinShuReductionFactor
galpy.potential.LinShuReductionFactor(axiPot, R, sigmar, nonaxiPot=None, k=None,
                                                   m=None, OmegaP=None)
     NAME:
```

```
LinShuReductionFactor
     PURPOSE:
           Calculate the Lin & Shu (1966) reduction factor: the reduced linear response of a kinematically-warm
           stellar disk to a perturbation
     INPUT:
           axiPot - The background, axisymmetric potential
           R - Cylindrical radius (can be Quantity)
           sigmar - radial velocity dispersion of the population (can be Quantity)
           Then either provide:
            1) m= m in the perturbation's m x phi (number of arms for a spiral)
               k= wavenumber (see Binney & Tremaine 2008)
               OmegaP= pattern speed (can be Quantity)
            2) nonaxiPot= a non-axisymmetric Potential instance (such as SteadyLogSpiralPotential) that has
               functions that return OmegaP, m, and wavenumber
     OUTPUT:
           reduction factor
     HISTORY:
           2014-08-23 - Written - Bovy (IAS)
galpy.potential.plotplanarPotentials
galpy.potential.plotplanarPotentials (Pot, *args, **kwargs)
     NAME:
           plotplanarPotentials
     PURPOSE:
           plot a planar potential
     INPUT:
           Rrange - range (can be Quantity)
           xrange, yrange - if relevant (can be Quantity)
           grid, gridx, gridy - number of points to plot
           savefilename - save to or restore from this savefile (pickle)
           ncontours - number of contours to plot (if applicable)
           +bovy_plot(*args,**kwargs) or bovy_dens2d(**kwargs)
     OUTPUT:
```

plot to output device

2010-07-13 - Written - Bovy (NYU)

HISTORY:

Specific potentials

All of the 3D potentials above can be used as two-dimensional potentials in the mid-plane.

galpy.potential.toPlanarPotential

```
galpy.potential.toPlanarPotential(Pot)
     NAME:
          toPlanarPotential
     PURPOSE:
          convert an Potential to a planarPotential in the mid-plane (z=0)
     INPUT:
          Pot - Potential instance or list of such instances (existing planarPotential instances are just copied to
          the output)
     OUTPUT:
          planarPotential instance(s)
     HISTORY:
          2016-06-11 - Written - Bovy (UofT)
galpy.potential.RZToplanarPotential
galpy.potential.RZToplanarPotential(RZPot)
     NAME:
          RZToplanarPotential
     PURPOSE:
          convert an RZPotential to a planarPotential in the mid-plane (z=0)
     INPUT:
          RZPot - RZPotential instance or list of such instances (existing planarPotential instances are just
          copied to the output)
     OUTPUT:
```

In addition, a two-dimensional bar potential, two spiral potentials, the Henon & Heiles (1964) potential, and some static non-axisymmetric perturbations are included

Cos(m phi) disk potential

HISTORY:

Generalization of the *lopsided* and *elliptical* disk potentials to any m and to allow for a break radius within which the radial dependence of the potential changes from R^p to R^-p.

planarPotential instance(s)

2010-07-13 - Written - Bovy (NYU)

class galpy.potential.CosmphiDiskPotential (amp=1.0, phib=0.4363323129985824, p=1.0, phio=0.01, m=4, r1=1.0, rb=None, cp=None, sp=None, ro=None, vo=None)

Class that implements the disk potential

$$\Phi(R,\phi) = \operatorname{amp} \phi_0 \ \cos\left[m\left(\phi - \phi_b\right)\right] \times \left\{ \begin{bmatrix} \left(\frac{R}{R_1}\right)^p, & \text{for } R \ge R_b \\ \left[2 - \left(\frac{R_b}{R}\right)^p\right] \times \left(\frac{R_b}{R_1}\right)^p, & \text{for } R \le R_b. \end{bmatrix} \right\}$$

This potential can be grown between t_{form} and $t_{\text{form}} + T_{\text{steady}}$ in a similar way as DehnenBarPotential by wrapping it with a DehnenSmoothWrapperPotential

__init__ (amp=1.0, phib=0.4363323129985824, p=1.0, phio=0.01, m=4, r1=1.0, rb=None, cp=None, sp=None, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize an cosmphi disk potential

INPUT:

amp= amplitude to be applied to the potential (default: 1.), degenerate with phio below, but kept for overall consistency with potentials

m= cos(m * (phi - phib)), integer

p= power-law index of the $phi(R) = (R/Ro)^p$ part

r1=(1.) normalization radius for the amplitude (can be Quantity); amp x phio is only the potential at (R,phi) = (r1,pib) when r1 > rb; otherwise more complicated

rb= (None) if set, break radius for power-law: potential R^p at R > Rb, R^p at R < Rb, potential and force continuous at Rb

Either:

- a) phib= angle (in rad; default=25 degree; or can be Quantity)
 phio= potential perturbation (in terms of phio/vo^2 if vo=1 at Ro=1; or can be Quantity with units of velocity-squared)
- b) cp, sp= m * phio * cos(m * phib), m * phio * sin(m * phib); can be Quantity with units of velocity-squared)

OUTPUT:

(none)

HISTORY:

2011-10-27 - Started - Bovy (IAS)

2017-09-16 - Added break radius rb - Bovy (UofT)

Elliptical disk potential

Like in Kuijken & Tremaine. See *galpy.potential.CosmphiDiskPotential* for a more general version that allows for a break radius within which the radial dependence of the potential changes from R^p to R^-p (elliptical disk corresponds to m=2).

class galpy.potential.EllipticalDiskPotential (amp=1.0, phib=0.4363323129985824, p=1.0, twophio=0.01, r1=1.0, tform=None, tsteady=None, cp=None, sp=None, ro=None, vo=None)

Class that implements the Elliptical disk potential of Kuijken & Tremaine (1994)

$$\Phi(R,\phi) = \operatorname{amp} \phi_0 \left(\frac{R}{R_1}\right)^p \cos\left(2\left(\phi - \phi_b\right)\right)$$

This potential can be grown between t_{form} and $t_{\text{form}} + T_{\text{steady}}$ in a similar way as DehnenBarPotential, but times are given directly in galpy time units

__init__ (amp=1.0, phib=0.4363323129985824, p=1.0, twophio=0.01, r1=1.0, tform=None, tsteady=None, cp=None, sp=None, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize an Elliptical disk potential

 $phi(R,phi) = phio (R/Ro)^p cos[2(phi-phib)]$

INPUT:

amp= amplitude to be applied to the potential (default: 1.), see twophio below

tform= start of growth (to smoothly grow this potential (can be Quantity)

tsteady= time delay at which the perturbation is fully grown (default: 2.; can be Quantity)

p= power-law index of the $phi(R) = (R/Ro)^p$ part

r1=(1.) normalization radius for the amplitude (can be Quantity)

Either:

- a) phib= angle (in rad; default=25 degree; or can be Quantity)
 twophio= potential perturbation (in terms of 2phio/vo^2 if vo=1 at Ro=1; can be Quantity with units of velocity-squared)
- b) cp, sp= twophio * cos(2phib), twophio * sin(2phib) (can be Quantity with units of velocity-squared)

OUTPUT:

(none)

HISTORY:

2011-10-19 - Started - Bovy (IAS)

Henon-Heiles potential

class galpy.potential.**HenonHeilesPotential** (amp=1.0, ro=None, vo=None) Class that implements a the Henon & Heiles (1964) potential

$$\Phi(R,\phi) = \frac{\text{amp}}{2} \left[R^2 + \frac{2R^3}{3} \sin(3\phi) \right]$$

```
init__ (amp=1.0, ro=None, vo=None)

NAME:
    __init__

PURPOSE:
    initialize a Henon-Heiles potential

INPUT:
    amp - amplitude to be applied to the potential (default: 1.)

OUTPUT:
    (none)

HISTORY:
    2017-10-16 - Written - Bovy (UofT)
```

Lopsided disk potential

Like in Kuijken & Tremaine, but for m=1. See *galpy.potential.CosmphiDiskPotential* for a more general version that allows for a break radius within which the radial dependence of the potential changes from R^p to R^-p (lopsided disk corresponds to m=1).

```
class galpy.potential.LopsidedDiskPotential (amp=1.0, phib=0.4363323129985824, p=1.0, phio=0.01, r1=1.0, cp=None, sp=None, ro=None, vo=None)
```

Class that implements the disk potential

$$\Phi(R,\phi) = \operatorname{amp} \phi_0 \left(\frac{R}{R_1}\right)^p \cos\left(\phi - \phi_b\right)$$

Special case of CosmphiDiskPotential with m=1; see documentation for CosmphiDiskPotential

```
__init__ (amp=1.0, phib=0.4363323129985824, p=1.0, phio=0.01, r1=1.0, cp=None, sp=None, ro=None, vo=None)

NAME:
```

___init___

PURPOSE:

initialize an cosmphi disk potential

INPUT:

amp= amplitude to be applied to the potential (default: 1.), degenerate with phio below, but kept for overall consistency with potentials

m= cos(m * (phi - phib)), integer

p= power-law index of the phi(R) = (R/Ro)^p part

r1=(1.) normalization radius for the amplitude (can be Quantity); amp x phio is only the potential at (R,phi) = (r1,pib) when r1 > rb; otherwise more complicated

rb= (None) if set, break radius for power-law: potential R^p at R > Rb, R^p at R < Rb, potential and force continuous at Rb

Either:

- a) phib= angle (in rad; default=25 degree; or can be Quantity)
 - phio= potential perturbation (in terms of phio/vo^2 if vo=1 at Ro=1; or can be Quantity with units of velocity-squared)
- b) cp, sp= m * phio * cos(m * phib), m * phio * sin(m * phib); can be Quantity with units of velocity-squared)

OUTPUT:

(none)

HISTORY:

2011-10-27 - Started - Bovy (IAS)

2017-09-16 - Added break radius rb - Bovy (UofT)

Steady-state logarithmic spiral potential

class galpy.potential.SteadyLogSpiralPotential (amp=1.0, omegas=0.65, A=-0.035, alpha=-7.0, m=2, gamma=0.7853981633974483, p=None, tform=None, tsteady=None, ro=None, vo=None)

Class that implements a steady-state spiral potential

$$\Phi(R,\phi) = \frac{\text{amp} \times A}{\alpha} \cos(\alpha \ln R - m(\phi - \Omega_s t - \gamma))$$

Can be grown in a similar way as the DehnenBarPotential, but using $T_s = 2\pi/\Omega_s$ to normalize $t_{\rm form}$ and $T_{\rm steady}$.

__init__ (amp=1.0, omegas=0.65, A=-0.035, alpha=-7.0, m=2, gamma=0.7853981633974483, p=None, tform=None, tsteady=None, ro=None, vo=None)

NAME:

__init__

PURPOSE:

initialize a logarithmic spiral potential

INPUT:

amp - amplitude to be applied to the potential (default: 1., A below)

gamma - angle between sun-GC line and the line connecting the peak of the spiral pattern at the Solar radius (in rad; default=45 degree; or can be Quantity)

A - amplitude (alpha*potential-amplitude; default=0.035; can be Quantity

omegas= - pattern speed (default=0.65; can be Quantity)

m= number of arms

Either provide:

- a) alpha=
- b) p= pitch angle (rad; can be Quantity)

tform - start of spiral growth / spiral period (default: -Infinity)

tsteady - time from tform at which the spiral is fully grown / spiral period (default: 2 periods)

OUTPUT:

(none)

HISTORY:

2011-03-27 - Started - Bovy (NYU)

Transient logarithmic spiral potential

class galpy.potential.TransientLogSpiralPotential (amp=1.0, omegas=0.65, A=-0.035, alpha=-7.0, m=2, gamma=0.7853981633974483, p=None, sigma=1.0, to=0.0, ro=None, vo=None)

Class that implements a steady-state spiral potential

$$\Phi(R,\phi) = \frac{\operatorname{amp}(t)}{\alpha} \cos \left(\alpha \ln R - m \left(\phi - \Omega_s t - \gamma\right)\right)$$

where

$$amp(t) = amp \times A \exp\left(-\frac{[t - t_0]^2}{2\sigma^2}\right)$$

__init__ (amp=1.0, omegas=0.65, A=-0.035, alpha=-7.0, m=2, gamma=0.7853981633974483, p=None, sigma=1.0, to=0.0, ro=None, vo=None)

NAME:

___init__

PURPOSE:

initialize a transient logarithmic spiral potential localized around to

INPUT:

amp - amplitude to be applied to the potential (default: 1., A below)

gamma - angle between sun-GC line and the line connecting the peak of the spiral pattern at the Solar radius (in rad; default=45 degree; can be Quantity)

A - amplitude (alpha*potential-amplitude; default=0.035; can be Quantity)

omegas= - pattern speed (default=0.65; can be Quantity)

m= number of arms

to= time at which the spiral peaks (can be Quantity)

sigma= "spiral duration" (sigma in Gaussian amplitude; can be Quantity)

Either provide:

- a) alpha=
- b) p= pitch angle (rad; can be Quantity)

OUTPUT:

(none)

HISTORY:

2011-03-27 - Started - Bovy (NYU)

2.2.4 1D potentials

General instance routines

```
Use as Potential-instance.method(...)
galpy.potential.linearPotential. add
linearPotential._{add}(b)
     NAME:
          add
     PURPOSE:
          Add linearPotential instances together to create a multi-component potential (e.g., pot=
          pot1+pot2+pot3)
     INPUT:
          b - linearPotential instance or a list thereof
     OUTPUT:
          List of linearPotential instances that represents the combined potential
     HISTORY:
          2019-01-27 - Written - Bovy (UofT)
galpy.potential.linearPotential.__mul__
linearPotential._{\underline{\underline{}}}mul_{\underline{\underline{}}}(b)
     NAME:
          ___mul___
     PURPOSE:
          Multiply a linearPotential's amplitude by a number
     INPUT:
          b - number
     OUTPUT:
          New instance with amplitude = (old amplitude) x b
     HISTORY:
          2019-01-27 - Written - Bovy (UofT)
galpy.potential.linearPotential. call
linearPotential.__call__(x, t=0.0)
     NAME: __call__
     PURPOSE:
```

```
evaluate the potential
     INPUT:
          x - position (can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          Phi(x,t)
     HISTORY:
          2010-07-12 - Written - Bovy (NYU)
galpy.potential.linearPotential.force
linearPotential.force (x, t=0.0)
     NAME:
          force
     PURPOSE:
          evaluate the force
     INPUT:
          x - position (can be Quantity)
          t= time (optional; can be Quantity)
     OUTPUT:
          F(x,t)
     HISTORY:
          2010-07-12 - Written - Bovy (NYU)
galpy.potential.linearPotential.plot
linearPotential.plot(t=0.0, min=-15.0, max=15, ns=21, savefilename=None)
     NAME:
          plot
     PURPOSE:
          plot the potential
     INPUT:
          t - time to evaluate the potential at
          min - minimum x
          max - maximum x
          ns - grid in x
          savefilename - save to or restore from this savefile (pickle)
     OUTPUT:
```

```
plot to output device
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.linearPotential.turn_physical_off
linearPotential.turn_physical_off()
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
galpy.potential.linearPotential.turn_physical_on
linearPotential.turn_physical_on (ro=None, vo=None)
     NAME:
          turn_physical_on
     PURPOSE:
          turn on automatic returning of outputs in physical units
     INPUT:
          ro= reference distance (kpc; can be Quantity)
          vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
          (none)
     HISTORY:
          2016-01-30 - Written - Bovy (UofT)
          2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)
General 1D potential routines
Use as method (...)
```

galpy.potential.evaluatelinearForces

```
galpy.potential.evaluatelinearForces (Pot, x, t=0.0)
          evaluatelinearForces
     PURPOSE:
          evaluate the forces due to a list of potentials
     INPUT:
          Pot - (list of) linearPotential instance(s)
          x - evaluate forces at this position (can be Quantity)
          t - time to evaluate at (can be Quantity)
     OUTPUT:
          force(x,t)
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.evaluatelinearPotentials
galpy.potential.evaluatelinearPotentials (Pot, x, t=0.0)
     NAME:
          evaluatelinearPotentials
     PURPOSE:
          evaluate the sum of a list of potentials
     INPUT:
          Pot - (list of) linearPotential instance(s)
          x - evaluate potentials at this position (can be Quantity)
          t - time to evaluate at (can be Quantity)
     OUTPUT:
          pot(x,t)
     HISTORY:
          2010-07-13 - Written - Bovy (NYU)
galpy.potential.plotlinearPotentials
galpy.potential.plotlinearPotentials(Pot, t=0.0, min=-15.0, max=15, ns=21, savefile-
                                                  name=None)
     NAME:
          plotlinearPotentials
     PURPOSE:
```

```
plot a combination of potentials

INPUT:

t - time to evaluate potential at

min - minimum x

max - maximum x

ns - grid in x

savefilename - save to or restore from this savefile (pickle)

OUTPUT:

plot to output device

HISTORY:

2010-07-13 - Written - Bovy (NYU)
```

Specific potentials

Isothermal disk potential

class galpy.potential.IsothermalDiskPotential (amp=1.0, sigma=0.1, ro=None, vo=None)Class representing the one-dimensional self-gravitating isothermal disk

$$\rho(x) = \operatorname{amp} \operatorname{sech}^2\left(\frac{x}{2H}\right)$$

where the scale height $H^2 = \sigma^2/[8\pi G \, \mathrm{amp}]$. The parameter to setup the disk is the velocity dispersion σ .

Vertical Kuijken & Gilmore potential

class galpy.potential.KGPotential (K=1.15, F=0.03, D=1.8, amp=1.0, ro=None, vo=None) Class representing the Kuijken & Gilmore (1989) potential

$$\Phi(x) = \mathrm{amp}\,\left(K\,\left(\sqrt{x^2 + D^2} - D\right) + F\,x^2\right)$$

```
_init__ (K=1.15, F=0.03, D=1.8, amp=1.0, ro=None, vo=None)
   NAME:
        __init__
   PURPOSE:
        Initialize a KGPotential
   INPUT:
        K= K parameter (= 2\pi\Sigma_{\rm disk}; specify either as surface density or directly as force [i.e., including
        2\pi G]; can be Quantity)
        F= F parameter (= 4\pi\rho_{\rm halo}; specify either as density or directly as second potential derivative
        [i.e., including 4\pi G]; can be Quantity)
        D= D parameter (natural units or Quantity length units)
        amp - an overall amplitude
   OUTPUT:
        instance
   HISTORY:
        2010-07-12 - Written - Bovy (NYU)
```

One-dimensional potentials can also be derived from 3D axisymmetric potentials as the vertical potential at a certain Galactocentric radius

galpy.potential.toVerticalPotential

```
galpy.potential.toVerticalPotential(Pot, R, phi=None, t0=0.0)
      NAME:
           to Vertical Potential
      PURPOSE:
           convert a Potential to a vertical potential at a given R: Phi(z,phi,t) = Phi(R,z,phi,t)-Phi(R,0.,phi0,t0)
           where phi0 and t0 are the phi and t inputs
      INPUT:
           Pot - Potential instance or list of such instances
           R - Galactocentric radius at which to evaluate the vertical potential (can be Quantity)
           phi= (None) Galactocentric azimuth at which to evaluate the vertical potential (can be Quantity);
           required if Pot is non-axisymmetric
           t0=(0.) time at which to evaluate the vertical potential (can be Quantity)
      OUTPUT:
           (list of) linearPotential instance(s)
      HISTORY:
           2018-10-07 - Written - Bovy (UofT)
```

galpy.potential.RZToverticalPotential

galpy.potential.RZToverticalPotential(RZPot, R)

NAME:

RZToverticalPotential

PURPOSE:

convert a RZPotential to a vertical potential at a given R

INPUT:

RZPot - RZPotential instance or list of such instances

R - Galactocentric radius at which to evaluate the vertical potential (can be Quantity)

OUTPUT:

(list of) linearPotential instance(s)

HISTORY:

2010-07-21 - Written - Bovy (NYU)

2.2.5 Potential wrappers

Gravitational potentials in galpy can also be modified using wrappers, for example, to change their amplitude as a function of time. These wrappers can be applied to *any* galpy potential (although whether they can be used in C depends on whether the wrapper *and* all of the potentials that it wraps are implemented in C). Multiple wrappers can be applied to the same potential.

Specific wrappers

Corotating rotation wrapper potential

class galpy.potential.CorotatingRotationWrapperPotential (amp=1.0, pot=None, vpo=1.0, beta=0.0, to=0.0, pa=0.0, ro=None, vo=None)

Potential wrapper class that implements rotation with fixed R x pattern-speed around the z-axis. Can be used to make spiral structure that is everywhere co-rotating. The potential is rotated by replacing

$$\phi \to \phi + \frac{V_p(R)}{R} \times (t - t_0) + \text{pa}$$

with $V_p(R)$ the circular velocity curve, t_0 a reference time—time at which the potential is unchanged by the wrapper—and pa the position angle at t=0. The circular velocity is parameterized as

$$V_p(R) = V_{p,0} \left(\frac{R}{R_0}\right)^{\beta}.$$

__init__ (amp=1.0, pot=None, vpo=1.0, beta=0.0, to=0.0, pa=0.0, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a CorotatingRotationWrapper Potential

INPUT:

amp - amplitude to be applied to the potential (default: 1.)

pot - Potential instance or list thereof; this potential is made to rotate around the z axis by the wrapper

vpo= (1.) amplitude of the circular-velocity curve (can be a Quantity)

beta= (0.) power-law amplitude of the circular-velocity curve

to=(0.) reference time at which the potential == pot

pa= (0.) the position angle (can be a Quantity)

OUTPUT:

(none)

HISTORY:

2018-02-21 - Started - Bovy (UofT)

Dehnen-like smoothing wrapper potential

class galpy.potential.DehnenSmoothWrapperPotential(amp=1.0, pot=None, tform=-4.0,

tsteady=None, decay=False,

ro=None, vo=None)

Potential wrapper class that implements the growth of a gravitational potential following Dehnen (2000). The amplitude A applied to a potential wrapped by an instance of this class is changed as

$$A(t) = amp\left(\frac{3}{16}\xi^5 - \frac{5}{8}\xi^3 + \frac{15}{16}\xi + \frac{1}{2}\right)$$

where

$$\xi = \begin{cases} -1 & t < t_{\text{form}} \\ 2\left(\frac{t - t_{\text{form}}}{t_{\text{steady}}}\right) - 1, & t_{\text{form}} \le t \le t_{\text{form}} + t_{\text{steady}} \\ 1 & t > t_{\text{form}} + t_{\text{steady}} \end{cases}$$

if decay=True, the amplitude decays rather than grows as decay = 1 - grow.

___init___ (amp=1.0, pot=None, tform=-4.0, tsteady=None, decay=False, ro=None, vo=None)
NAME:

__init__

PURPOSE:

initialize a DehnenSmoothWrapper Potential

INPUT:

amp - amplitude to be applied to the potential (default: 1.)

pot - Potential instance or list thereof; the amplitude of this will be grown by this wrapper

tform - start of growth (can be a Quantity)

tsteady - time from tform at which the potential is fully grown (default: -tform/2, st the perturbation is fully grown at tform/2; can be a Quantity)

decay= (False) if True, decay the amplitude instead of growing it (as 1-grow)

OUTPUT:

(none)

HISTORY:

2017-06-26 - Started - Bovy (UofT)

2018-10-07 - Added 'decay' option - Bovy (UofT)

Gaussian-modulated amplitude wrapper potential

 $\begin{array}{ll} \textbf{class} \ \ \text{galpy.potential.} \textbf{GaussianAmplitudeWrapperPotential} \ (amp=1.0, \\ to=0.0, \end{array} \quad \begin{array}{ll} pot=None, \\ sigma=1.0, \end{array}$

ro=None, vo=None) Potential wrapper class that allows the amplitude of a Potential object to be modulated as a Gaussian. The

$$A(t) = amp \, \exp\left(-\frac{[t - t_0]^2}{2\,\sigma^2}\right)$$

__init__ (amp=1.0, pot=None, to=0.0, sigma=1.0, ro=None, vo=None)

amplitude A applied to a potential wrapped by an instance of this class is changed as

NAME:

__init__

PURPOSE:

initialize a GaussianAmplitudeWrapper Potential

INPUT:

amp - amplitude to be applied to the potential (default: 1.)

pot - Potential instance or list thereof; this potential is made to rotate around the z axis by the wrapper

to= (0.) time at which the Gaussian peaks

sigma= (1.) standard deviation of the Gaussian (can be a Quantity)

OUTPUT:

(none)

HISTORY:

2018-02-21 - Started - Bovy (UofT)

Solid-body rotation wrapper potential

class galpy.potential.SolidBodyRotationWrapperPotential(amp=1.0, pot=None, omega=1.0, pa=0.0,

ro=None, vo=None)

Potential wrapper class that implements solid-body rotation around the z-axis. Can be used to make a bar or other perturbation rotate. The potential is rotated by replacing

$$\phi \to \phi + \Omega \times t + pa$$

with Ω the fixed pattern speed and pa the position angle at t=0.

```
__init__ (amp=1.0, pot=None, omega=1.0, pa=0.0, ro=None, vo=None)

NAME:
    __init__
PURPOSE:
    initialize a SolidBodyRotationWrapper Potential
INPUT:
    amp - amplitude to be applied to the potential (default: 1.)
    pot - Potential instance or list thereof; this potential is made to rotate around the z axis by the wrapper
    omega= (1.) the pattern speed (can be a Quantity)
    pa= (0.) the position angle (can be a Quantity)

OUTPUT:
    (none)

HISTORY:
    2017-08-22 - Started - Bovy (UofT)
```

2.3 actionAngle (galpy.actionAngle)

2.3.1 $(x, v) \rightarrow (J, O, a)$

General instance routines

Not necessarily supported for all different types of actionAngle calculations. These have extra arguments for different actionAngle modules, so check the documentation of the module-specific functions for more info (e.g., ?actionAngleIsochrone.__call__)

galpy.actionAngle.actionAngle.__call__

```
actionAngle.__call__ (*args, **kwargs)
NAME:
    __call__
PURPOSE:
    evaluate the actions (jr,lz,jz)
INPUT:
    Either:
    a) R,vR,vT,z,vz[,phi]:
```

- 1) floats: phase-space value for single object (phi is optional) (each can be a Quantity)
- 2) numpy.ndarray: [N] phase-space values for N objects (each can be a Quantity)
- b) Orbit instance: initial condition used if that's it, orbit(t) if there is a time given as well as the second argument

```
OUTPUT:
           (jr,lz,jz)
     HISTORY:
           2014-01-03 - Written for top level - Bovy (IAS)
galpy.actionAngle.actionAngle.actionsFreqs
actionAngle.actionsFreqs(*args, **kwargs)
     NAME:
           actionsFreqs
     PURPOSE:
           evaluate the actions and frequencies (jr,lz,jz,Omegar,Omegaphi,Omegaz)
     INPUT:
           Either:
            a) R,vR,vT,z,vz[,phi]:
                1) floats: phase-space value for single object (phi is optional) (each can be a Quantity)
                2) numpy.ndarray: [N] phase-space values for N objects (each can be a Quantity)
            b) Orbit instance: initial condition used if that's it, orbit(t) if there is a time given as well as the
               second argument
     OUTPUT:
           (jr,lz,jz,Omegar,Omegaphi,Omegaz)
     HISTORY:
           2014-01-03 - Written for top level - Bovy (IAS)
galpy.actionAngle.actionAngle.actionsFreqsAngles
actionAngle.actionsFreqsAngles(*args, **kwargs)
     NAME:
          actionsFreqsAngles
     PURPOSE:
           evaluate the actions, frequencies, and angles (jr,lz,jz,Omegar,Omegaphi,Omegaz,angler,anglephi,anglez)
     INPUT:
           Either:
            a) R,vR,vT,z,vz,phi:
                1) floats: phase-space value for single object (phi is optional) (each can be a Quantity)
                2) numpy.ndarray: [N] phase-space values for N objects (each can be a Quantity)
            b) Orbit instance: initial condition used if that's it, orbit(t) if there is a time given as well as the
               second argument
     OUTPUT:
```

```
(jr,lz,jz,Omegar,Omegaphi,Omegaz,angler,anglephi,anglez)
     HISTORY:
          2014-01-03 - Written for top level - Bovy (IAS)
galpy.actionAngle.actionAngle.EccZmaxRperiRap
actionAngle.EccZmaxRperiRap(*args, **kwargs)
     NAME:
          EccZmaxRperiRap
     PURPOSE:
          evaluate the eccentricity, maximum height above the plane, peri- and apocenter
     INPUT:
          Either:
            a) R,vR,vT,z,vz[,phi]:
                1) floats: phase-space value for single object (phi is optional) (each can be a Quantity)
                2) numpy.ndarray: [N] phase-space values for N objects (each can be a Quantity)
           b) Orbit instance: initial condition used if that's it, orbit(t) if there is a time given as well as the
              second argument
     OUTPUT:
          (e,zmax,rperi,rap)
     HISTORY:
          2017-12-12 - Written - Bovy (UofT)
galpy.actionAngle.actionAngle.turn_physical_off
actionAngle.turn_physical_off()
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2017-06-05 - Written - Bovy (UofT)
```

galpy.actionAngle.actionAngle.turn_physical_on

```
actionAngle.turn_physical_on (ro=None, vo=None)

NAME:

turn_physical_on

PURPOSE:

turn on automatic returning of outputs in physical units

INPUT:

ro= reference distance (kpc; can be Quantity)

vo= reference velocity (km/s; can be Quantity)

OUTPUT:

(none)

HISTORY:

2016-06-05 - Written - Bovy (UofT)

2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)

Specific actionAngle modules

actionAnglelsochrone

class galpy.actionAngle.actionAngleIsochrone (*args, **kwargs)

Action-angle formalism for the isochrone potential, on the Jphi, Jtheta system of Binney & init (*args **kwargs)
```

```
class galpy.actionAngle.actionAngleIsochrone (*args, **kwargs)

Action-angle formalism for the isochrone potential, on the Jphi, Jtheta system of Binney & Tremaine (2008)

__init___(*args, **kwargs)

NAME: __init__

PURPOSE: initialize an actionAngleIsochrone object

INPUT: Either:

b= scale parameter of the isochrone parameter (can be Quantity)

ip= instance of a IsochronePotential

ro= distance from vantage point to GC (kpc; can be Quantity)

vo= circular velocity at ro (km/s; can be Quantity)

OUTPUT:

instance

HISTORY: 2013-09-08 - Written - Bovy (IAS)
```

_init___(*args, **kwargs)

Action-angle formalism for spherical potentials

class galpy.actionAngle.actionAngleSpherical(*args, **kwargs)

actionAngleSpherical

NAME:

```
__init__
          PURPOSE:
              initialize an actionAngleSpherical object
          INPUT:
              pot= a Spherical potential
              ro= distance from vantage point to GC (kpc; can be Quantity)
              vo= circular velocity at ro (km/s; can be Quantity)
          OUTPUT:
              instance
          HISTORY:
              2013-12-28 - Written - Bovy (IAS)
actionAngleAdiabatic
class galpy.actionAngle.actionAngleAdiabatic(*args, **kwargs)
     Action-angle formalism for axisymmetric potentials using the adiabatic approximation
       init (*args, **kwargs)
          NAME:
               __init__
          PURPOSE:
              initialize an actionAngleAdiabatic object
          INPUT:
              pot= potential or list of potentials (planarPotentials)
              gamma= (default=1.) replace Lz by Lz+gamma Jz in effective potential
              ro= distance from vantage point to GC (kpc; can be Quantity)
              vo= circular velocity at ro (km/s; can be Quantity)
          OUTPUT:
              instance
          HISTORY:
              2012-07-26 - Written - Bovy (IAS@MPIA)
actionAngleAdiabaticGrid
class galpy.actionAngle.actionAngleAdiabaticGrid(pot=None, zmax=1.0, gamma=1.0,
                                                                  Rmax=5.0, nR=16, nEz=16, nEr=31,
                                                                  nLz=31, numcores=1, **kwargs)
     Action-angle formalism for axisymmetric potentials using the adiabatic approximation, grid-based interpolation
     __init__ (pot=None, zmax=1.0, gamma=1.0, Rmax=5.0, nR=16, nEz=16, nEr=31, nLz=31, num-
                 cores=1, **kwargs)
```

```
NAME: __init__
           PURPOSE: initialize an actionAngleAdiabaticGrid object
           INPUT:
               pot= potential or list of potentials
               zmax= zmax for building Ez grid
               Rmax = Rmax for building grids
               gamma= (default=1.) replace Lz by Lz+gamma Jz in effective potential
               nEz=, nEr=, nLz, nR= grid size
               numcores= number of cpus to use to parallellize
               c= if True, use C to calculate actions
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
               +scipy.integrate.quad keywords
           OUTPUT:
               instance
           HISTORY:
               2012-07-27 - Written - Bovy (IAS@MPIA)
actionAngleStaeckel
class galpy.actionAngle.actionAngleStaeckel(*args, **kwargs)
     Action-angle formalism for axisymmetric potentials using Binney (2012)'s Staeckel approximation
     ___init___(*args, **kwargs)
          NAME: __init__
           PURPOSE: initialize an actionAngleStaeckel object
           INPUT: pot= potential or list of potentials (3D)
               delta= focus (can be Quantity)
               useu0 - use u0 to calculate dV (NOT recommended)
               c= if True, always use C for calculations
               order= (10) number of points to use in the Gauss-Legendre numerical integration of the relevant
               action, frequency, and angle integrals
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
           OUTPUT:
               instance
           HISTORY:
               2012-11-27 - Written - Bovy (IAS)
```

actionAngleStaeckelGrid

```
class galpy.actionAngle.actionAngleStaeckelGrid(pot=None, delta=None, Rmax=5.0,
                                                                  nE=25,
                                                                          npsi=25, nLz=30,
                                                                  cores=1, interpecc=False, **kwargs)
     Action-angle formalism for axisymmetric potentials using Binney (2012)'s Staeckel approximation, grid-based
     interpolation
     __init__ (pot=None, delta=None, Rmax=5.0, nE=25, npsi=25, nLz=30, numcores=1, inter-
                 pecc=False, **kwargs)
          NAME: __init__
          PURPOSE: initialize an actionAngleStaeckelGrid object
          INPUT: pot= potential or list of potentials
               delta= focus of prolate confocal coordinate system (can be Quantity)
               Rmax = Rmax for building grids (natural units)
               nE=, npsi=, nLz= grid size
               interpecc= (False) if True, also interpolate the approximate eccentricity, zmax, rperi, and rapo
               numcores= number of cpus to use to parallellize
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
          OUTPUT:
               instance
          HISTORY:
               2012-11-29 - Written - Bovy (IAS)
               2017-12-15 - Written - Bovy (UofT)
actionAngleIsochroneApprox
class galpy.actionAngle.actionAngleIsochroneApprox(*args, **kwargs)
     Action-angle formalism using an isochrone potential as an approximate potential and using a Fox & Binney
     (2014?) like algorithm to calculate the actions using orbit integrations and a torus-machinery-like angle-fit to
     get the angles and frequencies (Bovy 2014)
      ___init___(*args, **kwargs)
          NAME: __init__
          PURPOSE: initialize an actionAngleIsochroneApprox object
          INPUT:
               Either:
                  b= scale parameter of the isochrone parameter (can be Quantity)
                  ip= instance of a IsochronePotential
                   aAI= instance of an actionAngleIsochrone
```

```
pot= potential to calculate action-angle variables for
tintJ= (default: 100) time to integrate orbits for to estimate actions (can be Quantity)
ntintJ= (default: 10000) number of time-integration points
integrate_method= (default: 'dopr54_c') integration method to use
dt= (None) orbit.integrate dt keyword (for fixed stepsize integration)
maxn= (default: 3) Default value for all methods when using a grid in vec(n) up to this n (zero-based)
ro= distance from vantage point to GC (kpc; can be Quantity)
vo= circular velocity at ro (km/s; can be Quantity)
OUTPUT:
instance
HISTORY: 2013-09-10 - Written - Bovy (IAS)
```

2.3.2 (J, a) \rightarrow (x, v, O)

General instance routines

Warning: While the actionAngleTorus code below can compute the Jacobian and Hessian of the $(J, a) \rightarrow (x, v, O)$ transformation, the accuracy of these does not appear to be very good using the current interface to the TorusMapper code, so care should be taken when using these.

Currently, only the interface to the Torus Mapper code supports going from $(J, a) \rightarrow (x, v, 0)$. Instance methods are

```
[R,vR,vT,z,vz,phi]
     HISTORY:
           2015-08-07 - Written - Bovy (UofT)
galpy.actionAngle.actionAngleTorus.Freqs
actionAngleTorus.Freqs (jr, jphi, jz, **kwargs)
     NAME:
          Freqs
     PURPOSE:
           return the frequencies corresponding to a torus
     INPUT:
          jr - radial action (scalar)
          jphi - azimuthal action (scalar)
          jz - vertical action (scalar)
          tol= (object-wide value) goal for |dJ|/|J| along the torus
     OUTPUT:
           (OmegaR,Omegaphi,Omegaz)
     HISTORY:
           2015-08-07 - Written - Bovy (UofT)
galpy.actionAngle.actionAngleTorus.hessianFreqs
actionAngleTorus.hessianFreqs(jr, jphi, jz, **kwargs)
     NAME:
          hessianFreqs
     PURPOSE:
           return the Hessian d Omega / d J and frequencies Omega corresponding to a torus
     INPUT:
          jr - radial action (scalar)
          jphi - azimuthal action (scalar)
          jz - vertical action (scalar)
          tol= (object-wide value) goal for |dJ|/|J| along the torus
           dJ= (object-wide value) action difference when computing derivatives (Hessian or Jacobian)
           nosym= (False) if True, don't explicitly symmetrize the Hessian (good to check errors)
     OUTPUT:
           (dO/dJ,Omegar,Omegaphi,Omegaz,Autofit error message)
     HISTORY:
```

```
2016-07-15 - Written - Bovy (UofT)
```

galpy.actionAngle.actionAngleTorus.xvFreqs

```
actionAngleTorus.xvFreqs(jr, jphi, jz, angler, anglephi, anglez, **kwargs)
      NAME:
           xvFreqs
      PURPOSE:
           evaluate the phase-space coordinates (x,v) for a number of angles on a single torus as well as the
           frequencies
      INPUT:
           jr - radial action (scalar)
           jphi - azimuthal action (scalar)
           iz - vertical action (scalar)
           angler - radial angle (array [N])
           anglephi - azimuthal angle (array [N])
           anglez - vertical angle (array [N])
           tol= (object-wide value) goal for |dJ|/|J| along the torus
      OUTPUT:
           ([R,vR,vT,z,vz,phi],OmegaR,Omegaphi,Omegaz,AutoFit error message)
      HISTORY:
           2015-08-07 - Written - Bovy (UofT)
galpy.actionAngle.actionAngleTorus.xvJacobianFregs
actionAngleTorus.xvJacobianFreqs(jr, jphi, jz, angler, anglephi, anglez, **kwargs)
      NAME:
           xvJacobianFreqs
      PURPOSE:
           return [R,vR,vT,z,vz,phi], the Jacobian d [R,vR,vT,z,vz,phi] / d (J,angle), the Hessian dO/dJ, and
           frequencies Omega corresponding to a torus at multiple sets of angles
      INPUT:
           jr - radial action (scalar)
           jphi - azimuthal action (scalar)
           jz - vertical action (scalar)
           angler - radial angle (array [N])
           anglephi - azimuthal angle (array [N])
           anglez - vertical angle (array [N])
           tol= (object-wide value) goal for |dJ|/|J| along the torus
```

```
dJ= (object-wide value) action difference when computing derivatives (Hessian or Jacobian)
           nosym= (False) if True, don't explicitly symmetrize the Hessian (good to check errors)
     OUTPUT:
           ([R,vR,vT,z,vz,phi], [N,6] array
               d[R,vR,vT,z,vz,phi]/d[J,angle], \rightarrow (N,6,6) array
               dO/dJ, -> (3,3) array
               Omegar, Omegaphi, Omegaz, [N] arrays
               Autofit error message)
     HISTORY:
           2016-07-19 - Written - Bovy (UofT)
Specific actionAngle modules
actionAngleTorus
class galpy.actionAngle.actionAngleTorus(*args, **kwargs)
     Action-angle formalism using the Torus machinery
       __init___(*args, **kwargs)
           NAME:
               __init__
           PURPOSE:
               initialize an actionAngleTorus object
           INPUT:
               pot= potential or list of potentials (3D)
               tol= default tolerance to use when fitting tori (|dJ|/J)
               dJ= default action difference when computing derivatives (Hessian or Jacobian)
           OUTPUT:
               instance
           HISTORY:
               2015-08-07 - Written - Bovy (UofT)
```

galpy.df contains tools for dealing with distribution functions of stars in galaxies. It mainly contains a number of classes that define different types of distribution function, but galpy.df.jeans also has some tools for solving the Jeans equations for equilibrium systems.

2.4.1 Jeans modeling tools (galpy.df.jeans)

```
galpy.df.jeans.sigmar
galpy.df.jeans.sigmar(Pot, r, dens=None, beta=0.0)
      NAME:
           sigmar
      PURPOSE:
           Compute the radial velocity dispersion using the spherical Jeans equation
      INPUT:
           Pot - potential or list of potentials (evaluated at R=r/sqrt(2),z=r/sqrt(2), sphericity not checked)
           r - Galactocentric radius (can be Quantity)
           dens= (None) tracer density profile (function of r); if None, the density is assumed to be that corre-
           sponding to the potential
           beta= (0.) anisotropy; can be a constant or a function of r
      OUTPUT:
           sigma_r(r)
      HISTORY:
           2018-07-05 - Written - Bovy (UofT)
galpy.df.jeans.sigmalos
qalpy.df.jeans.sigmalos(Pot, R, dens=None, surfdens=None, beta=0.0, sigma r=None)
      NAME:
           sigmalos
      PURPOSE:
           Compute the line-of-sight velocity dispersion using the spherical Jeans equation
      INPUT:
           Pot - potential or list of potentials (evaluated at R=r/sqrt(2),z=r/sqrt(2), sphericity not checked)
           R - Galactocentric projected radius (can be Quantity)
           dens= (None) tracer density profile (function of r); if None, the density is assumed to be that corre-
           sponding to the potential
           surfdens= (None) tracer surface density profile (value at R or function of R); if None, the surface
           density is assumed to be that corresponding to the density
           beta= (0.) anisotropy; can be a constant or a function of r
           sigma_r= (None) if given, the solution of the spherical Jeans equation sigma_r(r) (used instead of
           solving the Jeans equation as part of this routine)
      OUTPUT:
           sigma_los(R)
      HISTORY:
```

```
2018-08-27 - Written - Bovy (UofT)
```

2.4.2 General instance routines for all df classes

```
galpy. action Angle. action Angle. turn\_physical\_off
```

```
actionAngle.turn_physical_off()
     NAME:
          turn_physical_off
     PURPOSE:
          turn off automatic returning of outputs in physical units
     INPUT:
          (none)
     OUTPUT:
          (none)
     HISTORY:
          2017-06-05 - Written - Bovy (UofT)
galpy.actionAngle.actionAngle.turn_physical_on
actionAngle.turn_physical_on(ro=None, vo=None)
     NAME:
          turn_physical_on
     PURPOSE:
          turn on automatic returning of outputs in physical units
     INPUT:
          ro= reference distance (kpc; can be Quantity)
          vo= reference velocity (km/s; can be Quantity)
     OUTPUT:
          (none)
     HISTORY:
          2016-06-05 - Written - Bovy (UofT)
          2020-04-22 - Don't turn on a parameter when it is False - Bovy (UofT)
```

2.4.3 Two-dimensional, axisymmetric disk distribution functions

Distribution function for orbits in the plane of a galactic disk.

General instance routines

```
galpy.df.diskdf.__call__
diskdf.__call__(*args, **kwargs)
     NAME:
           call
     PURPOSE:
           evaluate the distribution function
     INPUT:
           either an orbit instance, a list of such instances, or E,Lz
            1) Orbit instance or list: a) Orbit instance alone: use initial condition b) Orbit instance + t: call the
               Orbit instance (for list, each instance is called at t)
            2) E - energy (/vo^2; or can be Quantity) L - angular momentun (/ro/vo; or can be Quantity)
            3) array vxvv [3/4,nt] [must be in natural units /vo,/ro; use Orbit interface for physical-unit input)
     KWARGS:
           marginalizeVperp - marginalize over perpendicular velocity (only supported with 1a) for single orbits
           marginalizeVlos - marginalize over line-of-sight velocity (only supported with 1a) for single orbits
           above)
           nsigma= number of sigma to integrate over when marginalizing
           +scipy.integrate.quad keywords
     OUTPUT:
           DF(orbit/E,L)
     HISTORY:
           2010-07-10 - Written - Bovy (NYU)
galpy.df.diskdf.asymmetricdrift
diskdf.asymmetricdrift(R)
     NAME:
           asymmetricdrift
     PURPOSE:
           estimate the asymmetric drift (vc-mean-vphi) from an approximation to the Jeans equation
     INPUT:
           R - radius at which to calculate the asymmetric drift (can be Quantity)
     OUTPUT:
           asymmetric drift at R
     HISTORY:
           2011-04-02 - Written - Bovy (NYU)
```

galpy.df.diskdf.kurtosisvR

```
diskdf.kurtosisvR(R, romberg=False, nsigma=None, phi=0.0)
          kurtosisvR
     PURPOSE:
          calculate excess kurtosis in vR at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vR> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          kurtosisvR
     HISTORY:
          2011-12-07 - Written - Bovy (NYU)
galpy.df.diskdf.kurtosisvT
diskdf.kurtosisvT(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          kurtosisvT
     PURPOSE:
          calculate excess kurtosis in vT at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vR> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          kurtosisvT
     HISTORY:
          2011-12-07 - Written - Bovy (NYU)
```

galpy.df.diskdf.meanvR

```
diskdf.meanvR(R, romberg=False, nsigma=None, phi=0.0)
          meanvR
     PURPOSE:
          calculate <vR> at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vR> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          \langle vR \rangle at R
     HISTORY:
          2011-03-30 - Written - Bovy (NYU)
galpy.df.diskdf.meanvT
diskdf.meanvT(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          meanvT
     PURPOSE:
          calculate <vT> at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vT> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          < vT > at R
     HISTORY:
          2011-03-30 - Written - Bovy (NYU)
```

galpy.df.diskdf.oortA

```
diskdf.oortA(R, romberg=False, nsigma=None, phi=0.0)
          oortA
     PURPOSE:
          calculate the Oort function A
     INPUT:
          R - radius at which to calculate A (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          Oort A at R
     HISTORY:
          2011-04-19 - Written - Bovy (NYU)
     BUGS:
          could be made more efficient, e.g., surfacemass is calculated multiple times
galpy.df.diskdf.oortB
diskdf.oortB(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          oortB
     PURPOSE:
          calculate the Oort function B
     INPUT:
          R - radius at which to calculate B (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          Oort B at R
     HISTORY:
          2011-04-19 - Written - Bovy (NYU)
     BUGS:
```

could be made more efficient, e.g., surfacemass is calculated multiple times

galpy.df.diskdf.oortC

```
diskdf.oortC(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
           oortC
     PURPOSE:
          calculate the Oort function C
     INPUT:
           R - radius at which to calculate C (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          Oort C at R
     HISTORY:
           2011-04-19 - Written - Bovy (NYU)
     BUGS:
           could be made more efficient, e.g., surfacemass is calculated multiple times we know this is zero, but
          it is calculated anyway (bug or feature?)
galpy.df.diskdf.oortK
diskdf.oortK(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          oortK
     PURPOSE:
          calculate the Oort function K
     INPUT:
           R - radius at which to calculate K (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
           romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
           Oort K at R
```

```
HISTORY:
          2011-04-19 - Written - Bovy (NYU)
     BUGS:
          could be made more efficient, e.g., surfacemass is calculated multiple times we know this is zero, but
          it is calculated anyway (bug or feature?)
galpy.df.diskdf.sigma2surfacemass
diskdf.sigma2surfacemass(R, romberg=False, nsigma=None, relative=False)
     NAME:
          sigma2surfacemass
     PURPOSE:
          calculate the product sigma_R^2 x surface-mass at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate the sigma_R^2 x surfacemass density (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          sigma_R^2 x surface-mass at R
     HISTORY:
          2010-03-XX - Written - Bovy (NYU)
galpy.df.diskdf.sigma2
diskdf.sigma2(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          sigma2
     PURPOSE:
          calculate sigma_R^2 at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate sigma_R^2 density (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
```

```
sigma_R^2 at R
     HISTORY:
          2010-03-XX - Written - Bovy (NYU)
galpy.df.diskdf.sigmaR2
diskdf.sigmaR2 (R, romberg=False, nsigma=None, phi=0.0)
          sigmaR2 (duplicate of sigma2 for consistency)
     PURPOSE:
          calculate sigma_R^2 at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate sigma_R^2 (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          sigma_R^2 at R
     HISTORY:
          2011-03-30 - Written - Bovy (NYU)
galpy.df.diskdf.sigmaT2
diskdf.sigmaT2(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          sigmaT2
     PURPOSE:
          calculate sigma_T^2 at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate sigma_T^2 (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          sigma_T^2 at R
     HISTORY:
```

```
2011-03-30 - Written - Bovy (NYU)
```

```
galpy.df.diskdf.skewvR
```

```
diskdf.skewvR(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          skewvR
     PURPOSE:
          calculate skew in vR at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vR> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          skewvR
     HISTORY:
          2011-12-07 - Written - Bovy (NYU)
galpy.df.diskdf.skewvT
diskdf.skewvT(R, romberg=False, nsigma=None, phi=0.0)
     NAME:
          skewvT
     PURPOSE:
          calculate skew in vT at R by marginalizing over velocity
     INPUT:
          R - radius at which to calculate <vR> (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
          skewvT
     HISTORY:
          2011-12-07 - Written - Bovy (NYU)
```

galpy.df.diskdf.surfacemass

```
diskdf.surfacemass(R, romberg=False, nsigma=None, relative=False)
           surfacemass
     PURPOSE:
          calculate the surface-mass at R by marginalizing over velocity
     INPUT:
           R - radius at which to calculate the surfacemass density (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
           romberg - if True, use a romberg integrator (default: False)
     OUTPUT:
           surface mass at R
     HISTORY:
           2010-03-XX - Written - Bovy (NYU)
galpy.df.diskdf.surfacemassLOS
diskdf.surfacemassLOS(d, l, deg=True, target=True, romberg=False, nsigma=None, relative=None)
     NAME:
           surfacemassLOS
     PURPOSE:
           evaluate the surface mass along the LOS given l and d
     INPUT:
           d - distance along the line of sight (can be Quantity)
          1 - Galactic longitude (in deg, unless deg=False; can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
     KEYWORDS:
          target= if True, use target surfacemass (default)
           romberg - if True, use a romberg integrator (default: False)
          deg= if False, 1 is in radians
     OUTPUT:
           Sigma(d,l) x d
     HISTORY:
           2011-03-24 - Written - Bovy (NYU)
```

galpy.df.diskdf.targetSigma2

```
diskdf.targetSigma2(R, log=False)
     NAME:
          targetSigma2
     PURPOSE:
          evaluate the target Sigma_R^2(R)
     INPUT:
          R - radius at which to evaluate (can be Quantity)
     OUTPUT:
          target Sigma_R^2(R)
          log - if True, return the log (default: False)
     HISTORY:
          2010-03-28 - Written - Bovy (NYU)
galpy.df.diskdf.targetSurfacemass
diskdf.targetSurfacemass(R, log=False)
     NAME:
          targetSurfacemass
     PURPOSE:
          evaluate the target surface mass at R
     INPUT:
          R - radius at which to evaluate (can be Quantity)
          log - if True, return the log (default: False)
     OUTPUT:
          Sigma(R)
     HISTORY:
          2010-03-28 - Written - Bovy (NYU)
galpy.df.diskdf.targetSurfacemassLOS
diskdf.targetSurfacemassLOS(d, l, log=False, deg=True)
     NAME:
          targetSurfacemassLOS
     PURPOSE:
          evaluate the target surface mass along the LOS given l and d
     INPUT:
```

```
d - distance along the line of sight (can be Quantity)
           1 - Galactic longitude (in deg, unless deg=False; can be Quantity)
           deg= if False, 1 is in radians
           log - if True, return the log (default: False)
     OUTPUT:
           Sigma(d,l) x d
     HISTORY:
           2011-03-23 - Written - Bovy (NYU)
galpy.df.diskdf._vmomentsurfacemass
diskdf._vmomentsurfacemass (R, n, m, romberg=False, nsigma=None, relative=False, phi=0.0, de-
                                      riv=None)
     Non-physical version of vmomentsurfacemass, otherwise the same
Sampling routines
galpy.df.diskdf.sample
diskdf.sample(n=1, rrange=None, returnROrbit=True, returnOrbit=False, nphi=1.0, los=None, los-
                    deg=True, nsigma=None, maxd=None, target=True)
     NAME:
           sample
     PURPOSE:
           sample n*nphi points from this DF
     INPUT:
           n - number of desired sample (specifying this rather than calling this routine n times is more efficient)
           rrange - if you only want samples in this rrange, set this keyword (only works when asking for an
           (RZ)Orbit) (can be Quantity)
           returnROrbit - if True, return a planarROrbit instance: [R,vR,vT] (default)
           returnOrbit - if True, return a planarOrbit instance (including phi)
           nphi - number of azimuths to sample for each E,L
           los= line of sight sampling along this line of sight (can be Quantity)
           losdeg= los in degrees? (default=True)
           target= if True, use target surface mass and sigma2 profiles (default=True)
           nsigma= number of sigma to rejection-sample on
           maxd= maximum distance to consider (for the rejection sampling)
     OUTPUT:
           n*nphi list of [[E,Lz],...] or list of planar(R)Orbits
```

```
CAUTION: lists of EL need to be post-processed to account for the kappa/omega_R discrep-
              ancy
     HISTORY:
          2010-07-10 - Started - Bovy (NYU)
galpy.df.diskdf.sampledSurfacemassLOS
diskdf.sampledSurfacemassLOS(l, n=1, maxd=None, target=True)
     NAME:
          sampledSurfacemassLOS
     PURPOSE:
          sample a distance along the line of sight
     INPUT:
          1 - Galactic longitude (in rad; can be Quantity)
          n= number of distances to sample
          maxd= maximum distance to consider (for the rejection sampling) (can be Quantity)
          target= if True, sample from the 'target' surface mass density, rather than the actual surface mass
          density (default=True)
     OUTPUT:
          list of samples
     HISTORY:
          2011-03-24 - Written - Bovy (NYU)
hhgalpy.df.diskdf.sampleLOS
diskdf.sampleLOS(los, n=1, deg=True, maxd=None, nsigma=None, targetSurfmass=True, target-
                       Sigma2=True)
     NAME:
          sampleLOS
     PURPOSE:
          sample along a given LOS
     INPUT:
          los - line of sight (in deg, unless deg=False; can be Quantity)
          n= number of desired samples
          deg= los in degrees? (default=True)
          targetSurfmass, targetSigma2= if True, use target surface mass and sigma2 profiles, respectively (there is not much p
              (default=True)
     OUTPUT:
          returns list of Orbits
```

```
BUGS: target=False uses target distribution for derivatives (this is a detail)
     HISTORY:
         2011-03-24 - Started - Bovy (NYU)
galpy.df.diskdf.sampleVRVT
diskdf.sampleVRVT(R, n=1, nsigma=None, target=True)
     NAME:
         sampleVRVT
     PURPOSE:
         sample a radial and azimuthal velocity at R
     INPUT:
         R - Galactocentric distance (can be Quantity)
         n= number of distances to sample
         nsigma= number of sigma to rejection-sample on
         target= if True, sample using the 'target' sigma_R rather than the actual sigma_R (default=True)
     OUTPUT:
         list of samples
     BUGS:
         should use the fact that vR and vT separate
     HISTORY:
         2011-03-24 - Written - Bovy (NYU)
Specific distribution functions
Dehnen DF
class galpy.df.dehnendf(surfaceSigma=<class'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>,
                             **kwargs)
     Dehnen's 'new' df
     ___init___(surfaceSigma=<class
                                    'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>,
                                                                                       pro-
               NAME: __init__
         PURPOSE: Initialize a Dehnen 'new' DF
         INPUT:
             surfaceSigma - instance or class name of the target surface density and sigma_R profile (default:
                both exponential)
             profileParams - parameters of the surface and sigma_R profile: (xD,xS,Sro) where
```

xD - disk surface mass scalelength (can be Quantity)

xS - disk velocity dispersion scalelength (can be Quantity)

```
Sro - disk velocity dispersion at Ro (can be Quantity)
                    Directly given to the 'surfaceSigmaProfile class, so could be anything that class takes
              beta - power-law index of the rotation curve
              correct - if True, correct the DF
              ro= distance from vantage point to GC (kpc; can be Quantity)
              vo= circular velocity at ro (km/s; can be Quantity)
              +DFcorrection kwargs (except for those already specified)
          OUTPUT:
              instance
          HISTORY:
              2010-03-10 - Written - Bovy (NYU)
Schwarzschild DF
class galpy.df.schwarzschilddf (surfaceSigma=<class'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>,
                                        beta=0.0, **kwargs)
     Schwarzschild's df
     ___init___(surfaceSigma=<class
                                        'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>,
                                                                                              pro-
                 NAME: __init__
          PURPOSE: Initialize a Schwarzschild DF
          INPUT:
              surfaceSigma - instance or class name of the target surface density and sigma_R profile (default:
                  both exponential)
              profileParams - parameters of the surface and sigma_R profile: (xD,xS,Sro) where
                    xD - disk surface mass scalelength (can be Quantity)
                    xS - disk velocity dispersion scalelength (can be Quantity)
                    Sro - disk velocity dispersion at Ro (can be Quantity)
                    Directly given to the 'surfaceSigmaProfile class, so could be anything that class takes
              beta - power-law index of the rotation curve
              correct - if True, correct the DF
              ro= distance from vantage point to GC (kpc; can be Quantity)
              vo= circular velocity at ro (km/s; can be Quantity)
              +DFcorrection kwargs (except for those already specified)
          OUTPUT:
              instance
```

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```
HISTORY:
2017-09-17 - Written - Bovy (UofT)
```

Shu DF

```
class galpy.df.shudf (surfaceSigma=<class 'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>>,
                          **kwargs)
     Shu's df (1969)
     ___init___(surfaceSigma=<class
                                      'galpy.df.surfaceSigmaProfile.expSurfaceSigmaProfile'>,
                NAME: __init__
         PURPOSE: Initialize a Shu DF
         INPUT:
             surfaceSigma - instance or class name of the target surface density and sigma_R profile (default:
                 both exponential)
             profileParams - parameters of the surface and sigma_R profile: (xD,xS,Sro) where
                   xD - disk surface mass scalelength (can be Quantity)
                   xS - disk velocity dispersion scalelength (can be Quantity)
                   Sro - disk velocity dispersion at Ro (can be Quantity)
                   Directly given to the 'surfaceSigmaProfile class, so could be anything that class takes
             beta - power-law index of the rotation curve
             correct - if True, correct the DF
             ro= distance from vantage point to GC (kpc; can be Quantity)
             vo= circular velocity at ro (km/s; can be Quantity)
             +DFcorrection kwargs (except for those already specified)
         OUTPUT:
             instance
         HISTORY:
             2010-05-09 - Written - Bovy (NYU)
```

2.4.4 Two-dimensional, non-axisymmetric disk distribution functions

Distribution function for orbits in the plane of a galactic disk in non-axisymmetric potentials. These are calculated using the technique of Dehnen 2000, where the DF at the current time is obtained as the evolution of an initially-axisymmetric DF at time to in the non-axisymmetric potential until the current time.

General instance routines

```
galpy.df.evolveddiskdf. call
evolveddiskdf.___call___(*args, **kwargs)
      NAME:
           call
      PURPOSE:
           evaluate the distribution function
      INPUT:
           Orbit instance:
            a) Orbit instance alone: use initial state and t=0
            b) Orbit instance + t: Orbit instance NOT called (i.e., Orbit's initial condition is used, call Orbit
               yourself), t can be Quantity
               If t is a list of t, DF is returned for each t, times must be in descending order and equally spaced
               (does not work with marginalize...)
           marginalizeVperp - marginalize over perpendicular velocity (only supported with 1a) above) +
           nsigma, +scipy.integrate.quad keywords
           marginalizeVlos - marginalize over line-of-sight velocity (only supported with 1a) above) + nsigma,
           +scipy.integrate.quad keywords
           log= if True, return the log (not for deriv, bc that can be negative)
           integrate_method= method argument of orbit.integrate
           deriv= None, 'R', or 'phi': calculates derivative of the moment wrt R or phi not with the marginalize
           options
      OUTPUT:
           DF(orbit,t)
      HISTORY:
           2011-03-30 - Written - Bovy (NYU)
           2011-04-15 - Added list of times option - Bovy (NYU)
The DF of a two-dimensional, non-axisymmetric disk
class galpy.df.evolveddiskdf(initdf, pot, to=0.0)
      Class that represents a diskdf as initial DF + subsequent secular evolution
        \underline{\texttt{init}} (initdf, pot, to=0.0)
           NAME:
                __init__
           PURPOSE:
               initialize
```

INPUT:

```
pot - potential to integrate orbits in
                to= initial time (time at which initdf is evaluated; orbits are integrated from current t back to to)
                (can be Quantity)
           OUTPUT:
                instance
           HISTORY:
                2011-03-30 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.meanvR
evolveddiskdf.meanvR(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-05,
                              grid=None, gridpoints=101, returnGrid=False, surfacemass=None, hierarch-
                              grid=False, nlevels=2, integrate_method='dopr54_c')
      NAME:
           meanvR
      PURPOSE:
           calculate the mean vR of the velocity distribution at (R,phi)
      INPUT:
           R - radius at which to calculate the moment(/ro) (can be Quantity)
           phi= azimuth (rad unless deg=True; can be Quantity)
           t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
           descending order and equally spaced) (can be Quantity)
           surfacemass= if set use this pre-calculated surfacemass
           nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
           deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
           epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that
           of the initial DF)
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
           gridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid object (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           nlevels= number of hierarchical levels for the hierarchical grid
           integrate_method= orbit.integrate method argument
      OUTPUT:
           mean vR
      HISTORY:
           2011-03-31 - Written - Bovy (NYU)
```

initdf - the df at the start of the evolution (at to) (units are transferred)

galpy.df.evolveddiskdf.meanvT

```
evolveddiskdf.meanvT (R, t=0.0, nsigma=None, deg=False, phi=0.0, epsel=0.01, epsabs=1e-05,
                              grid=None, gridpoints=101, returnGrid=False, surfacemass=None, hierarch-
                              grid=False, nlevels=2, integrate_method='dopr54_c')
     NAME:
           meanvT
     PURPOSE:
           calculate the mean vT of the velocity distribution at (R,phi)
     INPUT:
           R - radius at which to calculate the moment (can be Quantity)
           phi= azimuth (rad unless deg=True; can be Quantity)
           t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
           descending order and equally spaced) (can be Quantity)
           surfacemass= if set use this pre-calculated surfacemass
           nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
           deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
           epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that
           of the initial DF)
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
           gridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid object (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           nlevels= number of hierarchical levels for the hierarchical grid
           integrate_method= orbit.integrate method argument
     OUTPUT:
           mean vT
     HISTORY:
           2011-03-31 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.oortA
evolveddiskdf.oortA(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-
                            05, grid=None, gridpoints=101, returnGrids=False, derivRGrid=None, de-
                            rivphiGrid=None, derivGridpoints=101, derivHierarchgrid=False, hierarch-
                            grid=False, nlevels=2, integrate_method='dopr54_c')
     NAME:
           oortA
     PURPOSE:
           calculate the Oort function A at (R,phi,t)
```

INPUT: R - radius at phi= azimuth t= time at w descending of nsigma - nur deg= azimuth

R - radius at which to calculate A (can be Quantity)

phi= azimuth (rad unless deg=True; can be Quantity

t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in descending order and equally spaced) (can be Quantity)

nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)

deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity

epsrel, epsabs - scipy.integrate keywords

grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as returned by this procedure), use this grid

derivRGrid, derivphiGrid= if set to True, build a grid and use that to evaluate integrals of the derivatives of the DF;if set to a grid-objects (such as returned by this procedure), use this grid

gridpoints= number of points to use for the grid in 1D (default=101)

derivGridpoints= number of points to use for the grid in 1D (default=101)

returnGrid= if True, return the grid objects (default=False)

hierarchgrid= if True, use a hierarchical grid (default=False)

derivHierarchgrid= if True, use a hierarchical grid (default=False)

nlevels= number of hierarchical levels for the hierarchical grid

integrate_method= orbit.integrate method argument

OUTPUT:

Oort A at R,phi,t

HISTORY:

2011-10-16 - Written - Bovy (NYU)

galpy.df.evolveddiskdf.oortB

```
evolveddiskdf.oortB(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-05, grid=None, gridpoints=101, returnGrids=False, derivRGrid=None, derivPhiGrid=None, derivGridpoints=101, derivHierarchgrid=False, hierarchgrid=False, nlevels=2, integrate_method='dopr54_c')
```

NAME:

oortB

PURPOSE:

calculate the Oort function B at (R,phi,t)

INPUT:

R - radius at which to calculate B (can be Quantity)

phi= azimuth (rad unless deg=True; can be Quantity)

t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in descending order and equally spaced) (can be Quantity)

nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)

```
epsrel, epsabs - scipy.integrate keywords
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
           derivRGrid, derivphiGrid= if set to True, build a grid and use that to evaluat integrals of the deriva-
           tives of the DF: if set to a grid-objects (such as returned by this procedure), use this grid
           gridpoints= number of points to use for the grid in 1D (default=101)
           derivGridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid objects (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           derivHierarchgrid= if True, use a hierarchical grid (default=False)
           nlevels= number of hierarchical levels for the hierarchical grid
           integrate_method= orbit.integrate method argument
      OUTPUT:
           Oort B at R,phi,t
      HISTORY:
           2011-10-16 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.oortC
evolveddiskdf.oortC(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-
                             05, grid=None, gridpoints=101, returnGrids=False, derivRGrid=None, de-
                             rivphiGrid=None, derivGridpoints=101, derivHierarchgrid=False, hierarch-
                             grid=False, nlevels=2, integrate_method='dopr54_c')
      NAME:
           oortC
      PURPOSE:
           calculate the Oort function C at (R,phi,t)
      INPUT:
           R - radius at which to calculate C (can be Quantity)
           phi= azimuth (rad unless deg=True; can be Quantity)
           t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
           descending order and equally spaced) (can be Quantity)
           nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
           deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
           epsrel, epsabs - scipy.integrate keywords
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
```

derivRGrid, derivphiGrid= if set to True, build a grid and use that to evaluate integrals of the deriva-

tives of the DF; if set to a grid-objects (such as returned by this procedure), use this grid

deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity

```
gridpoints= number of points to use for the grid in 1D (default=101)
           derivGridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid objects (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           derivHierarchgrid= if True, use a hierarchical grid (default=False)
           nlevels= number of hierarchical levels for the hierarchical grid
           integrate_method= orbit.integrate method argument
      OUTPUT:
           Oort C at R,phi,t
      HISTORY:
           2011-10-16 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.oortK
evolveddiskdf.oortK(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-
                             05, grid=None, gridpoints=101, returnGrids=False, derivRGrid=None, de-
                             rivphiGrid=None, derivGridpoints=101, derivHierarchgrid=False, hierarch-
                             grid=False, nlevels=2, integrate method='dopr54 c')
      NAME:
           oortK
      PURPOSE:
           calculate the Oort function K at (R,phi,t)
      INPUT:
           R - radius at which to calculate K (can be Quantity)
           phi= azimuth (rad unless deg=True; can be Quantity)
           t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
           descending order and equally spaced) (can be Quantity)
           nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
           deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
           epsrel, epsabs - scipy.integrate keywords
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
           derivRGrid, derivphiGrid= if set to True, build a grid and use that to evaluate integrals of the deriva-
           tives of the DF; if set to a grid-objects (such as returned by this procedure), use this grid
           gridpoints= number of points to use for the grid in 1D (default=101)
           derivGridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid objects (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           derivHierarchgrid= if True, use a hierarchical grid (default=False)
```

```
nlevels= number of hierarchical levels for the hierarchical grid
          integrate_method= orbit.integrate method argument
     OUTPUT:
          Oort K at R,phi,t
     HISTORY:
          2011-10-16 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.sigmaR2
grid=None,
                                                    gridpoints=101,
                                                                        returnGrid=False,
                                            meanvR=None,
                                                              hierarchgrid=False,
                              mass=None,
                                                                                    nlevels=2,
                              grate_method='dopr54_c')
     NAME:
          sigmaR2
     PURPOSE:
          calculate the radial variance of the velocity distribution at (R,phi)
     INPUT:
          R - radius at which to calculate the moment (can be Quantity)
          phi= azimuth (rad unless deg=True; can be Quantity)
          t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
          descending order and equally spaced) (can be Quantity)
          surfacemass, meanvR= if set use this pre-calculated surfacemass and mean vR
          nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
          deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
          epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that
          of the initial DF)
          grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
          returned by this procedure), use this grid
          gridpoints= number of points to use for the grid in 1D (default=101)
          returnGrid= if True, return the grid object (default=False)
          hierarchgrid= if True, use a hierarchical grid (default=False)
          nlevels= number of hierarchical levels for the hierarchical grid
          integrate_method= orbit.integrate method argument
     OUTPUT:
          variance of vR
     HISTORY:
```

2011-03-31 - Written - Bovy (NYU)

galpy.df.evolveddiskdf.sigmaRT

PURPOSE:

```
evolveddiskdf.sigmaRT(R, t=0.0, nsigma=None, deg=False, epsrel=0.01, epsabs=1e-0.00, phi=0.00,
                                             gridpoints=101, returnGrid=False,
                               grid=None,
                                                                                      surfacemass=None,
                               meanvR=None, meanvT=None, hierarchgrid=False, nlevels=2, inte-
                               grate_method='dopr54_c')
     NAME:
           sigmaRT
     PURPOSE:
           calculate the radial-tangential co-variance of the velocity distribution at (R,phi)
     INPUT:
           R - radius at which to calculate the moment (can be Quantity)
           phi= azimuth (rad unless deg=True; can be Quantity)
           t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in
           descending order and equally spaced) (can be Quantity)
           surfacemass, meanvR, meavT= if set use this pre-calculated surfacemass and mean vR and vT
           nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)
           deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity
           epsrel, epsabs - scipy.integrate keywords (the integration calculates the ration of this vmoment to that
           of the initial DF)
           grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as
           returned by this procedure), use this grid
           gridpoints= number of points to use for the grid in 1D (default=101)
           returnGrid= if True, return the grid object (default=False)
           hierarchgrid= if True, use a hierarchical grid (default=False)
           nlevels= number of hierarchical levels for the hierarchical grid
           integrate_method= orbit.integrate method argument
     OUTPUT:
           covariance of vR and vT
     HISTORY:
           2011-03-31 - Written - Bovy (NYU)
galpy.df.evolveddiskdf.sigmaT2
evolveddiskdf.sigmaT2(R, t=0.0, nsigma=None, deg=False, phi=0.0, epsrel=0.01, epsabs=1e-
                                       grid=None,
                               05,
                                                      gridpoints=101,
                                                                           returnGrid=False,
                                                                                                 surface-
                                             meanvT=None,
                                                                hierarchgrid=False,
                               mass=None,
                                                                                       nlevels=2,
                               grate_method='dopr54_c')
     NAME:
           sigmaT2
```

```
calculate the tangential variance of the velocity distribution at (R,phi)
```

INPUT:

```
R - radius at which to calculate the moment (can be Quantity)
```

phi= azimuth (rad unless deg=True; can be Quantity)

t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in descending order and equally spaced) (can be Quantity)

surfacemass, meanvT= if set use this pre-calculated surfacemass and mean tangential velocity

nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)

deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity

epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that of the initial DF)

grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as returned by this procedure), use this grid

gridpoints= number of points to use for the grid in 1D (default=101)

returnGrid= if True, return the grid object (default=False)

hierarchgrid= if True, use a hierarchical grid (default=False)

nlevels= number of hierarchical levels for the hierarchical grid

integrate_method= orbit.integrate method argument

OUTPUT:

variance of vT

HISTORY:

2011-03-31 - Written - Bovy (NYU)

galpy.df.evolveddiskdf.vertexdev

```
evolveddiskdf.\mathbf{vertexdev} (R, t=0.0, nsigma=None, deg=False, epsrel=0.01, epsabs=1e-05, phi=0.0, grid=None, gridpoints=101, returnGrid=False, sigmaR2=None, sigmaR7=None, sigmaR7=None, sigracemass=None, hierarchgrid=False, nlevels=2, integrate_method='dopr54_c')
```

NAME:

vertexdev

PURPOSE:

calculate the vertex deviation of the velocity distribution at (R,phi)

INPUT:

R - radius at which to calculate the moment (can be Quantity)

phi= azimuth (rad unless deg=True; can be Quantity)

t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in descending order and equally spaced) (can be Quantity)

sigmaR2, sigmaT2, sigmaRT= if set the vertex deviation is simply calculated using these

nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous)

351

deg= azimuth is in degree (default=False); do not set this when giving phi as a Quantity

epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that of the initial DF)

grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as returned by this procedure), use this grid

gridpoints= number of points to use for the grid in 1D (default=101)

returnGrid= if True, return the grid object (default=False)

hierarchgrid= if True, use a hierarchical grid (default=False)

nlevels= number of hierarchical levels for the hierarchical grid

integrate_method= orbit.integrate method argument

OUTPUT:

vertex deviation in rad

HISTORY:

2011-03-31 - Written - Bovy (NYU)

galpy.df.evolveddiskdf.vmomentsurfacemass

```
evolveddiskdf.vmomentsurfacemass (R, n, m, t=0.0, nsigma=None, deg=False, epsrel=0.01, epsabs=1e-05, phi=0.0, grid=None, gridpoints=101, returnGrid=False, hierarchgrid=False, nlevels=2, print_progress=False, integrate_method='dopr54_c', deriv=None')
```

NAME:

vmomentsurfacemass

PURPOSE:

calculate the an arbitrary moment of the velocity distribution at (R,phi) times the surfacmass

INPUT:

R - radius at which to calculate the moment (in natural units)

phi= azimuth (rad unless deg=True)

n - vR^n

 $m - vT^m$

t= time at which to evaluate the DF (can be a list or ndarray; if this is the case, list needs to be in descending order and equally spaced)

nsigma - number of sigma to integrate the velocities over (based on an estimate, so be generous, but not too generous)

deg= azimuth is in degree (default=False)

epsrel, epsabs - scipy.integrate keywords (the integration calculates the ratio of this vmoment to that of the initial DF)

grid= if set to True, build a grid and use that to evaluate integrals; if set to a grid-objects (such as returned by this procedure), use this grid; if this was created for a list of times, moments are calculated for each time

```
gridpoints= number of points to use for the grid in 1D (default=101)

returnGrid= if True, return the grid object (default=False)

hierarchgrid= if True, use a hierarchical grid (default=False)

nlevels= number of hierarchical levels for the hierarchical grid

print_progress= if True, print progress updates

integrate_method= orbit.integrate method argument

deriv= None, 'R', or 'phi': calculates derivative of the moment wrt R or phi onnly with grid options

OUTPUT:

<vR^n vT^m x surface-mass> at R,phi (no support for units)

COMMENT:

grid-based calculation is the only one that is heavily tested (although the test suite also tests the direct calculation)

HISTORY:

2011-03-30 - Written - Bovy (NYU)
```

2.4.5 Three-dimensional disk distribution functions

Distribution functions for orbits in galactic disks, including the vertical motion for stars reaching large heights above the plane. Currently only the *quasi-isothermal DF*.

General instance routines

```
OUTPUT:
           value of DF
     HISTORY:
           2012-07-25 - Written - Bovy (IAS@MPIA)
     NOTE:
           For Miyamoto-Nagai/adiabatic approximation this seems to take about 30 ms / evaluation in the
           extended Solar neighborhood For a MWPotential/adiabatic approximation this takes about 50 ms /
           evaluation in the extended Solar neighborhood
           For adiabatic-approximation grid this seems to take about 0.67 to 0.75 ms / evaluation in the extended
           Solar neighborhood (includes some out of the grid)
           up to 200x faster when called with vector R,vR,vT,z,vz
galpy.df.quasiisothermaldf.density
quasiisothermaldf.density(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10,
                                    **kwargs)
     NAME:
           density
     PURPOSE:
           calculate the density at R,z by marginalizing over velocity
     INPUT:
           R - radius at which to calculate the density (can be Quantity)
           z - height at which to calculate the density (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
           gl= if True, calculate using Gauss-Legendre integration
           ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
           density at (R,z)
     HISTORY:
           2012-07-26 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.estimate_hr
quasiisothermaldf.estimate_hr (R, z=0.0, dR=1e-08, **kwargs)
     NAME:
```

estimate hr

```
PURPOSE:
          estimate the exponential scale length at R
     INPUT:
          R - Galactocentric radius (can be Quantity)
          z= height (default: 0 pc) (can be Quantity)
          dR- range in R to use (can be Quantity)
          density kwargs
     OUTPUT:
          estimated hR
     HISTORY:
          2012-09-11 - Written - Bovy (IAS)
          2013-01-28 - Re-written - Bovy
galpy.df.quasiisothermaldf.estimate_hsr
quasiisothermaldf.estimate_hsr(R, z=0.0, dR=1e-08, **kwargs)
     NAME:
          estimate_hsr
     PURPOSE:
          estimate the exponential scale length of the radial dispersion at R
     INPUT:
          R - Galactocentric radius (can be Quantity)
          z= height (default: 0 pc) (can be Quantity)
          dR- range in R to use (can be Quantity)
          density kwargs
     OUTPUT:
          estimated hsR
     HISTORY:
          2013-03-08 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.estimate hsz
quasiisothermaldf.estimate_hsz (R, z=0.0, dR=1e-08, **kwargs)
     NAME:
          estimate_hsz
     PURPOSE:
          estimate the exponential scale length of the vertical dispersion at R
     INPUT:
```

```
R - Galactocentric radius (can be Quantity)
          z= height (default: 0 pc) (can be Quantity)
          dR- range in R to use (can be Quantity)
          density kwargs
     OUTPUT:
          estimated hsz
     HISTORY:
          2013-03-08 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.estimate_hz
quasiisothermaldf.estimate_hz (R, z, dz=1e-08, **kwargs)
     NAME:
          estimate hz
     PURPOSE:
          estimate the exponential scale height at R
     INPUT:
          R - Galactocentric radius (can be Quantity)
          dz - z range to use (can be Quantity)
          density kwargs
     OUTPUT:
          estimated hz
     HISTORY:
          2012-08-30 - Written - Bovy (IAS)
          2013-01-28 - Re-written - Bovy
galpy.df.quasiisothermaldf._jmomentdensity
quasiisothermaldf._jmomentdensity(R, z, n, m, o, nsigma=None, mc=True, nmc=10000,
                                             _returnmc=False, _vrs=None, _vts=None, _vzs=None,
                                             **kwargs)
     Non-physical version of jmomentdensity, otherwise the same
galpy.df.quasiisothermaldf.meanjr
quasiisothermaldf.meanjr(R, z, nsigma=None, mc=True, nmc=10000, **kwargs)
     NAME:
          meanjr
     PURPOSE:
          calculate the mean radial action by marginalizing over velocity
```

```
INPUT:
          R - radius at which to calculate this (can be Quantity)
          z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
          scipy.integrate.tplquad kwargs epsabs and epsrel
          mc= if True, calculate using Monte Carlo integration
          nmc= if mc, use nmc samples
     OUTPUT:
          meanjr
     HISTORY:
          2012-08-09 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.meanjz
quasiisothermaldf.meanjz(R, z, nsigma=None, mc=True, nmc=10000, **kwargs)
     NAME:
          meanjz
     PURPOSE:
          calculate the mean vertical action by marginalizing over velocity
     INPUT:
          R - radius at which to calculate this (can be Quantity)
          z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
          scipy.integrate.tplquad kwargs epsabs and epsrel
          mc= if True, calculate using Monte Carlo integration
          nmc= if mc, use nmc samples
     OUTPUT:
          meanjz
     HISTORY:
          2012-08-09 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.meanlz
quasiisothermaldf.meanlz(R, z, nsigma=None, mc=True, nmc=10000, **kwargs)
     NAME:
          meanlz
```

```
PURPOSE:
           calculate the mean angular momentum by marginalizing over velocity
     INPUT:
           R - radius at which to calculate this (can be Quantity)
           z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
     OUTPUT:
           meanlz
     HISTORY:
           2012-08-09 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.meanvR
quasiisothermaldf.meanvR(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10, **kwargs)
     NAME:
           meanvR
     PURPOSE:
           calculate the mean radial velocity by marginalizing over velocity
     INPUT:
           R - radius at which to calculate this (can be Quantity)
           z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
           gl= if True, calculate using Gauss-Legendre integration
           ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
           meanvR
     HISTORY:
           2012-12-23 - Written - Bovy (IAS)
```

galpy.df.quasiisothermaldf.meanvT

```
quasiisothermaldf.meanvT(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10, **kwargs)
           meanvT
     PURPOSE:
           calculate the mean rotational velocity by marginalizing over velocity
     INPUT:
           R - radius at which to calculate this (can be Quantity)
           z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
           gl= if True, calculate using Gauss-Legendre integration
           ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
           meanvT
     HISTORY:
           2012-07-30 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.meanvz
quasiisothermaldf.meanvz(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10, **kwargs)
     NAME:
           meanvz
     PURPOSE:
           calculate the mean vertical velocity by marginalizing over velocity
     INPUT:
           R - radius at which to calculate this (can be Quantity)
           z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
           gl= if True, calculate using Gauss-Legendre integration
```

```
ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
           meanvz
     HISTORY:
           2012-12-23 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.pvR
quasiisothermaldf.pvR(vR, R, z, gl=True, ngl=20, nsigma=4.0, vTmax=1.5)
     NAME:
           pvR
     PURPOSE:
           calculate the marginalized vR probability at this location (NOT normalized by the density)
     INPUT:
           vR - radial velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
           gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           nsigma - sets integration limits to [-1,+1]*nsigma*sigma_z(R) for integration over vz (default: 4)
           vTmax - sets integration limits to [0,vTmax] for integration over vT (default: 1.5)
     OUTPUT:
           p(vR,R,z)
     HISTORY:
           2012-12-22 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.pvRvT
quasiisothermaldf.pvRvT (vR, vT, R, z, gl=True, ngl=20, nsigma=4.0)
     NAME:
           pvRvT
     PURPOSE:
           calculate the marginalized (vR,vT) probability at this location (NOT normalized by the density)
     INPUT:
           vR - radial velocity (can be Quantity)
           vT - tangential velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
```

```
gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           nsigma - sets integration limits to [-1,+1]*nsigma*sigma_z(R) for integration over vz (default: 4)
     OUTPUT:
          p(vR,vT,R,z)
     HISTORY:
           2013-01-02 - Written - Bovy (IAS) 2018-01-12 - Added Gauss-Legendre integration prefactor
           nsigma/2 - Trick (MPA)
galpy.df.quasiisothermaldf.pvRvz
quasiisothermaldf.pvRvz(vR, vz, R, z, gl=True, ngl=20, vTmax=1.5)
     NAME:
          pvR
     PURPOSE:
           calculate the marginalized (vR,vz) probability at this location (NOT normalized by the density)
     INPUT:
           vR - radial velocity (can be Quantity)
           vz - vertical velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
           gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           vTmax - sets integration limits to [0,vTmax] for integration over vT (default: 1.5)
     OUTPUT:
          p(vR,vz,R,z)
     HISTORY:
           2013-01-02 - Written - Bovy (IAS) 2018-01-12 - Added Gauss-Legendre integration prefactor vT-
           max/2 - Trick (MPA)
galpy.df.quasiisothermaldf.pvT
quasiisothermaldf.pvT(vT, R, z, gl=True, ngl=20, nsigma=4.0)
     NAME:
          pvT
     PURPOSE:
           calculate the marginalized vT probability at this location (NOT normalized by the density)
     INPUT:
```

```
vT - tangential velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
           gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           nsigma - sets integration limits to [-1,+1]*nsigma*sigma(R) for integration over vz and vR (default:
           4)
     OUTPUT:
           p(vT,R,z)
     HISTORY:
           2012-12-22 - Written - Bovy (IAS) 2018-01-12 - Added Gauss-Legendre integration prefactor
           nsigma^2/4 - Trick (MPA)
galpy.df.quasiisothermaldf.pvTvz
quasiisothermaldf.pvTvz(vT, vz, R, z, gl=True, ngl=20, nsigma=4.0)
     NAME:
          pvTvz
     PURPOSE:
           calculate the marginalized (vT,vz) probability at this location (NOT normalized by the density)
     INPUT:
           vT - tangential velocity (can be Quantity)
           vz - vertical velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
           gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           nsigma - sets integration limits to [-1,+1]*nsigma*sigma_R(R) for integration over vR (default: 4)
     OUTPUT:
           p(vT,vz,R,z)
     HISTORY:
           2012-12-22 - Written - Bovy (IAS) 2018-01-12 - Added Gauss-Legendre integration prefactor
           nsigma/2 - Trick (MPA)
galpy.df.quasiisothermaldf.pvz
quasiisothermaldf.pvz (vz, R, z, gl=True, ngl=20, nsigma=4.0, vTmax=1.5, _return_actions=False,
                              _jr=None, _lz=None, _jz=None, _return_freqs=False,
                              kappa=None, nu=None, Omega=None, sigmaR1=None)
     NAME:
```

```
pvz
     PURPOSE:
           calculate the marginalized vz probability at this location (NOT normalized by the density)
     INPUT:
           vz - vertical velocity (can be Quantity)
           R - radius (can be Quantity)
           z - height (can be Quantity)
           gl - use Gauss-Legendre integration (True, currently the only option)
           ngl - order of Gauss-Legendre integration
           nsigma - sets integration limits to [-1,+1]*nsigma*sigma_R(R) for integration over vR (default: 4)
           vTmax - sets integration limits to [0,vTmax] for integration over vT (default: 1.5)
     OUTPUT:
          p(vz,R,z)
     HISTORY:
           2012-12-22 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.sampleV
quasiisothermaldf.sampleV (R, z, n=1, **kwargs)
     NAME:
          sampleV
     PURPOSE:
           sample a radial, azimuthal, and vertical velocity at R,z
     INPUT:
           R - Galactocentric distance (can be Quantity)
           z - height (can be Quantity)
          n= number of distances to sample
     OUTPUT:
          list of samples
     HISTORY:
           2012-12-17 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.sampleV_interpolate
quasiisothermaldf.sampleV_interpolate(R, z, R_pixel, z_pixel, num_std=3, R_min=None,
                                                    R_{max}=None, z_{max}=None, **kwargs)
     NAME:
           sampleV_interpolate
     PURPOSE:
```

```
Given an array of R and z coordinates of stars, return the positions and their radial, azimuthal, and vertical velocity.
```

INPUT:

```
R - array of Galactocentric distance (can be Quantity)
```

z - array of height (can be Quantity)

R_pixel, **z_pixel= the pixel size for creating the grid for** interpolation (in natural unit)

num_std= number of standard deviation to be considered outliers sampled separately from interpolation

R_min, R_max, z_max= optional edges of the grid

OUTPUT:

coord_v= a numpy array containing the sampled velocity, (vR, vT, vz), where each row correspond to the row of (R,z)

HISTORY:

2018-08-10 - Written - Samuel Wong (University of Toronto)

galpy.df.quasiisothermaldf.sigmaR2

calculate sigma_R^2 by marginalizing over velocity

INPUT:

PURPOSE:

R - radius at which to calculate this (can be Quantity)

z - height at which to calculate this (can be Quantity)

OPTIONAL INPUT:

```
nsigma - number of sigma to integrate the velocities over
```

scipy.integrate.tplquad kwargs epsabs and epsrel

mc= if True, calculate using Monte Carlo integration

nmc= if mc, use nmc samples

gl= if True, calculate using Gauss-Legendre integration

ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension

OUTPUT:

sigma_R^2

HISTORY:

2012-07-30 - Written - Bovy (IAS@MPIA)

galpy.df.quasiisothermaldf.sigmaRz

```
quasiisothermaldf.sigmaRz(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10,
                                    **kwargs)
     NAME:
          sigmaRz
     PURPOSE:
          calculate sigma_RZ^2 by marginalizing over velocity
     INPUT:
          R - radius at which to calculate this (can be Quantity)
          z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
          scipy.integrate.tplquad kwargs epsabs and epsrel
          mc= if True, calculate using Monte Carlo integration
          nmc= if mc, use nmc samples
          gl= if True, calculate using Gauss-Legendre integration
          ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
          sigma Rz^2
     HISTORY:
          2012-07-30 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.sigmaT2
quasiisothermaldf.sigmaT2(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10,
                                    **kwargs)
     NAME:
          sigmaT2
     PURPOSE:
          calculate sigma_T^2 by marginalizing over velocity
     INPUT:
          R - radius at which to calculate this (can be Quantity)
          z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
          scipy.integrate.tplquad kwargs epsabs and epsrel
          mc= if True, calculate using Monte Carlo integration
          nmc= if mc, use nmc samples
```

```
gl= if True, calculate using Gauss-Legendre integration
          ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
          sigma_T^2
     HISTORY:
          2012-07-30 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.sigmaz2
quasiisothermaldf.sigmaz2(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10,
                                   **kwargs)
     NAME:
          sigmaz2
     PURPOSE:
          calculate sigma z^2 by marginalizing over velocity
     INPUT:
          R - radius at which to calculate this (can be Quantity)
          z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
          nsigma - number of sigma to integrate the velocities over
          scipy.integrate.tplquad kwargs epsabs and epsrel
          mc= if True, calculate using Monte Carlo integration
          nmc= if mc, use nmc samples
          gl= if True, calculate using Gauss-Legendre integration
          ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
          sigma z^2
     HISTORY:
          2012-07-30 - Written - Bovy (IAS@MPIA)
galpy.df.quasiisothermaldf.surfacemass_z
quasiisothermaldf.surfacemass_z(R, nz=7,
                                                       zmax=1.0, fixed\_quad=True, fixed\_order=8,
                                           **kwargs)
     NAME:
          surfacemass_z
     PURPOSE:
          calculate the vertically-integrated surface density
     INPUT:
```

```
R - Galactocentric radius (can be Quantity)
           fixed_quad= if True (default), use Gauss-Legendre integration
           fixed_order= (20), order of GL integration to use
           nz= number of zs to use to estimate
           zmax= maximum z to use (can be Quantity)
           density kwargs
     OUTPUT:
           Sigma(R)
     HISTORY:
           2012-08-30 - Written - Bovy (IAS)
galpy.df.quasiisothermaldf.tilt
quasiisothermaldf.tilt(R, z, nsigma=None, mc=False, nmc=10000, gl=True, ngl=10, **kwargs)
     NAME:
           tilt
     PURPOSE:
           calculate the tilt of the velocity ellipsoid by marginalizing over velocity
     INPUT:
           R - radius at which to calculate this (can be Quantity)
           z - height at which to calculate this (can be Quantity)
     OPTIONAL INPUT:
           nsigma - number of sigma to integrate the velocities over
           scipy.integrate.tplquad kwargs epsabs and epsrel
           mc= if True, calculate using Monte Carlo integration
           nmc= if mc, use nmc samples
           gl= if True, calculate using Gauss-Legendre integration
           ngl= if gl, use ngl-th order Gauss-Legendre integration for each dimension
     OUTPUT:
           tilt in rad
     HISTORY:
           2012-12-23 - Written - Bovy (IAS)
           2017-10-28 - Changed return unit to rad - Bovy (UofT)
```

galpy.df.quasiisothermaldf._vmomentdensity

Non-physical version of vmomentdensity, otherwise the same

Specific distribution functions

Quasi-isothermal DF

```
class galpy.df.quasiisothermaldf(hr, sr, sz, hsr, hsz, pot=None, aA=None, cutcounter=False,
                                               _precomputerg=True, _precomputergrmax=None, _pre-
                                               computergnLz=51, refr=1.0, lo=0.005681818181818182,
                                               ro=None, vo=None)
     Class that represents a 'Binney' quasi-isothermal DF
        _init__ (hr, sr, sz, hsr, hsz, pot=None, aA=None, cutcounter=False, _precomputerg=True, _precom-
                  putergrmax=None, _precomputergnLz=51, refr=1.0, lo=0.0056818181818182, ro=None,
                  vo=None)
           NAME:
               init
           PURPOSE:
               Initialize a quasi-isothermal DF
           INPUT:
               hr - radial scale length (can be Quantity)
               sr - radial velocity dispersion at the solar radius (can be Quantity)
               sz - vertical velocity dispersion at the solar radius (can be Quantity)
               hsr - radial-velocity-dispersion scale length (can be Quantity)
               hsz - vertial-velocity-dispersion scale length (can be Quantity)
               pot= Potential instance or list thereof
               aA= actionAngle instance used to convert (x,v) to actions [must be an instance of an actionAngle
               class that computes (J,Omega,angle) for a given (x,v)
               cutcounter= if True, set counter-rotating stars' DF to zero
               refr= reference radius for dispersions (can be different from ro) (can be Quantity)
               lo= reference angular momentum below where there are significant numbers of retrograde stars
               (can be Quantity)
               ro= distance from vantage point to GC (kpc; can be Quantity)
               vo= circular velocity at ro (km/s; can be Quantity)
           OTHER INPUTS:
```

```
_precomputerg= if True (default), pre-compute the rL(L)
_precomputergrmax= if set, this is the maximum R for which to pre-compute rg (default: 5*hr)
_precomputergnLz if set, number of Lz to pre-compute rg for (default: 51)

OUTPUT:
object

HISTORY:
2012-07-25 - Started - Bovy (IAS@MPIA)
```

2.4.6 The distribution function of a tidal stream

From Bovy 2014; see Dynamical modeling of tidal streams.

General instance routines

```
galpy.df.streamdf. call
streamdf.___call___(*args, **kwargs)
     NAME:
           __call__
     PURPOSE:
           evaluate the DF
     INPUT:
           Either:
                a) R,vR,vT,z,vz,phi ndarray [nobjects]
                b) (Omegar,Omegaphi,Omegaz,angler,anglephi,anglez) tuple if aAInput
                   where:
                     Omegar - radial frequency
                     Omegaphi - azimuthal frequency
                     Omegaz - vertical frequency
                     angler - radial angle
                     anglephi - azimuthal angle
                     anglez - vertical angle
                c) Orbit instance or list thereof
           log= if True, return the natural log
           aaInput= (False) if True, option b above
     OUTPUT:
           value of DF
```

HISTORY:

```
2013-12-03 - Written - Bovy (IAS)
```

The stream DF

class galpy.df.streamdf(sigv, progenitor=None, pot=None, aA=None, useTM=False, tdisrupt=None, sigMeanOffset=6.0, leading=True, sigangle=None, deltaAngleTrack=None, nTrackChunks=None, nTrackIterations=None, progIsTrack=False, ro=None, vo=None, Vnorm=None, Rnorm=None, R0=8.0,
Zsun=0.0208, vsun=[-11.1, 241.92, 7.25], multi=None, interpTrack=True,
useInterp=True, nosetup=False, nospreadsetup=False, approxConstTrackFreq=False, useTMHessian=False, custom_transform=None)

The DF of a tidal stream

___init___(sigv, progenitor=None, pot=None, aA=None, useTM=False, tdisrupt=None, sigMeanOff-set=6.0, leading=True, sigangle=None, deltaAngleTrack=None, nTrackChunks=None, nTrackIterations=None, progIsTrack=False, ro=None, vo=None, Vnorm=None, Rnorm=None, R0=8.0, Zsun=0.0208, vsun=[-11.1, 241.92, 7.25], multi=None, interpTrack=True, useInterp=True, nosetup=False, nospreadsetup=False, approxConstTrack-Freq=False, useTMHessian=False, custom_transform=None)

NAME:

init

PURPOSE:

Initialize a quasi-isothermal DF

INPUT:

sigv - radial velocity dispersion of the progenitor (can be Quantity)

tdisrupt= (5 Gyr) time since start of disruption (can be Quantity)

leading= (True) if True, model the leading part of the stream if False, model the trailing part

progenitor= progenitor orbit as Orbit instance (will be re-integrated, so don't bother integrating the orbit before)

progIsTrack= (False) if True, then the progenitor (x,v) is actually the (x,v) of the stream track at zero angle separation; useful when initializing with an orbit fit; the progenitor's position will be calculated

pot= Potential instance or list thereof

aA = actionAngle instance used to convert (x, v) to actions

useTM= (False) if set to an actionAngleTorus instance, use this to speed up calculations

sigMeanOffset= (6.) offset between the mean of the frequencies and the progenitor, in units of the largest eigenvalue of the frequency covariance matrix (along the largest eigenvector), should be positive; to model the trailing part, set leading=False

sigangle= (**sigv/122/**[**1km/s**]**=1.8sigv in natural coordinates**) estimate of the angle spread of the debris initially (can be Quantity)

deltaAngleTrack= (None) angle to estimate the stream track over (rad; or can be Quantity)

nTrackChunks= (floor(deltaAngleTrack/0.15)+1) number of chunks to divide the progenitor track in

nTrackIterations= Number of iterations to perform when establishing the track; each iteration starts from a previous approximation to the track in (x,v) and calculates a new track based on the deviation between the previous track and the desired track in action-angle coordinates; if not set, an appropriate value is determined based on the magnitude of the misalignment between stream and orbit, with larger numbers of iterations for larger misalignments

interpTrack= (might change), interpolate the stream track while setting up the instance (can be done by hand by calling self._interpolate_stream_track() and self._interpolate_stream_track_aA())

useInterp= (**might change**), **use interpolation by default when** calculating approximated frequencies and angles

nosetup= (False) if True, don't setup the stream track and anything else that is expensive

nospreadsetup= (False) if True, don't setup the spread around the stream track (only for nosetup is False)

multi= (None) if set, use multi-processing

Coordinate transformation inputs:

vo= (220) circular velocity to normalize velocities with [used to be Vnorm; can be Quantity]

ro= (8) Galactocentric radius to normalize positions with [used to be Rnorm; can be Quantity]

R0= (8) Galactocentric radius of the Sun (kpc) [can be different from ro; can be Quantity]

Zsun= (0.0208) Sun's height above the plane (kpc; can be Quantity)

vsun= ([-11.1,241.92,7.25]) Sun's motion in cylindrical coordinates (vR positive away from center) (can be Quantity)

custom_transform= (None) matrix implementing the rotation from (ra,dec) to a custom set of sky coordinates

approxConstTrackFreq= (False) if True, approximate the stream assuming that the frequency is constant along the stream (only works with useTM, for which this leads to a significant speed-up)

useTMHessian= (False) if True, compute the basic Hessian dO/dJ_prog using TM; otherwise use aA

```
OUTPUT:
```

object

HISTORY:

```
2013-09-16 - Started - Bovy (IAS)
2013-11-25 - Started over - Bovy (IAS)
```

galpy.df.streamdf.calc_stream_lb

```
convert the stream track to observational coordinates and store INPUT:
```

Coordinate transformation inputs (all default to the instance-wide values):

vo= circular velocity to normalize velocities with

ro= Galactocentric radius to normalize positions with

R0= Galactocentric radius of the Sun (kpc)

Zsun= Sun's height above the plane (kpc)

vsun= Sun's motion in cylindrical coordinates (vR positive away from center)

OUTPUT:

(none)

HISTORY:

2013-12-02 - Written - Bovy (IAS)

galpy.df.streamdf.callMarg

```
streamdf.callMarg(xy, **kwargs)
NAME:
callMarg
```

PURPOSE: evaluate the DF, marginalizing over some directions, in Galactocentric rectangular coordinates (or in observed 1,b,D,vlos,pmll,pmbb) coordinates)

INPUT:

xy - phase-space point [X,Y,Z,vX,vY,vZ]; the distribution of the dimensions set to None is returned

interp= (object-wide interp default) if True, use the interpolated stream track

cindx= index of the closest point on the (interpolated) stream track if not given, determined from the dimensions given

nsigma= (3) number of sigma to marginalize the DF over (approximate sigma)

ngl= (5) order of Gauss-Legendre integration

lb= (False) if True, xy contains [l,b,D,vlos,pmll,pmbb] in [deg,deg,kpc,km/s,mas/yr,mas/yr] and the marginalized PDF in these coordinates is returned

vo= (220) circular velocity to normalize with when lb=True

ro= (8) Galactocentric radius to normalize with when lb=True

R0= (8) Galactocentric radius of the Sun (kpc)

Zsun= (0.0208) Sun's height above the plane (kpc)

vsun= ([-11.1,241.92,7.25]) Sun's motion in cylindrical coordinates (vR positive away from center)

OUTPUT:

p(xy) marginalized over missing directions in xy

HISTORY:

```
2013-12-16 - Written - Bovy (IAS)
galpy.df.streamdf.density par
streamdf.density_par(dangle, coord='apar', tdisrupt=None, **kwargs)
     NAME:
          density_par
     PURPOSE:
          calculate the density as a function of a parallel coordinate
     INPUT:
          dangle - parallel angle offset for this coordinate value
          coord - coordinate to return the density in ('apar' [default], 'll','ra','customra','phi')
     OUTPUT:
          density(angle)
     HISTORY:
          2015-11-17 - Written - Bovy (UofT)
galpy.df.streamdf.estimateTdisrupt
streamdf.estimateTdisrupt(deltaAngle)
     NAME:
          estimateT disrupt \\
     PURPOSE:
          estimate the time of disruption
     INPUT:
          deltaAngle- spread in angle since disruption
     OUTPUT:
          time in natural units
     HISTORY:
          2013-11-27 - Written - Bovy (IAS)
galpy.df.streamdf.find closest trackpoint
streamdf.find_closest_trackpoint(R, vR, vT, z, vz, phi, interp=True, xy=False, usev=False)
     NAME:
          find_closest_trackpoint
     PURPOSE:
          find the closest point on the stream track to a given point
     INPUT:
```

```
R,vR,vT,z,vz,phi - phase-space coordinates of the given point
           interp= (True), if True, return the index of the interpolated track
           xy= (False) if True, input is X,Y,Z,vX,vY,vZ in Galactocentric rectangular coordinates; if xy, some
           coordinates may be missing (given as None) and they will not be used
           usev= (False) if True, also use velocities to find the closest point
     OUTPUT:
           index into the track of the closest track point
     HISTORY:
           2013-12-04 - Written - Bovy (IAS)
galpy.df.streamdf.find_closest_trackpointLB
streamdf.find_closest_trackpointLB(l, b, D, vlos, pmll, pmbb, interp=True, usev=False)
     NAME:
           find closest trackpointLB
     PURPOSE: find the closest point on the stream track to a given point in (1,b,...) coordinates
     INPUT:
           l,b,D,vlos,pmll,pmbb- coordinates in (deg,deg,kpc,km/s,mas/yr,mas/yr)
           interp= (True) if True, return the closest index on the interpolated track
           usev= (False) if True, also use the velocity components (default is to only use the positions)
     OUTPUT:
           index of closest track point on the interpolated or not-interpolated track
     HISTORY:
           2013-12-17- Written - Bovy (IAS)
galpy.df.streamdf.freqEigvalRatio
streamdf.freqEigvalRatio(isotropic=False)
     NAME:
           freqEigvalRatio
     PURPOSE:
           calculate the ratio between the largest and 2nd-to-largest (in abs) eigenvalue of sqrt(dO/dJ^T V_J
           dO/dJ) (if this is big, a 1D stream will form)
     INPUT:
           isotropic= (False), if True, return the ratio assuming an isotropic action distribution (i.e., just of
           dO/dJ)
     OUTPUT:
           ratio between eigenvalues of fabs(dO / dJ)
```

```
HISTORY:
           2013-12-05 - Written - Bovy (IAS)
galpy.df.streamdf.gaussApprox
streamdf.gaussApprox(xy, **kwargs)
      NAME:
           gaussApprox
      PURPOSE:
           return the mean and variance of a Gaussian approximation to the stream DF at a given phase-space
           point in Galactocentric rectangular coordinates (distribution is over missing directions)
      INPUT:
           xy - phase-space point [X,Y,Z,vX,vY,vZ]; the distribution of the dimensions set to None is returned
           interp= (object-wide interp default) if True, use the interpolated stream track
           cindx= index of the closest point on the (interpolated) stream track if not given, determined from the
           dimensions given
           lb= (False) if True, xy contains [l,b,D,vlos,pmll,pmbb] in [deg,deg,kpc,km/s,mas/yr,mas/yr] and the
           Gaussian approximation in these coordinates is returned
      OUTPUT:
           (mean, variance) of the approximate Gaussian DF for the missing directions in xy
      HISTORY:
           2013-12-12 - Written - Bovy (IAS)
galpy.df.streamdf.length
streamdf.length(threshold=0.2, phys=False, ang=False, tdisrupt=None, **kwargs)
      NAME:
           length
      PURPOSE:
           calculate the length of the stream
      INPUT:
           threshold - threshold down from the density near the progenitor at which to define the 'end' of the
           stream
           phys= (False) if True, return the length in physical kpc
           ang= (False) if True, return the length in sky angular arc length in degree
           coord - coordinate to return the density in ('apar' [default], 'll', 'ra', 'customra', 'phi')
      OUTPUT:
           length (rad for parallel angle; kpc for physical length; deg for sky arc length)
```

HISTORY:

```
2015-12-22 - Written - Bovy (UofT)
```

galpy.df.streamdf.meanangledAngle

```
streamdf.meanangledAngle(dangle, smallest=False)
     NAME:
          meanangledAngle
     PURPOSE:
          calculate the mean perpendicular angle at a given angle
     INPUT:
          dangle - angle offset along the stream
          smallest= (False) calculate for smallest eigenvalue direction rather than for middle
     OUTPUT:
          mean perpendicular angle
     HISTORY:
          2013-12-06 - Written - Bovy (IAS)
galpy.df.streamdf.meanOmega
streamdf.meanOmega(dangle, oned=False, offset_sign=None, tdisrupt=None)
     NAME:
          meanOmega
     PURPOSE:
          calculate the mean frequency as a function of angle, assuming a uniform time distribution up to a
          maximum time
     INPUT:
          dangle - angle offset
          oned= (False) if True, return the 1D offset from the progenitor (along the direction of disruption)
          offset_sign= sign of the frequency offset (shouldn't be set)
     OUTPUT:
          mean Omega
     HISTORY:
          2013-12-01 - Written - Bovy (IAS)
galpy.df.streamdf.meantdAngle
streamdf.meantdAngle(dangle)
     NAME:
          meantdAngle
```

```
PURPOSE:
           calculate the mean stripping time at a given angle
     INPUT:
           dangle - angle offset along the stream
     OUTPUT:
           mean stripping time at this dangle
     HISTORY:
           2013-12-05 - Written - Bovy (IAS)
galpy.df.streamdf.misalignment
streamdf.misalignment(isotropic=False, **kwargs)
     NAME:
          misalignment
     PURPOSE:
           calculate the misalignment between the progenitor's frequency and the direction along which the
           stream disrupts
     INPUT:
          isotropic= (False), if True, return the misalignment assuming an isotropic action distribution
     OUTPUT:
          misalignment in rad
     HISTORY:
           2013-12-05 - Written - Bovy (IAS)
           2017-10-28 - Changed output unit to rad - Bovy (UofT)
galpy.df.streamdf.pangledAngle
streamdf.pangledAngle(angleperp, dangle, smallest=False)
     NAME:
           pangledAngle
     PURPOSE: return the probability of a given perpendicular angle at a given angle along the stream
     INPUT:
           angleperp - perpendicular angle
           dangle - angle offset along the stream
           smallest= (False) calculate for smallest eigenvalue direction rather than for middle
     OUTPUT:
          p(angle_perpldangle)
     HISTORY:
```

```
2013-12-06 - Written - Bovy (IAS)
```

galpy.df.streamdf.plotCompareTrackAAModel

```
streamdf.plotCompareTrackAAModel(**kwargs)
     NAME:
           plotCompareTrackAAModel
     PURPOSE:
           plot the comparison between the underlying model's dOmega_perp vs. dangle_r (line) and the track
          in (x,v)'s dOmega_perp vs. dangle_r (dots; explicitly calculating the track's action-angle coordinates)
     INPUT:
          bovy_plot.bovy_plot kwargs
     OUTPUT:
          plot
     HISTORY:
           2014-08-27 - Written - Bovy (IAS)
galpy.df.streamdf.plotProgenitor
streamdf.plotProgenitor(d1='x', d2='z', *args, **kwargs)
     NAME:
          plotProgenitor
     PURPOSE:
           plot the progenitor orbit
     INPUT:
           d1= plot this on the X axis ('x','y','z','R','phi','vx','vy','vz','vR','vt','ll','bb','dist','pmll','pmbb','vlos')
           d2= plot this on the Y axis (same list as for d1)
           scaleToPhysical= (False), if True, plot positions in kpc and velocities in km/s
          bovy_plot.bovy_plot args and kwargs
     OUTPUT:
           plot to output device
     HISTORY:
           2013-12-09 - Written - Bovy (IAS)
galpy.df.streamdf.plotTrack
streamdf.plotTrack(dI='x', d2='z', interp=True, spread=0, simple=True, *args, **kwargs)
     NAME:
           plotTrack
```

```
PURPOSE:
           plot the stream track
      INPUT:
           d1= plot this on the X axis ('x', 'y', 'z', 'R', 'phi', 'vx', 'vy', 'vz', 'vR', 'vt', 'll', 'bb', 'dist', 'pmll', 'pmbb', 'vlos')
           d2= plot this on the Y axis (same list as for d1)
           interp= (True) if True, use the interpolated stream track
           spread= (0) if int > 0, also plot the spread around the track as spread x sigma
           scaleToPhysical= (False), if True, plot positions in kpc and velocities in km/s
           simple= (False), if True, use a simple estimate for the spread in perpendicular angle
           bovy_plot.bovy_plot args and kwargs
      OUTPUT:
           plot to output device
      HISTORY:
           2013-12-09 - Written - Bovy (IAS)
galpy.df.streamdf.pOparapar
streamdf.pOparapar(Opar, apar, tdisrupt=None)
      NAME:
           pOparapar
      PURPOSE:
           return the probability of a given parallel (frequency, angle) offset pair
     INPUT:
           Opar - parallel frequency offset (array) (can be Quantity)
           apar - parallel angle offset along the stream (scalar) (can be Quantity)
      OUTPUT:
           p(Opar,apar)
      HISTORY:
           2015-12-07 - Written - Bovy (UofT)
galpy.df.streamdf.ptdAngle
streamdf.ptdAngle(t, dangle)
      NAME:
           ptdangle
      PURPOSE:
           return the probability of a given stripping time at a given angle along the stream
      INPUT:
```

```
t - stripping time
           dangle - angle offset along the stream
     OUTPUT:
           p(tdldangle)
     HISTORY:
           2013-12-05 - Written - Bovy (IAS)
galpy.df.streamdf.sample
streamdf.sample(n, returnaAdt=False, returndt=False, interp=None, xy=False, lb=False)
     NAME:
           sample
     PURPOSE:
           sample from the DF
     INPUT:
           n - number of points to return
           returnaAdt= (False) if True, return (Omega,angle,dt)
           returndT= (False) if True, also return the time since the star was stripped
           interp= (object-wide default) use interpolation of the stream track
           xy= (False) if True, return Galactocentric rectangular coordinates
           lb= (False) if True, return Galactic l,b,d,vlos,pmll,pmbb coordinates
     OUTPUT:
           (R,vR,vT,z,vz,phi) of points on the stream in 6,N array
     HISTORY:
           2013-12-22 - Written - Bovy (IAS)
galpy.df.streamdf.sigangledAngle
streamdf.sigangledAngle (dangle, assumeZeroMean=True, smallest=False, simple=False)
     NAME:
           sigangledAngle
     PURPOSE:
           calculate the dispersion in the perpendicular angle at a given angle
     INPUT:
           dangle - angle offset along the stream
           assumeZeroMean= (True) if True, assume that the mean is zero (should be)
           smallest= (False) calculate for smallest eigenvalue direction rather than for middle
           simple= (False), if True, return an even simpler estimate
```

```
OUTPUT:
          dispersion in the perpendicular angle at this angle
     HISTORY:
          2013-12-06 - Written - Bovy (IAS)
galpy.df.streamdf.sigOmega
streamdf.sigOmega(dangle)
     NAME:
          sigmaOmega
     PURPOSE:
          calculate the 1D sigma in frequency as a function of angle, assuming a uniform time distribution up
          to a maximum time
     INPUT:
          dangle - angle offset
     OUTPUT:
          sigma Omega
     HISTORY:
          2013-12-05 - Written - Bovy (IAS)
galpy.df.streamdf.sigtdAngle
streamdf.sigtdAngle(dangle)
     NAME:
          sigtdAngle
     PURPOSE:
          calculate the dispersion in the stripping times at a given angle
     INPUT:
          dangle - angle offset along the stream
     OUTPUT:
          dispersion in the stripping times at this angle
     HISTORY:
          2013-12-05 - Written - Bovy (IAS)
galpy.df.streamdf.subhalo_encounters
streamdf.subhalo_encounters(venc=inf, sigma=0.6818181818181818, nsubhalo=0.3, bmax=0.025,
                                    yoon=False)
     NAME:
          subhalo_encounters
```

PURPOSE:

estimate the number of encounters with subhalos over the lifetime of this stream, using a formalism similar to that of Yoon et al. (2011)

INPUT:

```
venc= (numpy.inf) count encounters with (relative) speeds less than this (relative radial velocity in cylindrical stream frame, unless youn is True) (can be Quantity)
sigma= (150/220) velocity dispersion of the DM subhalo population (can be Quantity)
```

nsubhalo= (0.3) spatial number density of subhalos (can be Quantity)

bmax= (0.025) maximum impact parameter (if larger than width of stream) (can be Quantity)

yoon= (False) if True, use erroneous Yoon et al. formula

OUTPUT:

number of encounters

HISTORY:

2016-01-19 - Written - Bovy (UofT)

2.4.7 The distribution function of a gap in a tidal stream

From Sanders, Bovy, & Erkal 2015; see *Modeling gaps in streams*. Implemented as a subclass of streamdf. No full implementation is available currently, but the model can be set up and sampled as in the above paper.

General instance routines

The stream gap DF

```
class galpy.df.streamgapdf(*args, **kwargs)
     The DF of a gap in a tidal stream
      ___init___(*args, **kwargs)
           NAME:
               init
           PURPOSE:
               Initialize the DF of a gap in a stellar stream
           INPUT:
               streamdf args and kwargs
               Subhalo and impact parameters:
                   impactb= impact parameter (can be Quantity)
                   subhalovel= velocity of the subhalo shape=(3) (can be Quantity)
                   timpact time since impact (can be Quantity)
                   impact_angle= angle offset from progenitor at which the impact occurred (rad) (can be
                   Quantity)
                   Subhalo: specify either 1( mass and size of Plummer sphere or 2( general spherical-
                   potential object (kick is numerically computed)
```

```
1( GM= mass of the subhalo (can be Quantity)
                        rs= size parameter of the subhalo (can be Quantity)
                      2( subhalopot= galpy potential object or list thereof (should be spherical)
                      3( hernquist= (False) if True, use Hernquist kicks for GM/rs
               deltaAngleTrackImpact= (None) angle to estimate the stream track over to determine the effect
               of the impact [similar to deltaAngleTrack] (rad)
               nTrackChunksImpact= (floor(deltaAngleTrack/0.15)+1) number of chunks to divide the progen-
               itor track in near the impact [similar to nTrackChunks]
               nKickPoints=(30xnTrackChunksImpact) number of points along the stream to compute the kicks
               at (kicks are then interpolated); '30' chosen such that higherorderTrack can be set to False and
               get calculations accurate to > 99%
               nokicksetup= (False) if True, only run as far as setting up the coordinate transformation at the
               time of impact (useful when using this in streampepperdf)
               spline_order= (3) order of the spline to interpolate the kicks with
               higherorderTrack= (False) if True, calculate the track using higher-order terms
           OUTPUT:
               object
           HISTORY:
               2015-06-02 - Started - Bovy (IAS)
Helper routines to compute kicks
galpy.df.impulse_deltav_plummer
galpy.df.impulse_deltav_plummer(v, y, b, w, GM, rs)
      NAME:
           impulse deltav plummer
      PURPOSE:
           calculate the delta velocity to due an encounter with a Plummer sphere in the impulse approximation;
           allows for arbitrary velocity vectors, but y is input as the position along the stream
      INPUT:
           v - velocity of the stream (nstar,3)
           y - position along the stream (nstar)
           b - impact parameter
           w - velocity of the Plummer sphere (3)
           GM - mass of the Plummer sphere (in natural units)
           rs - size of the Plummer sphere
      OUTPUT:
           deltav (nstar,3)
      HISTORY:
```

```
2015-04-30 - Written based on Erkal's expressions - Bovy (IAS)
galpy.df.impulse_deltav_plummer_curvedstream
galpy.df.impulse_deltav_plummer_curvedstream(v, x, b, w, x0, v0, GM, rs)
     NAME:
           impulse deltav plummer curvedstream
     PURPOSE:
           calculate the delta velocity to due an encounter with a Plummer sphere in the impulse approximation;
           allows for arbitrary velocity vectors, and arbitrary position along the stream
     INPUT:
           v - velocity of the stream (nstar,3)
           x - position along the stream (nstar,3)
           b - impact parameter
           w - velocity of the Plummer sphere (3)
           x0 - point of closest approach
           v0 - velocity of point of closest approach
           GM - mass of the Plummer sphere (in natural units)
           rs - size of the Plummer sphere
     OUTPUT:
           deltav (nstar,3)
     HISTORY:
           2015-05-04 - Written based on above - SANDERS
galpy.df.impulse_deltav_hernquist
galpy.df.impulse_deltav_hernquist(v, y, b, w, GM, rs)
     NAME:
           impulse_deltav_hernquist
     PURPOSE:
           calculate the delta velocity to due an encounter with a Hernquist sphere in the impulse approximation;
           allows for arbitrary velocity vectors, but y is input as the position along the stream
     INPUT:
           v - velocity of the stream (nstar,3)
           y - position along the stream (nstar)
           b - impact parameter
           w - velocity of the Hernquist sphere (3)
```

GM - mass of the Hernquist sphere (in natural units)

rs - size of the Hernquist sphere

```
OUTPUT:
           deltav (nstar,3)
     HISTORY:
           2015-08-13 SANDERS, using Wyn Evans calculation
galpy.df.impulse_deltav_hernquist_curvedstream
galpy.df.impulse_deltav_hernquist_curvedstream (v, x, b, w, x0, v0, GM, rs)
     NAME:
           impulse_deltav_plummer_hernquist
     PURPOSE:
           calculate the delta velocity to due an encounter with a Hernquist sphere in the impulse approximation;
           allows for arbitrary velocity vectors, and arbitrary position along the stream
     INPUT:
           v - velocity of the stream (nstar,3)
           x - position along the stream (nstar,3)
           b - impact parameter
           w - velocity of the Hernquist sphere (3)
           x0 - point of closest approach
           v0 - velocity of point of closest approach
           GM - mass of the Hernquist sphere (in natural units)
           rs - size of the Hernquist sphere
     OUTPUT:
           deltav (nstar,3)
     HISTORY:
           2015-08-13 - SANDERS, using Wyn Evans calculation
galpy.df.impulse deltav general
galpy.df.impulse_deltav_general(v, y, b, w, pot)
     NAME:
           impulse_deltav_general
     PURPOSE:
           calculate the delta velocity to due an encounter with a general spherical potential in the impulse
           approximation; allows for arbitrary velocity vectors, but y is input as the position along the stream
     INPUT:
           v - velocity of the stream (nstar,3)
           y - position along the stream (nstar)
           b - impact parameter
```

```
w - velocity of the subhalo (3)
          pot - Potential object or list thereof (should be spherical)
     OUTPUT:
          deltav (nstar,3)
     HISTORY:
          2015-05-04 - SANDERS
          2015-06-15 - Tweak to use galpy' potential objects - Bovy (IAS)
galpy.df.impulse_deltav_general_curvedstream
galpy.df.impulse_deltav_general_curvedstream (v, x, b, w, x0, v0, pot)
     NAME:
          impulse_deltav_general_curvedstream
     PURPOSE:
          calculate the delta velocity to due an encounter with a general spherical potential in the impulse
          approximation; allows for arbitrary velocity vectors and arbitrary shaped streams
     INPUT:
          v - velocity of the stream (nstar,3)
          x - position along the stream (nstar,3)
          b - impact parameter
          w - velocity of the subhalo (3)
          x0 - position of closest approach (3)
          v0 - velocity of stream at closest approach (3)
          pot - Potential object or list thereof (should be spherical)
     OUTPUT:
          deltav (nstar,3)
     HISTORY:
          2015-05-04 - SANDERS
          2015-06-15 - Tweak to use galpy' potential objects - Bovy (IAS)
galpy.df.impulse_deltav_general_orbitintegration
qalpy.df.impulse_deltav_general_orbitintegration(v, x, b, w, x0, v0, pot, tmax,
                                                                   galpot, tmaxfac=10.0, nsamp=1000,
                                                                   integrate_method='symplec4_c')
     NAME:
          impulse_deltav_general_orbitintegration
     PURPOSE:
```

calculate the delta velocity to due an encounter with a general spherical potential NOT in the impulse approximation by integrating each particle in the underlying galactic potential; allows for arbitrary velocity vectors and arbitrary shaped streams.

```
velocity vectors and arbitrary shaped streams.
     INPUT:
           v - velocity of the stream (nstar,3)
           x - position along the stream (nstar,3)
           b - impact parameter
           w - velocity of the subhalo (3)
           x0 - position of closest approach (3)
           v0 - velocity of stream at closest approach (3)
           pot - Potential object or list thereof (should be spherical)
           tmax - maximum integration time
           galpot - galpy Potential object or list thereof
           nsamp(1000) - number of forward integration points
           integrate_method= ('symplec4_c') orbit integrator to use (see Orbit.integrate)
     OUTPUT:
           deltav (nstar,3)
     HISTORY:
           2015-08-17 - SANDERS
galpy.df.impulse_deltav_general_fullplummerintegration
galpy.df.impulse_deltav_general_fullplummerintegration(v, x,
                                                                                       b, w, x0,
                                                                                                      v0,
                                                                              galpot, GM, rs, tmax-
                                                                              fac=10.0, N=1000, inte-
                                                                              grate method='symplec4 c')
     NAME:
           impulse_deltav_general_fullplummerintegration
     PURPOSE:
           calculate the delta velocity to due an encounter with a moving Plummer sphere and galactic potential
           relative to just in galactic potential
     INPUT:
           v - velocity of the stream (nstar,3)
           x - position along the stream (nstar,3)
           b - impact parameter
           w - velocity of the subhalo (3)
```

x0 - position of closest approach (3)

galpot - Galaxy Potential object

v0 - velocity of stream at closest approach (3)

```
GM - mass of Plummer
rs - scale of Plummer
tmaxfac(10) - multiple of rs/fabs(w - v0) to use for time integration interval
N(1000) - number of forward integration points
integrate_method('symplec4_c') - orbit integrator to use (see Orbit.integrate)
OUTPUT:
deltav (nstar,3)
HISTORY:
2015-08-18 - SANDERS
```

2.5 Utilities (galpy.util)

2.5.1 galpy.util.config

Configuration module

galpy.util.config.set_ro

```
NAME: set_ro
PURPOSE: set the global configuration value of ro (distance scale)
INPUT: ro - scale in kpc or astropy Quantity
OUTPUT: (none)
HISTORY: 2016-01-05 - Written - Bovy (UofT)
```

galpy.util.config.set_vo

```
galpy.util.config.set_vo (vo)
    NAME: set_vo
    PURPOSE: set the global configuration value of vo (velocity scale)
    INPUT: vo - scale in km/s or astropy Quantity
    OUTPUT: (none)
    HISTORY: 2016-01-05 - Written - Bovy (UofT)
```

2.5.2 galpy.util.bovy_plot

Warning: Importing galpy.util.bovy_plot (or having it be imported by other galpy routines) with seaborn installed may change the seaborn plot style. If you don't like this, set the configuration parameter seaborn-plotting-defaults to False in the configuration file

Various plotting routines:

```
galpy.util.bovy plot.bovy dens2d
galpy.util.bovy_plot.bovy_dens2d(X, **kwargs)
      NAME:
           bovy_dens2d
      PURPOSE:
           plot a 2d density with optional contours
      INPUT:
           first argument is the density
           matplotlib.pyplot.imshow
                                       keywords (see
                                                         http://matplotlib.sourceforge.net/api/axes_api.html#
           matplotlib.axes.Axes.imshow)
           xlabel - (raw string!) x-axis label, LaTeX math mode, no $s needed
           ylabel - (raw string!) y-axis label, LaTeX math mode, no $s needed
           xrange
           yrange
           noaxes - don't plot any axes
           overplot - if True, overplot
           colorbar - if True, add colorbar
           shrink= colorbar argument: shrink the colorbar by the factor (optional)
           conditional - normalize each column separately (for probability densities, i.e., cntrmass=True)
           gcf=True does not start a new figure (does change the ranges and labels)
           Contours:
           justcontours - if True, only draw contours
           contours - if True, draw contours (10 by default)
           levels - contour-levels
           cntrmass - if True, the density is a probability and the levels are probability masses contained within
           the contour
           cntrcolors - colors for contours (single color or array)
           cntrlabel - label the contours
           cntrlw, cntrls - linewidths and linestyles for contour
           cntrlabelsize, cntrlabelcolors, cntrinline - contour arguments
           cntrSmooth - use ndimage.gaussian_filter to smooth before contouring
           onedhists - if True, make one-d histograms on the sides
```

onedhistcolor - histogram color retAxes= return all Axes instances retCont= return the contour instance

```
OUTPUT:
          plot to output device, Axes instances depending on input
     HISTORY:
          2010-03-09 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_end_print
galpy.util.bovy_plot.bovy_end_print (filename, **kwargs)
     NAME:
          bovy_end_print
     PURPOSE:
          saves the current figure(s) to filename
     INPUT:
          filename - filename for plot (with extension)
     OPTIONAL INPUTS:
          format - file-format
     OUTPUT:
          (none)
     HISTORY:
          2009-12-23 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_hist
galpy.util.bovy_plot.bovy_hist(x, xlabel=None, ylabel=None, overplot=False, **kwargs)
     NAME:
          bovy_hist
     PURPOSE:
          wrapper around matplotlib's hist function
     INPUT:
          x - array to histogram
          xlabel - (raw string!) x-axis label, LaTeX math mode, no $s needed
          ylabel - (raw string!) y-axis label, LaTeX math mode, no $s needed
          yrange - set the y-axis range
          +all pyplot.hist keywords
     OUTPUT: (from the matplotlib docs: http://matplotlib.sourceforge.net/api/pyplot_api.html#matplotlib.pyplot.
          The return value is a tuple (n, bins, patches) or ([n0, n1, ...], bins, [patches0, patches1, ...]) if the input
          contains multiple data
     HISTORY:
```

```
2009-12-23 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_plot
galpy.util.bovy_plot.bovy_plot(*args, **kwargs)
     NAME:
           bovy_plot
     PURPOSE:
           wrapper around matplotlib's plot function
     INPUT:
           see http://matplotlib.sourceforge.net/api/pyplot_api.html#matplotlib.pyplot.plot
           xlabel - (raw string!) x-axis label, LaTeX math mode, no $s needed
           ylabel - (raw string!) y-axis label, LaTeX math mode, no $s needed
           xrange
           yrange
           scatter= if True, use pyplot.scatter and its options etc.
           colorbar= if True, and scatter==True, add colorbar
           crange - range for colorbar of scatter==True
           clabel= label for colorbar
           overplot=True does not start a new figure and does not change the ranges and labels
           gcf=True does not start a new figure (does change the ranges and labels)
           onedhists - if True, make one-d histograms on the sides
           onedhistcolor, onedhistfc, onedhistec
           onedhistxnormed, onedhistynormed - normed keyword for one-d histograms
           onedhistxweights, onedhistyweights - weights keyword for one-d histograms
           bins= number of bins for onedhists
           semilogx=, semilogy=, loglog= if True, plot logs
     OUTPUT:
           plot to output device, returns what pyplot.plot returns, or 3 Axes instances if onedhists=True
     HISTORY:
           2009-12-28 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_print
galpy.util.bovy_plot.bovy_print (fig_width=5,
                                                                 fig height=5,
                                                                                      axes\ labelsize=16,
                                             text_fontsize=11, legend_fontsize=12, xtick_labelsize=10,
                                             ytick_labelsize=10, xtick_minor_size=2, ytick_minor_size=2,
                                             xtick_major_size=4, ytick_major_size=4)
     NAME:
           bovy_print
```

```
PURPOSE:
           setup a figure for plotting
      INPUT:
           fig_width - width in inches
           fig_height - height in inches
           axes_labelsize - size of the axis-labels
           text_fontsize - font-size of the text (if any)
           legend_fontsize - font-size of the legend (if any)
           xtick_labelsize - size of the x-axis labels
           ytick_labelsize - size of the y-axis labels
           xtick_minor_size - size of the minor x-ticks
           ytick_minor_size - size of the minor y-ticks
      OUTPUT:
           (none)
      HISTORY:
           2009-12-23 - Written - Bovy (NYU)
galpy.util.bovy_plot.bovy_text
galpy.util.bovy_plot.bovy_text(*args, **kwargs)
      NAME:
           bovy_text
      PURPOSE:
           thin wrapper around matplotlib's text and annotate
           use keywords:
                'bottom_left=True'
                'bottom_right=True'
                'top_left=True'
                'top_right=True'
                'title=True'
           to place the text in one of the corners or use it as the title
      INPUT:
           see matplotlib's text (http://matplotlib.sourceforge.net/api/pyplot_api.html#matplotlib.pyplot.text)
      OUTPUT:
           prints text on the current figure
      HISTORY:
           2010-01-26 - Written - Bovy (NYU)
```

galpy.util.bovy_plot.scatterplot

```
galpy.util.bovy_plot.scatterplot(x, y, *args, **kwargs)
      NAME:
           scatterplot
      PURPOSE:
           make a 'smart' scatterplot that is a density plot in high-density regions and a regular scatterplot for
           outliers
      INPUT:
           x, y
           xlabel - (raw string!) x-axis label, LaTeX math mode, no $s needed
           ylabel - (raw string!) y-axis label, LaTeX math mode, no $s needed
           xrange
           yrange
           bins - number of bins to use in each dimension
           weights - data-weights
           aspect - aspect ratio
           conditional - normalize each column separately (for probability densities, i.e., cntrmass=True)
           gcf=True does not start a new figure (does change the ranges and labels)
           contours - if False, don't plot contours
           justcontours - if True, only draw contours, no density
           cntrcolors - color of contours (can be array as for bovy_dens2d)
           cntrlw, cntrls - linewidths and linestyles for contour
           cntrSmooth - use ndimage.gaussian_filter to smooth before contouring
           levels - contour-levels; data points outside of the last level will be individually shown (so, e.g., if this
           list is descending, contours and data points will be overplotted)
           onedhists - if True, make one-d histograms on the sides
           onedhistx - if True, make one-d histograms on the side of the x distribution
           onedhisty - if True, make one-d histograms on the side of the y distribution
           onedhistcolor, onedhistfc, onedhistec
           onedhistxnormed, onedhistynormed - normed keyword for one-d histograms
           onedhistxweights, onedhistyweights - weights keyword for one-d histograms
           cmap= cmap for density plot
           hist= and edges= - you can supply the histogram of the data yourself, this can be useful if you want
           to censor the data, both need to be set and calculated using scipy.histogramdd with the given range
           retAxes= return all Axes instances
      OUTPUT:
```

plot to output device, Axes instance(s) or not, depending on input

HISTORY:

```
2010-04-15 - Written - Bovy (NYU)
```

galpy also contains a new matplotlib projection 'galpolar' that can be used when working with older versions of matplotlib like 'polar' to create a polar plot in which the azimuth increases clockwise (like when looking at the Milky Way from the north Galactic pole). In newer versions of matplotlib, this does not work, but the 'polar' projection now supports clockwise azimuths by doing, e.g.,

```
>>> ax= pyplot.subplot(111,projection='polar')
>>> ax.set_theta_direction(-1)
```

2.5.3 galpy.util.bovy_conversion

Utility functions that provide conversions between galpy's *natural* units and *physical* units. These can be used to translate galpy outputs in natural coordinates to physical units by multiplying with the appropriate function.

These could also be used to figure out the conversion between different units. For example, if you want to know how many $\text{GeV} \, \text{cm}^{-3}$ correspond to $1 \, M_{\odot} \, \text{pc}^{-3}$, you can calculate

```
>>> from galpy.util import bovy_conversion
>>> bovy_conversion.dens_in_gevcc(1.,1.)/bovy_conversion.dens_in_msolpc3(1.,1.)
# 37.978342941703616
```

```
or 1 M_{\odot} \,\mathrm{pc}^{-3} \approx 40 \,\mathrm{GeV} \,\mathrm{cm}^{-3}.
```

Also contains a few utility functions to deal with the ro and vo conversion parameters for galpy object or lists thereof.

Utility functions:

galpy.util.bovy_conversion.get_physical

PURPOSE:

return the velocity and length units for converting between physical and internal units as a dictionary for any galpy object, so they can easily be fed to galpy routines

INPUT:

obj - a galpy object or list of such objects (e.g., a Potential, list of Potentials, Orbit, actionAngle instance, DF instance)

include_set= (False) if True, also include roSet and voSet, flags of whether the unit is explicitly set in the object

OUTPUT:

Dictionary { 'ro':length unit in kpc,'vo':velocity unit in km/s}; note that this routine will *always* return these conversion units, even if the obj you provide does not have units turned on

HISTORY:

```
2019-08-03 - Written - Bovy (UofT)
```

galpy.util.bovy_conversion.physical_compatible

```
galpy.util.bovy_conversion.physical_compatible(obj, other_obj)
     NAME:
          physical_compatible
     PURPOSE:
          test whether the velocity and length units for converting between physical and internal units are
          compatible for two galpy objects
     INPUT:
          obj - a galpy object or list of such objects (e.g., a Potential, list of Potentials, Orbit, actionAngle
          instance, DF instance)
          other_obj - another galpy object or list of such objects (e.g., a Potential, list of Potentials, Orbit,
          actionAngle instance, DF instance)
     OUTPUT:
          True if the units are compatible, False if not (compatible means that the units are the same when they
          are set for both objects)
     HISTORY:
          2020-04-22 - Written - Bovy (UofT)
Conversion functions:
galpy.util.bovy conversion.dens in criticaldens
galpy.util.bovy_conversion.dens_in_criticaldens(vo, ro, H=70.0)
     NAME:
          dens_in_criticaldens
     PURPOSE:
          convert density to units of the critical density
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
          H= (default: 70) Hubble constant in km/s/Mpc
     OUTPUT:
          conversion from units where vo=1. at ro=1. to units of the critical density
     HISTORY:
          2014-01-28 - Written - Bovy (IAS)
galpy.util.bovy_conversion.dens_in_gevcc
galpy.util.bovy_conversion.dens_in_gevcc(vo, ro)
```

NAME:

```
dens_in_gevcc
     PURPOSE:
          convert density to GeV / cm^3
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1. to GeV/cm^3
     HISTORY:
          2014-06-16 - Written - Bovy (IAS)
galpy.util.bovy_conversion.dens_in_meanmatterdens
galpy.util.bovy_conversion.dens_in_meanmatterdens(vo, ro, H=70.0, Om=0.3)
     NAME:
          dens in meanmatterdens
     PURPOSE:
          convert density to units of the mean matter density
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
          H= (default: 70) Hubble constant in km/s/Mpc
          Om= (default: 0.3) Omega matter
     OUTPUT:
          conversion from units where vo=1. at ro=1. to units of the mean matter density
     HISTORY:
          2014-01-28 - Written - Bovy (IAS)
galpy.util.bovy conversion.dens in msolpc3
galpy.util.bovy_conversion.dens_in_msolpc3(vo, ro)
     NAME:
          dens_in_msolpc3
     PURPOSE:
          convert density to Msolar / pc^3
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
```

```
OUTPUT:
          conversion from units where vo=1. at ro=1. to Msolar/pc^3
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_2piGmsolpc2
galpy.util.bovy_conversion.force_in_2piGmsolpc2(vo, ro)
     NAME:
          force_in_2piGmsolpc2
     PURPOSE:
          convert a force or acceleration to 2piG x Msolar / pc^2
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_pcMyr2
galpy.util.bovy_conversion.force_in_pcMyr2 (vo, ro)
     NAME:
          force_in_pcMyr2
     PURPOSE:
          convert a force or acceleration to pc/Myr^2
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
```

```
galpy.util.bovy conversion.force in 10m13kms2
galpy.util.bovy_conversion.force_in_10m13kms2(vo, ro)
     NAME:
          force_in_10m13kms2
     PURPOSE:
          convert a force or acceleration to 10^(-13) km/s^2
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2014-01-22 - Written - Bovy (IAS)
galpy.util.bovy_conversion.force_in_kmsMyr
galpy.util.bovy_conversion.force_in_kmsMyr(vo, ro)
     NAME:
          force_in_kmsMyr
     PURPOSE:
          convert a force or acceleration to km/s/Myr
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy conversion.freq in Gyr
galpy.util.bovy_conversion.freq_in_Gyr(vo, ro)
     NAME:
          freq_in_Gyr
     PURPOSE:
          convert a frequency to 1/Gyr
```

INPUT:

```
vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy conversion.freq in kmskpc
galpy.util.bovy_conversion.freq_in_kmskpc(vo, ro)
     NAME:
          freq_in_kmskpc
     PURPOSE:
          convert a frequency to km/s/kpc
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy_conversion.surfdens_in_msolpc2
galpy.util.bovy_conversion.surfdens_in_msolpc2 (vo, ro)
     NAME:
          surfdens_in_msolpc2
     PURPOSE:
          convert a surface density to Msolar / pc^2
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
```

galpy.util.bovy conversion.mass in msol galpy.util.bovy_conversion.mass_in_msol(vo, ro) NAME: mass_in_msol PURPOSE: convert a mass to Msolar INPUT: vo - velocity unit in km/s ro - length unit in kpc **OUTPUT**: conversion from units where vo=1. at ro=1. HISTORY: 2013-09-01 - Written - Bovy (IAS) galpy.util.bovy_conversion.mass_in_1010msol galpy.util.bovy_conversion.mass_in_1010msol(vo, ro) NAME: mass in 1010msol PURPOSE: convert a mass to 10^10 x Msolar INPUT: vo - velocity unit in km/s ro - length unit in kpc OUTPUT: conversion from units where vo=1. at ro=1. HISTORY: 2013-09-01 - Written - Bovy (IAS) galpy.util.bovy_conversion.time_in_Gyr galpy.util.bovy_conversion.time_in_Gyr(vo, ro) NAME: time_in_Gyr PURPOSE:

INPUT:

convert a time to Gyr

```
vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2013-09-01 - Written - Bovy (IAS)
galpy.util.bovy conversion.velocity in kpcGyr
galpy.util.bovy_conversion.velocity_in_kpcGyr(vo, ro)
     NAME:
          velocity_in_kpcGyr
     PURPOSE:
          convert a velocity to kpc/Gyr
     INPUT:
          vo - velocity unit in km/s
          ro - length unit in kpc
     OUTPUT:
          conversion from units where vo=1. at ro=1.
     HISTORY:
          2014-12-19 - Written - Bovy (IAS)
```

2.5.4 galpy.util.bovy coords

Warning: galpy uses a left-handed Galactocentric coordinate frame, as is common in studies of the kinematics of the Milky Way. Care should be taken when using the coordinate transformation routines below for coordinates in a right-handed frame, the routines do not always apply and are only tested for the standard galpy left-handed frame.

Various coordinate transformation routines with fairly self-explanatory names:

```
d - distance [kpc, as/mas for plx]
           e_d - distance uncertainty [kpc, [as/mas] for plx]
           e_vr - low velocity uncertainty [km/s]
           pmll - proper motion in l (*cos(b)) [ [as/mas]/yr ]
           pmbb - proper motion in b [ [as/mas]/yr ]
           cov_pmllbb - uncertainty covariance for proper motion [pmll is pmll x cos(b)]
           1 - Galactic longitude
           b - Galactic lattitude
     KEYWORDS:
           plx - if True, d is a parallax, and e_d is a parallax uncertainty
           degree - if True, 1 and b are given in degree
     OUTPUT:
           cov(vx,vy,vz) [3,3] or [:,3,3]
     HISTORY:
           2010-04-12 - Written - Bovy (NYU)
galpy.util.bovy coords.cov pmrapmdec to pmllpmbb
galpy.util.bovy_coords.cov_pmrapmdec_to_pmllpmbb(cov_pmradec, ra, dec, degree=False,
                                                                      epoch=2000.0)
     NAME:
           cov_pmrapmdec_to_pmllpmbb
     PURPOSE:
           propagate the proper motions errors through the rotation from (ra,dec) to (l,b)
     INPUT:
           covar_pmradec - uncertainty covariance matrix of the proper motion in ra (multplied with cos(dec))
           and dec [2,2] or [:,2,2]
           ra - right ascension
           dec - declination
           degree - if True, ra and dec are given in degrees (default=False)
           epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported when not using astropy's
           transformations internally; when internally using astropy's coordinate transformations, epoch can be
           None for ICRS, 'JXXXX' for FK5, and 'BXXXX' for FK4)
     OUTPUT:
           covar_pmllbb [2,2] or [:,2,2] [pmll here is pmll x cos(b)]
     HISTORY:
           2010-04-12 - Written - Bovy (NYU)
```

```
galpy.util.bovy_coords.custom_to_pmrapmdec
```

```
galpy.util.bovy_coords.custom_to_pmrapmdec(pmphi1, pmphi2, phi1, phi2, T=None, de-
                                                            gree=False)
     NAME:
           custom_to_pmrapmdec
     PURPOSE:
           rotate proper motions in a custom set of sky coordinates (phi1,phi2) to ICRS (ra,dec)
     INPUT:
           pmphi1 - proper motion in custom (multplied with cos(phi2)) [mas/yr]
           pmphi2 - proper motion in phi2 [mas/yr]
           phi1 - custom longitude
           phi2 - custom latitude
           T = matrix defining the transformation in cartesian coordinates: new_rect = T dot old_rect
               where old rect = [cos(dec)cos(ra), cos(dec)sin(ra), sin(dec)] and similar for new rect
           degree= (False) if True, phi1 and phi2 are given in degrees (default=False)
     OUTPUT:
           (pmra x cos(dec), dec) for vector inputs [:,2]
     HISTORY:
           2019-03-02 - Written - Nathaniel Starkman (UofT)
galpy.util.bovy_coords.custom_to_radec
galpy.util.bovy_coords.custom_to_radec(phil, phi2, T=None, degree=False)
     NAME:
           custom_to_radec
     PURPOSE:
           rotate a custom set of sky coordinates (phi1, phi2) to (ra, dec) given the rotation matrix T for (ra, dec)
           -> (phi1, phi2)
     INPUT:
           phi1 - custom sky coord
           phi2 - custom sky coord
          T - matrix defining the transformation (ra, dec) -> (phi1, phi2)
           degree - default: False. If True, phi1 and phi2 in degrees
     OUTPUT:
           (ra, dec) for vector inputs [:, 2]
     HISTORY:
           2018-10-23 - Written - Nathaniel (UofT)
```

```
galpy.util.bovy coords.cyl to rect
galpy.util.bovy_coords.cyl_to_rect(R, phi, Z)
      NAME:
           cyl_to_rect
      PURPOSE:
           convert from cylindrical to rectangular coordinates
      INPUT:
           R, phi, Z - cylindrical coordinates
      OUTPUT:
           X,Y,Z
      HISTORY:
           2011-02-23 - Written - Bovy (NYU)
galpy.util.bovy_coords.cyl_to_rect_vec
galpy.util.bovy_coords.cyl_to_rect_vec(vr, vt, vz, phi)
     NAME:
           cyl_to_rect_vec
      PURPOSE:
           transform vectors from cylindrical to rectangular coordinate vectors
      INPUT:
           vr - radial velocity
           vt - tangential velocity
           vz - vertical velocity
           phi - azimuth
      OUTPUT:
           vx,vy,vz
      HISTORY:
           2011-02-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.dl_to_rphi_2d
\verb|galpy.util.bovy_coords.dl_to_rphi_2d| (\textit{d}, \textit{l}, \textit{degree} = \textit{False}, \textit{ro} = \textit{1.0}, \textit{phio} = \textit{0.0})|
      NAME:
           dl_to_rphi_2d
      PURPOSE:
           convert Galactic longitude and distance to Galactocentric radius and azimuth
```

INPUT:

```
d - distance
          1 - Galactic longitude [rad/deg if degree]
     KEYWORDS:
           degree= (False): 1 is in degrees rather than rad
           ro= (1) Galactocentric radius of the observer
           phio= (0) Galactocentric azimuth of the observer [rad/deg if degree]
     OUTPUT:
          (R,phi); phi in degree if degree
     HISTORY:
           2012-01-04 - Written - Bovy (IAS)
galpy.util.bovy_coords.galcencyl_to_XYZ
galpy.util.bovy_coords.galcencyl_to_XYZ(R, phi, Z, Xsun=1.0, Zsun=0.0, _extra_rot=True)
     NAME:
           galcencyl_to_XYZ
     PURPOSE:
           transform cylindrical Galactocentric coordinates to XYZ coordinates (wrt Sun)
     INPUT:
           R, phi, Z - Galactocentric cylindrical coordinates
           Xsun - cylindrical distance to the GC (can be array of same length as R)
           Zsun - Sun's height above the midplane (can be array of same length as R)
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
          X,Y,Z
     HISTORY:
           2011-02-23 - Written - Bovy (NYU)
           2017-10-24 - Allowed Xsun/Zsun to be arrays - Bovy (UofT)
galpy.util.bovy coords.galcencyl to vxvyvz
galpy.util.bovy_coords.galcencyl_to_vxvyvz(vR, vT, vZ, phi, vsun=[0.0, 1.0, 0.0], Xsun=1.0,
                                                            Zsun=0.0, _extra_rot=True)
     NAME:
          galcencyl_to_vxvyvz
     PURPOSE:
           transform cylindrical Galactocentric coordinates to XYZ (wrt Sun) coordinates for velocities
     INPUT:
```

```
vT - Galactocentric tangential velocity
           vZ - Galactocentric vertical velocity
           phi - Galactocentric azimuth
           vsun - velocity of the sun in the GC frame ndarray[3] (can be array of same length as vRg; shape
           [3,N]
           Xsun - cylindrical distance to the GC (can be array of same length as vRg)
           Zsun - Sun's height above the midplane (can be array of same length as vRg)
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
           vx,vy,vz
     HISTORY:
           2011-02-24 - Written - Bovy (NYU)
           2017-10-24 - Allowed vsun/Xsun/Zsun to be arrays - Bovy (NYU)
galpy.util.bovy coords.galcenrect to XYZ
galpy.util.bovy_coords.galcenrect_to_XYZ(X, Y, Z, Xsun=1.0, Zsun=0.0, _extra_rot=True)
     NAME:
           galcenrect_to_XYZ
     PURPOSE:
           transform rectangular Galactocentric to XYZ coordinates (wrt Sun) coordinates
     INPUT:
           X, Y, Z - Galactocentric rectangular coordinates
           Xsun - cylindrical distance to the GC (can be array of same length as X)
           Zsun - Sun's height above the midplane (can be array of same length as X)
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
           (X, Y, Z)
     HISTORY:
           2011-02-23 - Written - Bovy (NYU)
           2016-05-12 - Edited to properly take into account the Sun's vertical position; dropped Ysun keyword
           - Bovy (UofT)
           2017-10-24 - Allowed Xsun/Zsun to be arrays - Bovy (UofT)
           2018-04-18 - Tweaked to be consistent with astropy's Galactocentric frame - Bovy (UofT)
```

vR - Galactocentric radial velocity

```
galpy.util.bovy_coords.galcenrect_to_vxvyvz
```

```
galpy.util.bovy_coords.galcenrect_to_vxvyvz(vXg, vYg, vZg, vsun=[0.0, 1.0, 0.0],
                                                              Xsun=1.0, Zsun=0.0, _extra_rot=True)
     NAME:
           galcenrect_to_vxvyvz
     PURPOSE:
           transform rectangular Galactocentric coordinates to XYZ coordinates (wrt Sun) for velocities
     INPUT:
           vXg - Galactocentric x-velocity
           vYg - Galactocentric y-velocity
           vZg - Galactocentric z-velocity
           vsun - velocity of the sun in the GC frame ndarray[3] (can be array of same length as vXg; shape
           [3,N]
           Xsun - cylindrical distance to the GC (can be array of same length as vXg)
           Zsun - Sun's height above the midplane (can be array of same length as vXg)
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
          [:,3] = vx, vy, vz
     HISTORY:
           2011-02-24 - Written - Bovy (NYU)
           2016-05-12 - Edited to properly take into account the Sun's vertical position; dropped Ysun keyword
           - Bovy (UofT)
           2017-10-24 - Allowed vsun/Xsun/Zsun to be arrays - Bovy (UofT)
           2018-04-18 - Tweaked to be consistent with astropy's Galactocentric frame - Bovy (UofT)
galpy.util.bovy coords.lb to radec
galpy.util.bovy_coords.lb_to_radec(l, b, degree=False, epoch=2000.0)
     NAME:
          lb_to_radec
     PURPOSE:
           transform from Galactic coordinates to equatorial coordinates
     INPUT:
           1 - Galactic longitude
           b - Galactic lattitude
           degree - (Bool) if True, I and b are given in degree and ra and dec will be as well
```

```
None for ICRS, 'JXXXX' for FK5, and 'BXXXX' for FK4)
     OUTPUT:
          ra,dec
          For vector inputs [:,2]
     HISTORY:
          2010-04-07 - Written - Bovy (NYU)
          2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
          2016-05-13 - Added support for using astropy's coordinate transformations and for non-standard
          epochs - Bovy (UofT)
galpy.util.bovy_coords.lb_to_radec
galpy.util.bovy_coords.lbd_to_XYZ(l, b, d, degree=False)
     NAME:
          lbd_to_XYZ
     PURPOSE:
          transform from spherical Galactic coordinates to rectangular Galactic coordinates (works with vector
          inputs)
     INPUT:
          1 - Galactic longitude (rad)
          b - Galactic lattitude (rad)
          d - distance (arbitrary units)
          degree - (bool) if True, 1 and b are in degrees
     OUTPUT:
          [X,Y,Z] in whatever units d was in
          For vector inputs [:,3]
     HISTORY:
          2009-10-24- Written - Bovy (NYU)
          2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy coords.pmllpmbb to pmrapmdec
galpy.util.bovy_coords.pmllpmbb_to_pmrapmdec(pmll,
                                                                              l,
                                                                     pmbb,
                                                                                   b,
                                                                                        degree=False,
                                                             epoch=2000.0)
     NAME:
          pmllpmbb_to_pmrapmdec
     PURPOSE:
          rotate proper motions in (l,b) into proper motions in (ra,dec)
```

epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported when not using astropy's transformations internally; when internally using astropy's coordinate transformations, epoch can be

```
INPUT:
           pmll - proper motion in l (multplied with cos(b)) [mas/yr]
           pmbb - proper motion in b [mas/yr]
           1 - Galactic longitude
           b - Galactic lattitude
           degree - if True, I and b are given in degrees (default=False)
           epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported when not using astropy's
           transformations internally; when internally using astropy's coordinate transformations, epoch can be
           None for ICRS, 'JXXXX' for FK5, and 'BXXXX' for FK4)
     OUTPUT:
           (pmra x cos(dec),pmdec), for vector inputs [:,2]
     HISTORY:
           2010-04-07 - Written - Bovy (NYU)
           2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy coords.pmrapmdec to pmllpmbb
galpy.util.bovy_coords.pmrapmdec_to_pmllpmbb(pmra, pmdec, ra, dec, degree=False,
                                                               epoch=2000.0)
     NAME:
           pmrapmdec_to_pmllpmbb
     PURPOSE:
           rotate proper motions in (ra,dec) into proper motions in (l,b)
     INPUT:
           pmra - proper motion in ra (multplied with cos(dec)) [mas/yr]
           pmdec - proper motion in dec [mas/yr]
           ra - right ascension
           dec - declination
           degree - if True, ra and dec are given in degrees (default=False)
           epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported when not using astropy's
           transformations internally; when internally using astropy's coordinate transformations, epoch can be
           None for ICRS, 'JXXXX' for FK5, and 'BXXXX' for FK4)
     OUTPUT:
           (pmll \times cos(b), pmbb) for vector inputs [:,2]
     HISTORY:
           2010-04-07 - Written - Bovy (NYU)
           2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
```

galpy.util.bovy_coords.pmrapmdec_to_custom

```
galpy.util.bovy_coords.pmrapmdec_to_custom(pmra, pmdec, ra, dec, T=None, degree=False)
     NAME:
          pmrapmdec_to_custom
     PURPOSE:
          rotate proper motions in (ra,dec) to proper motions in a custom set of sky coordinates (phi1,phi2)
     INPUT:
          pmra - proper motion in ra (multplied with cos(dec)) [mas/yr]
          pmdec - proper motion in dec [mas/yr]
          ra - right ascension
          dec - declination
          T= matrix defining the transformation:
                                                      new rect= T dot old rect, where old rect =
          [cos(dec)cos(ra),cos(dec)sin(ra),sin(dec)] and similar for new_rect
          degree= (False) if True, ra and dec are given in degrees (default=False)
     OUTPUT:
          (pmphi1 x cos(phi2),pmph2) for vector inputs [:,2]
     HISTORY:
          2016-10-24 - Written - Bovy (UofT/CCA)
          2019-03-09 - uses custom_to_radec - Nathaniel Starkman (UofT)
galpy.util.bovy_coords.pupv_to_vRvz
galpy.util.bovy_coords.pupv_to_vRvz(pu, pv, u, v, delta=1.0, oblate=False)
     NAME:
          pupv_to_vRvz
     PURPOSE:
          calculate cylindrical vR and vz from momenta in prolate or oblate confocal u and v coordinates for a
          given focal length delta
     INPUT:
          pu - u momentum
          pv - v momentum
          u - u coordinate
          v - v coordinate
          delta= focus
          oblate= (False) if True, compute oblate confocal coordinates instead of prolate
     OUTPUT:
          (vR,vz)
     HISTORY:
```

```
2017-12-04 - Written - Bovy (UofT)
galpy.util.bovy_coords.radec_to_lb
galpy.util.bovy_coords.radec_to_lb(ra, dec, degree=False, epoch=2000.0)
     NAME:
          radec_to_lb
     PURPOSE:
          transform from equatorial coordinates to Galactic coordinates
     INPUT:
          ra - right ascension
          dec - declination
          degree - (Bool) if True, ra and dec are given in degree and l and b will be as well
          epoch - epoch of ra,dec (right now only 2000.0 and 1950.0 are supported when not using astropy's
          transformations internally; when internally using astropy's coordinate transformations, epoch can be
          None for ICRS, 'JXXXX' for FK5, and 'BXXXX' for FK4)
     OUTPUT:
          1,b
          For vector inputs [:,2]
     HISTORY:
          2009-11-12 - Written - Bovy (NYU)
          2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
          2016-05-13 - Added support for using astropy's coordinate transformations and for non-standard
          epochs - Bovy (UofT)
galpy.util.bovy_coords.radec_to_custom
qalpy.util.bovy_coords.radec_to_custom(ra, dec, T=None, degree=False)
     NAME:
          radec_to_custom
     PURPOSE:
          transform from equatorial coordinates to a custom set of sky coordinates
     INPUT:
          ra - right ascension
          dec - declination
          T= matrix defining the transformation:
                                                      new rect= T dot old rect, where old rect =
          [cos(dec)cos(ra),cos(dec)sin(ra),sin(dec)] and similar for new rect
          degree - (Bool) if True, ra and dec are given in degree and l and b will be as well
     OUTPUT:
```

```
custom longitude, custom latitude (with longitude -180 to 180)
           For vector inputs [:,2]
     HISTORY:
           2009-11-12 - Written - Bovy (NYU)
           2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
           2019-03-02 - adjusted angle ranges - Nathaniel (UofT)
galpy.util.bovy coords.rectgal to sphergal
galpy.util.bovy_coords.rectgal_to_sphergal(X, Y, Z, vx, vy, vz, degree=False)
     NAME:
           rectgal_to_sphergal
     PURPOSE:
           transform phase-space coordinates in rectangular Galactic coordinates to spherical Galactic coordi-
           nates (can take vector inputs)
     INPUT:
           X - component towards the Galactic Center (kpc)
           Y - component in the direction of Galactic rotation (kpc)
          Z - component towards the North Galactic Pole (kpc)
           vx - velocity towards the Galactic Center (km/s)
           vy - velocity in the direction of Galactic rotation (km/s)
           vz - velocity towards the North Galactic Pole (km/s)
           degree - (Bool) if True, return 1 and b in degrees
     OUTPUT:
           (l,b,d,vr,pmll x cos(b),pmbb) in (rad,rad,kpc,km/s,mas/yr,mas/yr)
     HISTORY:
           2009-10-25 - Written - Bovy (NYU)
galpy.util.bovy_coords.rect_to_cyl
galpy.util.bovy_coords.rect_to_cyl(X, Y, Z)
     NAME:
          rect_to_cyl
     PURPOSE:
           convert from rectangular to cylindrical coordinates
     INPUT:
           X, Y, Z - rectangular coordinates
     OUTPUT:
           R,phi,z
```

```
HISTORY:
          2010-09-24 - Written - Bovy (NYU)
          2019-06-21 - Changed such that phi in [-pi,pi] - Bovy (UofT)
galpy.util.bovy_coords.rect_to_cyl_vec
galpy.util.bovy_coords.rect_to_cyl_vec(vx, vy, vz, X, Y, Z, cyl=False)
     NAME:
          rect_to_cyl_vec
     PURPOSE:
          transform vectors from rectangular to cylindrical coordinates vectors
     INPUT:
          vx -
          vy -
          vz -
          X - X
          Y - Y
          Z - Z
          cyl - if True, X,Y,Z are already cylindrical
     OUTPUT:
          vR,vT,vz
     HISTORY:
          2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy_coords.rphi_to_dl_2d
galpy.util.bovy_coords.rphi_to_dl_2d(R, phi, degree=False, ro=1.0, phio=0.0)
     NAME:
          rphi_to_dl_2d
     PURPOSE:
          convert Galactocentric radius and azimuth to distance and Galactic longitude
     INPUT:
          R - Galactocentric radius
          phi - Galactocentric azimuth [rad/deg if degree]
     KEYWORDS:
          degree= (False): phi is in degrees rather than rad
          ro= (1) Galactocentric radius of the observer
          phio= (0) Galactocentric azimuth of the observer [rad/deg if degree]
```

```
OUTPUT:
          (d,l); phi in degree if degree
     HISTORY:
          2012-01-04 - Written - Bovy (IAS)
galpy.util.bovy_coords.Rz_to_coshucosv
galpy.util.bovy_coords.Rz_to_coshucosv(R, z, delta=1.0, oblate=False)
     NAME:
          Rz to coshucosy
     PURPOSE:
          calculate prolate confocal cosh(u) and cos(v) coordinates from R,z, and delta
     INPUT:
          R - radius
          z - height
          delta= focus
          oblate= (False) if True, compute oblate confocal coordinates instead of prolate
     OUTPUT:
          (\cosh(u), \cos(v))
     HISTORY:
          2012-11-27 - Written - Bovy (IAS)
          2017-10-11 - Added oblate coordinates - Bovy (UofT)
galpy.util.bovy_coords.Rz_to_uv
galpy.util.bovy_coords.Rz_to_uv (R, z, delta=1.0, oblate=False)
     NAME:
          Rz_to_uv
     PURPOSE:
          calculate prolate or oblate confocal u and v coordinates from R,z, and delta
     INPUT:
          R - radius
          z - height
          delta= focus
          oblate= (False) if True, compute oblate confocal coordinates instead of prolate
     OUTPUT:
          (u,v)
     HISTORY:
```

```
2012-11-27 - Written - Bovy (IAS)
           2017-10-11 - Added oblate coordinates - Bovy (UofT)
galpy.util.bovy coords.sphergal to rectgal
galpy.util.bovy_coords.sphergal_to_rectgal(l, b, d, vr, pmll, pmbb, degree=False)
     NAME:
          sphergal_to_rectgal
     PURPOSE:
           transform phase-space coordinates in spherical Galactic coordinates to rectangular Galactic coordi-
           nates (can take vector inputs)
     INPUT:
          1 - Galactic longitude (rad)
           b - Galactic lattitude (rad)
           d - distance (kpc)
           vr - line-of-sight velocity (km/s)
           pmll - proper motion in the Galactic longitude direction (mu_l*cos(b) ) (mas/yr)
           pmbb - proper motion in the Galactic lattitude (mas/yr)
           degree - (bool) if True, I and b are in degrees
     OUTPUT:
           (X,Y,Z,vx,vy,vz) in (kpc,kpc,kpc,km/s,km/s,km/s)
     HISTORY:
           2009-10-25 - Written - Bovy (NYU)
galpy.util.bovy_coords.uv_to_Rz
galpy.util.bovy_coords.uv_to_Rz (u, v, delta=1.0, oblate=False)
     NAME:
          uv_to_Rz
     PURPOSE:
           calculate R and z from prolate confocal u and v coordinates
     INPUT:
          u - confocal u
          v - confocal v
           delta= focus
           oblate= (False) if True, compute oblate confocal coordinates instead of prolate
     OUTPUT:
           (R,z)
     HISTORY:
```

```
2012-11-27 - Written - Bovy (IAS)
           2017-10-11 - Added oblate coordinates - Bovy (UofT)
galpy.util.bovy coords.vrpmllpmbb to vxvyvz
galpy.util.bovy_coords.vrpmllpmbb_to_vxvyvz(vr, pmll, pmbb, l, b, d, XYZ=False, de-
                                                              gree=False)
     NAME:
           vrpmllpmbb_to_vxvyvz
     PURPOSE:
           Transform velocities in the spherical Galactic coordinate frame to the rectangular Galactic coordinate
           frame (can take vector inputs)
     INPUT:
           vr - line-of-sight velocity (km/s)
           pmll - proper motion in the Galactic longitude (mu_l * cos(b))(mas/yr)
           pmbb - proper motion in the Galactic lattitude (mas/yr)
           1 - Galactic longitude
           b - Galactic lattitude
           d - distance (kpc)
           XYZ - (bool) If True, then l,b,d is actually X,Y,Z (rectangular Galactic coordinates)
           degree - (bool) if True, l and b are in degrees
     OUTPUT:
           (vx,vy,vz) in (km/s,km/s,km/s)
           For vector inputs [:,3]
     HISTORY:
           2009-10-24 - Written - Bovy (NYU)
           2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy coords.vRvz to pupv
galpy.util.bovy_coords.vRvz_to_pupv(vR, vz, R, z, delta=1.0, oblate=False, uv=False)
     NAME:
           vRvz_to_pupv
     PURPOSE:
           calculate momenta in prolate or oblate confocal u and v coordinates from cylindrical velocities vR,vz
           for a given focal length delta
     INPUT:
           vR - radial velocity in cylindrical coordinates
           vz - vertical velocity in cylindrical coordinates
           R - radius
```

```
z - height
           delta= focus
           oblate= (False) if True, compute oblate confocal coordinates instead of prolate
           uv= (False) if True, the given R,z are actually u,v
     OUTPUT:
           (pu,pv)
     HISTORY:
           2017-11-28 - Written - Bovy (UofT)
galpy.util.bovy_coords.vxvyvz_to_galcencyl
galpy.util.bovy_coords.vxvyvz_to_galcencyl(vx, vy, vz, X, Y, Z, vsun=[0.0, 1.0, 0.0],
                                                             Xsun=1.0, Zsun=0.0, galcen=False, _ex-
                                                             tra\_rot=True)
     NAME:
           vxvyvz_to_galcencyl
     PURPOSE:
           transform velocities in XYZ coordinates (wrt Sun) to cylindrical Galactocentric coordinates for ve-
           locities
     INPUT:
           vx - U
           vy - V
           vz - W
           X - X in Galactocentric rectangular coordinates
           Y - Y in Galactocentric rectangular coordinates
           Z - Z in Galactocentric rectangular coordinates
           vsun - velocity of the sun in the GC frame ndarray[3]
           Xsun - cylindrical distance to the GC
           Zsun - Sun's height above the midplane
           galcen - if True, then X,Y,Z are in cylindrical Galactocentric coordinates rather than rectangular
           coordinates
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
           vRg, vTg, vZg
     HISTORY:
           2010-09-24 - Written - Bovy (NYU)
```

galpy.util.bovy_coords.vxvyvz_to_galcenrect

```
galpy.util.bovy_coords.vxvyvz_to_galcenrect(vx, vy, vz, vsun=[0.0, 1.0, 0.0], Xsun=1.0,
                                                              Zsun=0.0, \_extra\_rot=True)
     NAME:
           vxvyvz_to_galcenrect
     PURPOSE:
           transform velocities in XYZ coordinates (wrt Sun) to rectangular Galactocentric coordinates for ve-
           locities
     INPUT:
           vx - U
           vy - V
           vz - W
           vsun - velocity of the sun in the GC frame ndarray[3]
           Xsun - cylindrical distance to the GC
           Zsun - Sun's height above the midplane
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
           [:,3]=vXg, vYg, vZg
     HISTORY:
           2010-09-24 - Written - Bovy (NYU)
           2016-05-12 - Edited to properly take into account the Sun's vertical position; dropped Ysun keyword
           - Bovy (UofT)
           2018-04-18 - Tweaked to be consistent with astropy's Galactocentric frame - Bovy (UofT)
galpy.util.bovy coords.vxvyvz to vrpmllpmbb
galpy.util.bovy_coords.vxvyvz_to_vrpmllpmbb(vx, vy, vz, l, b, d, XYZ=False, degree=False)
     NAME:
           vxvyvz to vrpmllpmbb
     PURPOSE:
           Transform velocities in the rectangular Galactic coordinate frame to the spherical Galactic coordinate
           frame (can take vector inputs)
     INPUT:
           vx - velocity towards the Galactic Center (km/s)
           vy - velocity in the direction of Galactic rotation (km/s)
           vz - velocity towards the North Galactic Pole (km/s)
           1 - Galactic longitude
           b - Galactic lattitude
```

```
d - distance (kpc)
          XYZ - (bool) If True, then l,b,d is actually X,Y,Z (rectangular Galactic coordinates)
          degree - (bool) if True, l and b are in degrees
     OUTPUT:
          (vr,pmll x cos(b),pmbb) in (km/s,mas/yr,mas/yr); pmll = mu 1 * cos(b)
          For vector inputs [:,3]
     HISTORY:
          2009-10-24 - Written - Bovy (NYU)
          2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
galpy.util.bovy_coords.XYZ_to_galcencyl
galpy.util.bovy_coords.XYZ_to_galcencyl(X, Y, Z, Xsun=1.0, Zsun=0.0, _extra_rot=True)
     NAME:
          XYZ_to_galcencyl
     PURPOSE:
          transform XYZ coordinates (wrt Sun) to cylindrical Galactocentric coordinates
     INPUT:
          X - X
          Y - Y
          Z - Z
          Xsun - cylindrical distance to the GC
          Zsun - Sun's height above the midplane
          _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
          with astropy's definition
     OUTPUT:
          R,phi,z
     HISTORY:
          2010-09-24 - Written - Bovy (NYU)
galpy.util.bovy coords.XYZ to galcenrect
galpy.util.bovy_coords.XYZ_to_galcenrect(X, Y, Z, Xsun=1.0, Zsun=0.0, _extra_rot=True)
     NAME:
          XYZ_to_galcenrect
     PURPOSE:
          transform XYZ coordinates (wrt Sun) to rectangular Galactocentric coordinates
     INPUT:
```

```
X - X
           Y - Y
           Z - Z
           Xsun - cylindrical distance to the GC
           Zsun - Sun's height above the midplane
           _extra_rot= (True) if True, perform an extra tiny rotation to align the Galactocentric coordinate frame
           with astropy's definition
     OUTPUT:
           (Xg, Yg, Zg)
     HISTORY:
           2010-09-24 - Written - Bovy (NYU)
           2016-05-12 - Edited to properly take into account the Sun's vertical position; dropped Ysun keyword
           - Bovy (UofT)
           2018-04-18 - Tweaked to be consistent with astropy's Galactocentric frame - Bovy (UofT)
galpy.util.bovy coords.XYZ to Ibd
galpy.util.bovy_coords.XYZ_to_lbd(X, Y, Z, degree=False)
     NAME:
           XYZ to 1bd
     PURPOSE:
           transform from rectangular Galactic coordinates to spherical Galactic coordinates (works with vector
           inputs)
     INPUT:
           X - component towards the Galactic Center (in kpc; though this obviously does not matter))
           Y - component in the direction of Galactic rotation (in kpc)
           Z - component towards the North Galactic Pole (kpc)
           degree - (Bool) if True, return 1 and b in degrees
     OUTPUT:
           [l,b,d] in (rad or degree,rad or degree,kpc)
           For vector inputs [:,3]
     HISTORY:
           2009-10-24 - Written - Bovy (NYU)
           2014-06-14 - Re-written w/ numpy functions for speed and w/ decorators for beauty - Bovy (IAS)
```

2.5.5 galpy.util.bovy ars.bovy ars

```
galpy.util.bovy_ars.bovy_ars(domain, isDomainFinite, abcissae, hx, hpx, nsamples=1, hx-
                                         params=(), maxn=100)
     bovy_ars: Implementation of the Adaptive-Rejection Sampling algorithm by Gilks & Wild (1992): Adaptive
     Rejection Sampling for Gibbs Sampling, Applied Statistics, 41, 337 Based on Wild & Gilks (1993), Algorithm
     AS 287: Adaptive Rejection Sampling from Log-concave Density Functions, Applied Statistics, 42, 701
     Input:
           domain - [.,.] upper and lower limit to the domain
           isDomainFinite - [.,.] is there a lower/upper limit to the domain?
           abcissae - initial list of abcissae (must lie on either side of the peak in hx if the domain is unbounded
           hx - function that evaluates h(x) = \ln g(x)
           hpx - function that evaluates hp(x) = d h(x) / d x
           nsamples - (optional) number of desired samples (default=1)
           hxparams - (optional) a tuple of parameters for h(x) and h'(x)
           maxn - (optional) maximum number of updates to the hull (default=100)
     Output:
           list with nsamples of samples from exp(h(x))
     External dependencies:
           math scipy scipy.stats
     History: 2009-05-21 - Written - Bovy (NYU)
```

CHAPTER 3

Acknowledging galpy

If you use galpy in a publication, please cite the following paper

• galpy: A Python Library for Galactic Dynamics, Jo Bovy (2015), Astrophys. J. Supp., 216, 29 (arXiv/1412.3451).

and link to http://github.com/jobovy/galpy. Some of the code's functionality is introduced in separate papers:

- galpy.actionAngle.EccZmaxRperiRap and galpy.orbit.Orbit methods with analytic=True: Fast method for computing orbital parameters from *this section*: please cite Mackereth & Bovy (2018).
- galpy.actionAngle.actionAngleAdiabatic: please cite Binney (2010).
- galpy.actionAngle.actionAngleStaeckel: please cite Bovy & Rix (2013) and Binney (2012).
- galpy.actionAngle.actionAngleIsochroneApprox: please cite Bovy (2014).
- galpy.df.streamdf: please cite Bovy (2014).
- galpy.df.streamgapdf: please cite Sanders, Bovy, & Erkal (2016).
- galpy.potential.ttensor and galpy.potential.rtide: please cite Webb et al. (2019a).
- galpy.potential.to_amuse: please cite Webb et al. (2019b).

Please also send me a reference to the paper or send a pull request including your paper in the list of galpy papers on this page (this page is at doc/source/index.rst). Thanks!

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Papers using galpy

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